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GENERALIZED RAINBOW CONNECTION OF GRAPHS AND THEIR COMPLEMENTS

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Abstract

Let G be an edge-colored connected graph. A path P in G is called ℓ-rainbow if each subpath of length at most $\ell+1$ is rainbow. The graph G is called (k,ℓ)-rainbow connected if there is an edge-coloring such that every pair of distinct vertices of G is connected by k pairwise internally vertex-disjoint ℓ-rainbow paths in G. The minimum number of colors needed to make G (k,ℓ)-rainbow connected is called the (k,ℓ)-rainbow connection number of G and denoted by $rc_{k,\ell}(G)$. In this paper, we first focus on the $(1,2)$-rainbow connection number of G depending on some constraints of $\overline{G}$. Then, we characterize the graphs of order n with $(1,2)$-rainbow connection number $n-1$ or $n-2$. Using this result, we investigate the Nordhaus-Gaddum-Type problem of $(1,2)$-rainbow connection number and prove that $rc_{1,2}(G)+rc_{1,2}(\overline{G}) \le n+2$ for connected graphs G and $\overline{G}$. The equality holds if and only if G or $\overline{G}$ is isomorphic to a double star.

Keywords: ℓ-rainbow path, (k,ℓ)-rainbow connected, (k,ℓ)-rainbow connection number.

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