CORES, JOINS AND THE FANO-FLOW CONJECTURES

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Abstract

The Fan-Raspaud Conjecture states that every bridgeless cubic graph has three 1-factors with empty intersection. A weaker one than this conjecture is that every bridgeless cubic graph has two 1-factors and one join with empty intersection. Both of these two conjectures can be related to conjectures on Fano-flows. In this paper, we show that these two conjectures are equivalent to some statements on cores and weak cores of a bridgeless cubic graph. In particular, we prove that the Fan-Raspaud Conjecture is equivalent to a conjecture proposed in [E. Steffen, 1-factor and cycle covers of cubic graphs, J. Graph Theory 78 (2015) 195–206]. Furthermore, we disprove a conjecture proposed in [G. Mazzuoccolo, New conjectures on perfect matchings in cubic graphs, Electron. Notes Discrete Math. 40 (2013) 235–238] and we propose a new version of it under a stronger connectivity assumption. The weak oddness of a cubic graph $G$ is the minimum number of odd components (i.e., with an odd number of vertices) in the complement of a join of $G$. We obtain an upper bound of weak oddness in terms of weak cores, and thus an upper bound of oddness in terms of cores as a by-product.

Keywords: cubic graphs, Fan-Raspaud Conjecture, cores, weak-cores.

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References


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