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MATCHINGS EXTEND TO HAMILTONIAN CYCLES IN 5-CUBE [1]

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Abstract

Ruskey and Savage asked the following question: Does every matching in a hypercube Q_n for $n \geq 2$ extend to a Hamiltonian cycle of Q_n? Fink confirmed that every perfect matching can be extended to a Hamiltonian cycle of Q_n, thus solved Kreweras' conjecture. Also, Fink pointed out that every matching can be extended to a Hamiltonian cycle of Q_n for $n \in \{2, 3, 4\}$. In this paper, we prove that every matching in Q_5 can be extended to a Hamiltonian cycle of Q_5.

Keywords: hypercube, Hamiltonian cycle, matching.

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