

## DISTINGUISHING CARTESIAN PRODUCTS OF COUNTABLE GRAPHS

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### Abstract

The distinguishing number  $D(G)$  of a graph  $G$  is the minimum number of colors needed to color the vertices of  $G$  such that the coloring is preserved only by the trivial automorphism. In this paper we improve results about the distinguishing number of Cartesian products of finite and infinite graphs by removing restrictions to prime or relatively prime factors.

**Keywords:** vertex coloring, distinguishing number, automorphisms, infinite graphs, Cartesian and weak Cartesian product.

**2010 Mathematics Subject Classification:** 05C25, 05C15, 03E10.

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<sup>1</sup>Rafal Kalinowski and Monika Pilśniak were partially supported by the Polish Ministry of Science and Higher Education.

<sup>2</sup>Thomas Tucker was supported by Grant 317689 from the Simons Foundation.

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Received 5 August 2015

Revised 7 March 2016

Accepted 7 March 2016