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# THE COMPLEXITY OF SECURE DOMINATION PROBLEM IN GRAPHS

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**Abstract**

A dominating set of a graph $G$ is a subset $D \subseteq V(G)$ such that every vertex not in $D$ is adjacent to at least one vertex in $D$. A dominating set $S$ of $G$ is called a secure dominating set if each vertex $u \in V(G) \setminus S$ has one neighbor $v$ in $S$ such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating set of $G$. The secure domination problem is to determine a minimum secure dominating set of $G$. In this paper, we first show that the decision version of the secure domination problem is NP-complete for star convex bipartite graphs and doubly chordal graphs. We also prove that the secure domination problem cannot be approximated within a factor of $(1-\varepsilon)\ln|V|$ for any $\varepsilon > 0$, unless NP$\subseteq$DTIME$\big(|V|^{O(\log\log|V|)}\big)$. Finally, we show that the secure domination problem is APX-complete for bounded degree graphs.

**Keywords:** secure domination, star convex bipartite graph, doubly chordal graph, NP-complete, APX-complete.

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