

LIGHT GRAPHS IN PLANAR GRAPHS OF LARGE GIRTH

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Abstract

A graph H is defined to be light in a graph family \mathcal{G} if there exist finite numbers $\varphi(H, \mathcal{G})$ and $w(H, \mathcal{G})$ such that each $G \in \mathcal{G}$ which contains H as a subgraph, also contains its isomorphic copy K with $\Delta_G(K) \leq \varphi(H, \mathcal{G})$ and $\sum_{x \in V(K)} \deg_G(x) \leq w(H, \mathcal{G})$. In this paper, we investigate light graphs in families of plane graphs of minimum degree 2 with prescribed girth and no adjacent 2-vertices, specifying several necessary conditions for their lightness and providing sharp bounds on φ and w for light $K_{1,3}$ and C_{10} .

Keywords: planar graph, girth, light graph.

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