Discussiones Mathematicae
Graph Theory 39 (2019) 81–93
doi:10.7151/dmgt.2050

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DMGT Page

FACIAL INCIDENCE COLORINGS OF EMBEDDED MULTIGRAPHS [1]

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Abstract

Let G be a cellular embedding of a multigraph in a 2-manifold. Two distinct edges $e_1, e_2 \in E(G)$ are facially adjacent if they are consecutive on a facial walk of a face $f \in F(G)$. An incidence of the multigraph G is a pair (v, e), where $v \in V(G)$, $e \in E(G)$ and v is incident with e in G. Two distinct incidences (v_1, e_1) and (v_2, e_2) of G are facially adjacent if either $e_1 = e_2$ or e_1, e_2 are facially adjacent and either $v_1 = v_2$ or $v_1 \neq v_2$ and there is $i \in \{1, 2\}$ such that e_i is incident with both v_1, v_2. A facial incidence coloring of G assigns a color to each incidence of G in such a way that facially adjacent incidences get distinct colors. In this note we show that any embedded multigraph has a facial incidence coloring with seven colors. This bound is improved to six for several wide families of plane graphs and to four for plane triangulations.

Keywords: embedded multigraph, incidence, facial incidence coloring.

2010 Mathematics Subject Classification: 05C15, 05C10.

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[1]The research was supported by the Slovak grant VEGA 1/0368/16, by the grant APVV-15-0116, and by the Agency of the Ministry of Education, Science, Research and Sport of the Slovak Republic for the Structural Funds of EU under the project ITMS 26220120007.

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Received 12 December 2016
Revised 17 May 2017
Accepted 17 May 2017