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ON INCIDENCE COLORING OF COMPLETE MULTIPARTITE AND SEMICUBIC BIPARTITE GRAPHS [1]

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Abstract

In the paper, we show that the incidence chromatic number χ_i of a complete k-partite graph is at most $\Delta+2$ (i.e., proving the *incidence coloring conjecture* for these graphs) and it is equal to $\Delta+1$ if and only if the smallest part has only one vertex (i.e., $\Delta = n-1$). Formally, for a complete k-partite graph $G = K_{r_1,r_2,\ldots,r_k}$ with the size of the smallest part equal to $r_1 \geq 1$ we have

$$\chi_i(G) = \begin{cases} \Delta(G)+1 & \text{if } r_1 = 1, \\ \Delta(G)+2 & \text{if } r_1 > 1. \end{cases}$$

In the paper we prove that the incidence 4-coloring problem for semicubic bipartite graphs is $\mathcal{NP}$-complete, thus we prove also the $\mathcal{NP}$-completeness of $L(1,1)$-labeling problem for semicubic bipartite graphs. Moreover, we observe that the incidence 4-coloring problem is $\mathcal{NP}$-complete for cubic graphs, which was proved in the paper [12] (in terms of generalized dominating sets).

Keywords: incidence coloring, complete multipartite graphs, semicubic graphs, subcubic graphs, $\mathcal{NP}$-completeness, $L(1,1)$-labelling.

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