

THE SATURATION NUMBER FOR THE LENGTH OF DEGREE MONOTONE PATHS

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AND

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Abstract

A degree monotone path in a graph G is a path P such that the sequence of degrees of the vertices in the order in which they appear on P is monotonic. The length (number of vertices) of the longest degree monotone path in G is denoted by $mp(G)$. This parameter, inspired by the well-known Erdős-Szekeres theorem, has been studied by the authors in two earlier papers. Here we consider a saturation problem for the parameter $mp(G)$. We call G saturated if, for every edge e added to G , $mp(G + e) > mp(G)$, and we define $h(n, k)$ to be the least possible number of edges in a saturated graph G on n vertices with $mp(G) < k$, while $mp(G + e) \geq k$ for every new edge e .

We obtain linear lower and upper bounds for $h(n, k)$, we determine exactly the values of $h(n, k)$ for $k = 3$ and 4 , and we present constructions of saturated graphs.

Keywords: paths, degrees, saturation.

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