Discussiones Mathematicae
Graph Theory 36 (2016) 339–353
doi:10.7151/dmgt.1859

3-PATHS IN GRAPHS WITH BOUNDED AVERAGE DEGREE

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Abstract

In this paper we study the existence of unavoidable paths on three vertices in sparse graphs. A path uvw on three vertices u, v, and w is of type (i, j, k) if the degree of u (respectively v, w) is at most i (respectively j, k). We prove that every graph with minimum degree at least 2 and average degree strictly less than m contains a path of one of the types

- $(2, \infty, 2)$, $(2, 8, 3)$, $(4, 3, 5)$ if $m = \frac{15}{4}$,
- $(2, \infty, 2)$, $(2, 5, 3)$, $(3, 2, 4)$, $(3, 3, 3)$ if $m = \frac{10}{3}$,
- $(2, 2, \infty)$, $(2, 3, 4)$, $(2, 5, 2)$ if $m = 3$,
- $(2, 2, 13)$, $(2, 3, 3)$, $(2, 4, 2)$ if $m = \frac{14}{5}$,
- $(2, 2, i)$, $(2, 3, 2)$ if $m = \frac{3(i+1)}{i+2}$ for $4 \leq i \leq 7$,
- $(2, 2, 3)$ if $m = \frac{12}{5}$, and
- $(2, 2, 2)$ if $m = \frac{9}{4}$.

Moreover, no parameter of this description can be improved.

Keywords: average degree, structural property, 3-path, degree sequence.
2010 Mathematics Subject Classification: 05C10.

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Received 30 January 2015
Revised 7 July 2015
Accepted 7 July 2015