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ON MINIMAL GEODETIC DOMINATION IN GRAPHS

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Abstract

Let G be a connected graph. For two vertices u and v in G, a u–v geodesic is any shortest path joining u and v. The closed geodetic interval $I_G[u,v]$ consists of all vertices of G lying on any u–v geodesic. For $S \subseteq V(G)$, S is a geodetic set in G if $\bigcup_{u,v\in S} I_G[u,v] = V(G)$.

Vertices u and v of G are neighbors if u and v are adjacent. The closed neighborhood $N_G[v]$ of vertex v consists of v and all neighbors of v. For $S \subseteq V(G)$, S is a dominating set in G if $\bigcup_{u\in S} N_G[u] = V(G)$. A geodetic dominating set in G is any geodetic set in G which is at the same time a dominating set in G. A geodetic dominating set in G is a minimal geodetic dominating set if it does not have a proper subset which is itself a geodetic dominating set in G. The maximum cardinality of a minimal geodetic dominating set in G is the upper geodetic domination number of G. This paper initiates the study of minimal geodetic dominating sets and upper geodetic domination numbers of connected graphs.

Keywords: minimal geodetic dominating set, upper geodetic domination number.

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