NEW FORMULAE FOR THE DECYCLING NUMBER OF GRAPHS

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Abstract

A set $S$ of vertices of a graph $G$ is called a decycling set if $G - S$ is acyclic. The minimum order of a decycling set is called the decycling number of $G$, and denoted by $\nabla(G)$. Our results include: (a) For any graph $G$,

$$\nabla(G) = n - \max_T \{\alpha(G - E(T))\},$$

where $T$ is taken over all the spanning trees of $G$ and $\alpha(G - E(T))$ is the independence number of the co-tree $G - E(T)$. This formula implies that computing the decycling number of a graph $G$ is equivalent to finding a spanning tree in $G$ such that its co-tree has the largest independence number. Applying the formula, the lower bounds for the decycling number of some (dense) graphs may be obtained. (b) For any decycling set $S$ of a $k$-regular graph $G$,

$$|S| = \frac{1}{k-1}(\beta(G) + m(S)),$$

where $\beta(G) = |E(G)| - |V(G)| + 1$ and $m(S) = c + |E(S)| - 1$, $c$ and $|E(S)|$ are, respectively, the number of components of $G - S$ and the number of $1$The corresponding author.
edges in $G[S]$. Hence $S$ is a $\nabla$-set if and only if $m(S)$ is minimum, where $\nabla$-set denotes a decycling set containing exactly $\nabla(G)$ vertices of $G$. This provides a new way to locate $\nabla(G)$ for $k$-regular graphs $G$. (c) 4-regular graphs $G$ with the decycling number $\nabla(G) = \lceil \frac{\alpha(G)}{3} \rceil$ are determined.

**Keywords:** decycling number, independence number, cycle rank, margin number.

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### References


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