

## BOUNDING THE OPEN $k$ -MONOPOLY NUMBER OF STRONG PRODUCT GRAPHS

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### Abstract

Let  $G = (V, E)$  be a simple graph without isolated vertices and minimum degree  $\delta$ , and let  $k \in \{1 - \lceil \delta/2 \rceil, \dots, \lfloor \delta/2 \rfloor\}$  be an integer. Given a set  $M \subset V$ , a vertex  $v$  of  $G$  is said to be  $k$ -controlled by  $M$  if  $\delta_M(v) \geq \frac{\delta_G(v)}{2} + k$ , where  $\delta_M(v)$  represents the number of neighbors of  $v$  in  $M$  and  $\delta_G(v)$  the degree of  $v$  in  $G$ . A set  $M$  is called an open  $k$ -monopoly if every vertex  $v$  of  $G$  is  $k$ -controlled by  $M$ . The minimum cardinality of any open  $k$ -monopoly is the open  $k$ -monopoly number of  $G$ . In this article we study the open  $k$ -monopoly number of strong product graphs. We present general lower and upper bounds for the open  $k$ -monopoly number of strong product graphs. Moreover, we study in addition the open 0-monopolies of several specific families of strong product graphs.

**Keywords:** open monopolies, strong product graphs, alliances, domination.

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