CORE INDEX OF PERFECT MATCHING POLYTOPE
FOR A 2-CONNECTED CUBIC GRAPH

XIUMEI WANG AND YIXUN LIN

School of Mathematics and Statistics
Zhengzhou University
Zhengzhou 450001, China

e-mail: wangxiumei@zzu.edu.cn.

Abstract

For a 2-connected cubic graph $G$, the perfect matching polytope $P(G)$ of $G$ contains a special point $x^c = \left(\frac{1}{3}, \frac{1}{3}, \ldots, \frac{1}{3}\right)$. The *core index* $\varphi(P(G))$ of the polytope $P(G)$ is the minimum number of vertices of $P(G)$ whose convex hull contains $x^c$. The Fulkerson’s conjecture asserts that every 2-connected cubic graph $G$ has six perfect matchings such that each edge appears in exactly two of them, namely, there are six vertices of $P(G)$ such that $x^c$ is the convex combination of them, which implies that $\varphi(P(G)) \leq 6$. It turns out that the latter assertion in turn implies the Fan-Raspaud conjecture: In every 2-connected cubic graph $G$, there are three perfect matchings $M_1$, $M_2$, and $M_3$ such that $M_1 \cap M_2 \cap M_3 = \emptyset$. In this paper we prove the Fan-Raspaud conjecture for $\varphi(P(G)) \leq 12$ with certain dimensional conditions.

Keywords: Fulkerson’s conjecture, Fan-Raspaud conjecture, cubic graph, perfect matching polytope, core index.

2010 Mathematics Subject Classification: 05C70.

References


doi:10.1016/j.jctb.2013.05.001

doi:10.1002/jgt.20036


Received 4 April 2016
Revised 31 October 2016
Accepted 31 October 2016