

## UPPER BOUNDS FOR THE STRONG CHROMATIC INDEX OF HALIN GRAPHS

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### Abstract

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The strong chromatic index of a graph  $G$ , denoted by  $\chi'_s(G)$ , is the minimum number of vertex induced matchings needed to partition the edge set of  $G$ . Let  $T$  be a tree without vertices of degree 2 and have at least one vertex of degree greater than 2. We construct a Halin graph  $G$  by drawing  $T$  on the plane and then drawing a cycle  $C$  connecting all its leaves in such a way that  $C$  forms the boundary of the unbounded face. We call  $T$  the characteristic tree of  $G$ . Let  $G$  denote a Halin graph with maximum degree  $\Delta$  and characteristic tree  $T$ . We prove that  $\chi'_s(G) \leq 2\Delta + 1$  when  $\Delta \geq 4$ . In addition, we show that if  $\Delta = 4$  and  $G$  is not a wheel, then  $\chi'_s(G) \leq \chi'_s(T) + 2$ . A similar result for  $\Delta = 3$  was established by Lih and Liu [21].

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