

HARDNESS RESULTS FOR TOTAL RAINBOW CONNECTION OF GRAPHS

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Abstract

A total-colored path is *total rainbow* if both its edges and internal vertices have distinct colors. The *total rainbow connection number* of a connected graph G , denoted by $trc(G)$, is the smallest number of colors that are needed in a total-coloring of G in order to make G *total rainbow connected*, that is, any two vertices of G are connected by a total rainbow path. In this paper, we study the computational complexity of total rainbow connection of graphs. We show that deciding whether a given total-coloring of a graph G makes it total rainbow connected is NP-Complete. We also prove that given a graph G , deciding whether $trc(G) = 3$ is NP-Complete.

Keywords: total rainbow connection, computational complexity.

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