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CORE INDEX OF PERFECT MATCHING POLYTOPE FOR A 2-CONNECTED CUBIC GRAPH

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Abstract

For a 2-connected cubic graph G, the perfect matching polytope $P(G)$ of G contains a special point $x^c = \left(\frac{1}{3}, \frac{1}{3}, \dots, \frac{1}{3}\right)$. The *core index* $\varphi(P(G))$ of the polytope $P(G)$ is the minimum number of vertices of $P(G)$ whose convex hull contains x^c. The Fulkerson's conjecture asserts that every 2-connected cubic graph G has six perfect matchings such that each edge appears in exactly two of them, namely, there are six vertices of $P(G)$ such that x^c is the convex combination of them, which implies that $\varphi(P(G)) \leq 6$. It turns out that the latter assertion in turn implies the Fan-Raspaud conjecture: In every 2-connected cubic graph G, there are three perfect matchings M_1, M_2, and M_3 such that $M_1 \cap M_2 \cap M_3 = \emptyset$. In this paper we prove the Fan-Raspaud conjecture for $\varphi(P(G)) \leq 12$ with certain dimensional conditions.

Keywords: Fulkerson's conjecture, Fan-Raspaud conjecture, cubic graph, perfect matching polytope, core index.

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