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BOUNDS ON THE LOCATING ROMAN DOMINATION NUMBER IN TREES

Nader Jafari Rad and Hadi Rahbani

Department of Mathematics
Shahrood University of Technology
Shahrood, Iran

e-mail: n.jafarirad@gmail.com

Abstract

A Roman dominating function (or just RDF) on a graph $G = (V, E)$ is a function $f : V \longrightarrow \{0, 1, 2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$. The weight of an RDF f is the value $f(V(G)) = \sum_{u \in V(G)} f(u)$. An RDF f can be represented as $f = (V_0, V_1, V_2)$, where $V_i = \{v \in V : f(v) = i\}$ for $i = 0, 1, 2$. An RDF $f = (V_0, V_1, V_2)$ is called a locating Roman dominating function (or just LRDF) if $N(u) \cap V_2 \neq N(v) \cap V_2$ for any pair u, v of distinct vertices of V_0. The locating Roman domination number $\gamma_R^L(G)$ is the minimum weight of an LRDF of G. In this paper, we study the locating Roman domination number in trees. We obtain lower and upper bounds for the locating Roman domination number of a tree in terms of its order and the number of leaves and support vertices, and characterize trees achieving equality for the bounds.

Keywords: Roman domination number, locating domination number, locating Roman domination number, tree.

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