

A LIMIT CONJECTURE ON THE NUMBER OF  
HAMILTONIAN CYCLES ON THIN TRIANGULAR  
GRID CYLINDER GRAPHS<sup>1</sup>

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**Abstract**

We continue our research in the enumeration of Hamiltonian cycles (HCs) on thin cylinder grid graphs  $C_m \times P_{n+1}$  by studying a triangular variant of the problem. There are two types of HCs, distinguished by whether they wrap around the cylinder. Using two characterizations of these HCs, we prove that, for fixed  $m$ , the number of HCs of both types satisfy some linear recurrence relations. For small  $m$ , computational results reveal that the two

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numbers are asymptotically the same. We conjecture that this is true for all  $m \geq 2$ .

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