GENERALIZED RAINBOW CONNECTION OF GRAPHS AND THEIR COMPLEMENTS

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Abstract

Let $G$ be an edge-colored connected graph. A path $P$ in $G$ is called $\ell$-rainbow if each subpath of length at most $\ell + 1$ is rainbow. The graph $G$ is called $(k, \ell)$-rainbow connected if there is an edge-coloring such that every pair of distinct vertices of $G$ is connected by $k$ pairwise internally vertex-disjoint $\ell$-rainbow paths in $G$. The minimum number of colors needed to make $G$ $(k, \ell)$-rainbow connected is called the $(k, \ell)$-rainbow connection number of $G$ and denoted by $rc_{k,\ell}(G)$. In this paper, we first focus on the $(1, 2)$-rainbow connection number of $G$ depending on some constraints of $G$. Then, we characterize the graphs of order $n$ with $(1,2)$-rainbow connection number $n-1$ or $n-2$. Using this result, we investigate the Nordhaus-Gaddum-Type problem of $(1,2)$-rainbow connection number and prove that $rc_{1,2}(G) + rc_{1,2}(\overline{G}) \leq n + 2$ for connected graphs $G$ and $\overline{G}$. The equality holds if and only if $G$ or $\overline{G}$ is isomorphic to a double star.

Keywords: $\ell$-rainbow path, $(k, \ell)$-rainbow connected, $(k, \ell)$-rainbow connection number.

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References


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