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INVERSE PROBLEM ON THE STEINER WIENER INDEX

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Abstract

The Wiener index $W(G)$ of a connected graph G, introduced by Wiener in 1947, is defined as $W(G) = \sum_{u,v\in V(G)} d_G(u,v)$, where $d_G(u,v)$ is the distance (the length a shortest path) between the vertices u and v in G. For $S \subseteq V(G)$, the *Steiner distance* $d(S)$ of the vertices of S, introduced by Chartrand *et al.* in 1989, is the minimum size of a connected subgraph of G whose vertex set contains S. The *k-th Steiner Wiener index* $SW_k(G)$ of G is defined as $SW_k(G) = \sum_{\substack{S\subseteq V(G)\\ |S|=k}} d(S)$. We investigate the following problem: Fixed a positive integer k, for what kind of positive integer w does there exist a connected graph G (or a tree T) of order $n \geq k$ such that $SW_k(G) = w$ (or $SW_k(T) = w$)? In this paper, we give some solutions to this problem.

Keywords: distance, Steiner distance, Wiener index, Steiner Wiener index.

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