

SHARP UPPER BOUNDS ON THE CLAR NUMBER OF FULLERENE GRAPHS¹

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Abstract

The Clar number of a fullerene graph with n vertices is bounded above by $\lfloor n/6 \rfloor - 2$ and this bound has been improved to $\lfloor n/6 \rfloor - 3$ when n is congruent to 2 modulo 6. We can construct at least one fullerene graph attaining the upper bounds for every even number of vertices $n \geq 20$ except $n = 22$ and $n = 30$.

Keywords: fullerene, Clar number, Clar set, leapfrog transformation.

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