SOME RESULTS ON 4-TRANSITIVE DIGRAPHS

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Abstract

Let $D$ be a digraph with set of vertices $V$ and set of arcs $A$. We say that $D$ is $k$-transitive if for every pair of vertices $u, v \in V$, the existence of a $uv$-path of length $k$ in $D$ implies that $(u, v) \in A$. A 2-transitive digraph is a transitive digraph in the usual sense.

A subset $N$ of $V$ is $k$-independent if for every pair of vertices $u, v \in N$, we have $d(u, v), d(v, u) \geq k$; it is $l$-absorbent if for every $u \in V \setminus N$ there exists $v \in N$ such that $d(u, v) \leq l$. A $k$-kernel of $D$ is a $k$-independent and $(k - 1)$-absorbent subset of $V$. The problem of determining whether a digraph has a $k$-kernel is known to be $\text{NP}$-complete for every $k \geq 2$.

In this work, we characterize 4-transitive digraphs having a 3-kernel and also 4-transitive digraphs having a 2-kernel. Using the latter result, a proof of the Laborde-Payan-Xuong conjecture for 4-transitive digraphs is given. This conjecture establishes that for every digraph $D$, an independent set can be found such that it intersects every longest path in $D$. Also, Seymour’s Second Neighborhood Conjecture is verified for 4-transitive digraphs and further problems are proposed.

Keywords: 4-transitive digraph, $k$-transitive digraph, 3-kernel, $k$-kernel, Laborde-Payan-Xuong Conjecture.

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