CRITICALITY OF SWITCHING CLASSES OF REVERSIBLE 2-STRUCTURES LABELED BY AN ABELIAN GROUP

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Abstract

Let $V$ be a finite vertex set and let $(\mathbb{A}, +)$ be a finite abelian group. An $\mathbb{A}$-labeled and reversible 2-structure defined on $V$ is a function $g : (V \times V) \setminus \{(v,v) : v \in V\} \rightarrow \mathbb{A}$ such that for distinct $u, v \in V$, $g(u,v) = -g(v,u)$. The set of $\mathbb{A}$-labeled and reversible 2-structures defined on $V$ is denoted by $\mathcal{L}(V, \mathbb{A})$. Given $g \in \mathcal{L}(V, \mathbb{A})$, a subset $X$ of $V$ is a clan of $g$ if for any $x, y \in X$ and $v \in V \setminus X$, $g(x,v) = g(y,v)$. For example, $\emptyset$, $V$ and $\{v\}$ (for $v \in V$) are clans of $g$, called trivial. An element $g$ of $\mathcal{L}(V, \mathbb{A})$ is primitive if $|V| \geq 3$ and all the clans of $g$ are trivial.

The set of the functions from $V$ to $\mathbb{A}$ is denoted by $\mathcal{S}(V, \mathbb{A})$. Given $g \in \mathcal{L}(V, \mathbb{A})$, with each $s \in \mathcal{S}(V, \mathbb{A})$ is associated the switch $g^s$ of $g$ by $s$ defined as follows: given distinct $x, y \in V$, $g^s(x,y) = s(x) + g(x,y) - s(y)$. The switching class of $g$ is $\{g^s : s \in \mathcal{S}(V, \mathbb{A})\}$. Given a switching class $\mathcal{G} \subseteq \mathcal{L}(V, \mathbb{A})$ and $X \subseteq V$, $\{g|_{(X \times X) \setminus \{(x,x) : x \in X\}} : g \in \mathcal{G}\}$ is a switching class, denoted by $\mathcal{G}[X]$. 


Given a switching class $\mathcal{S} \subseteq \mathcal{L}(V,A)$, a subset $X$ of $V$ is a clan of $\mathcal{S}$ if $X$ is a clan of some $g \in \mathcal{S}$. For instance, every $X \subseteq V$ such that $\min(|X|,|V \setminus X|) \leq 1$ is a clan of $\mathcal{S}$, called trivial. A switching class $\mathcal{S} \subseteq \mathcal{L}(V,A)$ is primitive if $|V| \geq 4$ and all the clans of $\mathcal{S}$ are trivial. Given a primitive switching class $\mathcal{S} \subseteq \mathcal{L}(V,A)$, $\mathcal{S}$ is critical if for each $v \in V$, $\mathcal{S} - v$ is not primitive. First, we translate the main results on the primitivity of $\mathcal{A}$-labeled and reversible 2-structures in terms of switching classes. For instance, we prove the following. For a primitive switching class $\mathcal{S} \subseteq \mathcal{L}(V,A)$ such that $|V| \geq 8$, there exist $u, v \in V$ such that $u \neq v$ and $\mathcal{S}[V \setminus \{u, v\}]$ is primitive.

Second, we characterize the critical switching classes by using some of the critical digraphs described in [Y. Boudabous and P. Ille, Indecomposability graph and critical vertices of an indecomposable graph, Discrete Math. 309 (2009) 2839–2846].

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References


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