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NEW FORMULAE FOR THE DECYCLING NUMBER OF GRAPHS

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Abstract

A set S of vertices of a graph G is called a decycling set if $G-S$ is acyclic. The minimum order of a decycling set is called the decycling number of G, and denoted by $\nabla(G)$. Our results include: (a) For any graph G,

$$\nabla(G) = n - \max_{T}\{\alpha(G - E(T))\},$$

where T is taken over all the spanning trees of G and $\alpha(G - E(T))$ is the independence number of the co-tree $G - E(T)$. This formula implies that computing the decycling number of a graph G is equivalent to finding a spanning tree in G such that its co-tree has the largest independence number. Applying the formula, the lower bounds for the decycling number of some (dense) graphs may be obtained. (b) For any decycling set S of a k-regular graph G,

$$|S| = \frac{1}{k-1}(\beta(G) + m(S)),$$

where $\beta(G) = |E(G)| - |V(G)| + 1$ and $m(S) = c + |E(S)| - 1$, c and $|E(S)|$ are, respectively, the number of components of $G - S$ and the number of

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edges in $G[S]$. Hence S is a ∇-set if and only if $m(S)$ is minimum, where ∇-set denotes a decycling set containing exactly $\nabla(G)$ vertices of G. This provides a new way to locate $\nabla(G)$ for k-regular graphs G. (c) 4-regular graphs G with the decycling number $\nabla(G) = \left\lceil \frac{\beta(G)}{3} \right\rceil$ are determined.

Keywords: decycling number, independence number, cycle rank, margin number.

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