

## THE GRAPHS WHOSE PERMANENTAL POLYNOMIALS ARE SYMMETRIC

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### Abstract

The permanental polynomial  $\pi(G, x) = \sum_{i=0}^n b_i x^{n-i}$  of a graph  $G$  is symmetric if  $b_i = b_{n-i}$  for each  $i$ . In this paper, we characterize the graphs with symmetric permanental polynomials. Firstly, we introduce the rooted product  $H(K)$  of a graph  $H$  by a graph  $K$ , and provide a way to compute the permanental polynomial of the rooted product  $H(K)$ . Then we give a sufficient and necessary condition for the symmetric polynomial, and we prove that the permanental polynomial of a graph  $G$  is symmetric if and only if  $G$  is the rooted product of a graph by a path of length one.

**Keywords:** permanental polynomial, rooted product, matching.

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