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# COMPLETELY INDEPENDENT SPANNING TREES IN (PARTIAL) $k$-TREES

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**Abstract**

Two spanning trees $T_1$ and $T_2$ of a graph $G$ are completely independent if, for any two vertices $u$ and $v$, the paths from $u$ to $v$ in $T_1$ and $T_2$ are internally disjoint. For a graph $G$, we denote the maximum number of pairwise completely independent spanning trees by $\text{cist}(G)$. In this paper, we consider $\text{cist}(G)$ when $G$ is a partial $k$-tree.

First we show that $\lceil k/2 \rceil \leq \text{cist}(G) \leq k-1$ for any $k$-tree $G$. Then we show that for any $p \in \{\lceil k/2 \rceil, \ldots, k-1\}$, there exist infinitely many $k$-trees $G$ such that $\text{cist}(G) = p$. Finally we consider algorithmic aspects for computing $\text{cist}(G)$. Using Courcelle's theorem, we show that there is a linear-time algorithm that computes $\text{cist}(G)$ for a partial $k$-tree, where $k$ is a fixed constant.

**Keywords:** completely independent spanning trees, partial $k$-trees.

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