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THE DICHROMATIC NUMBER OF INFINITE FAMILIES OF CIRCULANT TOURNAMENTS

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Abstract

The *dichromatic number* $dc(D)$ of a digraph D is defined to be the minimum number of colors such that the vertices of D can be colored in such a way that every chromatic class induces an acyclic subdigraph in D. The *cyclic* circulant tournament is denoted by $T = \overrightarrow{C}_{2n+1}(1, 2, \ldots, n)$, where $V(T) = \mathbb{Z}_{2n+1}$ and for every jump $j \in \{1, 2, \ldots, n\}$ there exist the arcs $(a, a+j)$ for every $a \in \mathbb{Z}_{2n+1}$. Consider the circulant tournament $\overrightarrow{C}_{2n+1}\langle k\rangle$ obtained from the cyclic tournament by reversing one of its jumps, that is, $\overrightarrow{C}_{2n+1}\langle k\rangle$ has the same arc set as $\overrightarrow{C}_{2n+1}(1, 2, \ldots, n)$ except for $j = k$ in which case, the arcs are $(a, a-k)$ for every $a \in \mathbb{Z}_{2n+1}$. In this paper, we prove that $dc(\overrightarrow{C}_{2n+1}\langle k\rangle) \in \{2, 3, 4\}$ for every $k \in \{1, 2, \ldots, n\}$. Moreover, we classify which circulant tournaments $\overrightarrow{C}_{2n+1}\langle k\rangle$ are vertex-critical r-dichromatic for every $k \in \{1, 2, \ldots, n\}$ and $r \in \{2, 3, 4\}$. Some previous results by Neumann-Lara are generalized.

Keywords: tournament, dichromatic number, vertex-critical r-dichromatic tournament.

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