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A NOTE ON MINIMUM DEGREE, BIPARTITE HOLES, AND HAMILTONIAN PROPERTIES*

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Abstract

We adopt the recently introduced concept of the bipartite-hole-number due to McDiarmid and Yolov, and extend their result on Hamiltonicity to other Hamiltonian properties of graphs with a large minimum degree in terms of this concept. An (s,t)-bipartite-hole in a graph G consists of two disjoint sets of vertices S and T with $|S| = s$ and $|T| = t$ such that $E(S,T) = \emptyset$. The bipartite-hole-number $\widetilde{\alpha}(G)$ is the maximum integer r such that G contains an (s,t)-bipartite-hole for every pair of nonnegative integers s and t with $s + t = r$. Our main results are that a graph G is traceable if $\delta(G) \geq \widetilde{\alpha}(G) - 1$, and Hamilton-connected if $\delta(G) \geq \widetilde{\alpha}(G) + 1$, both improving the analogues of Dirac's Theorem for traceable and Hamilton-connected graphs.

Keywords: Hamilton-connected graph, traceable graph, degree condition, bipartite-hole-number, minimum degree.

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1. Introduction

Our motivation for the presented results is a recent generalization of a classic result of Dirac [3] on Hamiltonicity (Theorem 1 below) due to McDiarmid and Yolov [8] (Theorem 2 below). We answer the natural question whether similar extensions can be established for analogues of Dirac's Theorem for traceability and Hamilton-connectivity. Throughout this note, we use Bondy and Murty [1] for terminology and notation not defined here and only consider finite simple graphs.

For a graph G, we use $V(G)$ and $E(G)$ to denote the vertex set and the edge set of G, respectively. For $v \in V(G)$, we use $N_G(v)$ to denote the set of neighbors of v in G, and we let $d(v) = d_G(v) = |N_G(v)|$ denote the degree of v in G. Moreover, we use $N_G[v] = N_G(v) \cup \{v\}$. If the graph G is clear from the context, we will usually drop the subscript G. Let $\delta(G)$ denote the minimum degree of (the vertices of) G. An independent set of G is a set of vertices no two of which are adjacent. The cardinality of a maximum independent set in G is called the independence number of G, and denoted by $\alpha(G)$. A spanning subgraph of a graph G is a subgraph obtained by edge deletions only. If H is a spanning subgraph of G, we use $G - H$ to denote the graph with vertex set $V(G)$ and edge set $E(G) \setminus E(H)$. For two disjoint nonempty subsets S and T of $V(G)$, $E[S, T]$ denotes the set of edges with one end in S and one end in T. The disjoint union of G and H, denoted by $G + H$, is the graph with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H)$. The join of G and H, denoted by $G \vee H$, is the graph obtained from the disjoint union of G and H by adding edges joining every vertex of G to every vertex of H. The complement $\overline{G}$ of G is the graph with vertex set $V(G)$ and the property that $uv \in E(\overline{G})$ if and only if $uv \notin E(G)$.

A connected graph G is said to be k-connected if it has more than k vertices and remains connected whenever fewer than k vertices are removed. The connectivity $\kappa(G)$ of G is the maximum value of k for which G is k-connected.

If C is a cycle in G, we let $\overrightarrow{C}$ denote the cycle C with a clockwise or anticlockwise orientation. For $u, v \in V(C)$ with a fixed chosen orientation for C, we let $\overrightarrow{C}[u, v]$ denote the consecutive vertices on C from u to v in the direction specified by $\overrightarrow{C}$. The same vertices, in reverse order, are given by $\overleftarrow{C}[v, u]$. Both $\overrightarrow{C}[u, v]$ and $\overleftarrow{C}[v, u]$ are considered as paths and as vertex sets in the sequel. Note that we do not exclude the possibility that $u = v$; in this case both $\overrightarrow{C}[u, v]$ and $\overleftarrow{C}[v, u]$ reduce to one vertex.

A cycle passing through all the vertices of a graph is called a Hamilton cycle. Similarly, a path passing through all the vertices of a graph is called a Hamilton path. A graph G is said to be Hamiltonian if G has a Hamilton cycle, traceable if G has a Hamilton path, and Hamilton-connected if every two vertices of G are connected by a Hamilton path.

Already back in 1952, Dirac [3] gave the following minimum degree condition for a graph to be Hamiltonian.

Theorem 1 [3]**.** *A graph G with $n \geq 3$ vertices is Hamiltonian if $\delta(G) \geq n/2$.*

There exist many generalizations of Dirac's Theorem. In this note we refrain from providing more details. For more information on some of these generalizations, we refer the reader to [2, 4–7, 9, 10].

Motivated by Dirac's Theorem, in a paper of 2017 McDiarmid and Yolov [8] introduced a new graph parameter which they named the bipartite-hole-number.

Definition [8]**.** An (s,t)-bipartite-hole in a graph G consists of two disjoint sets of vertices S and T with $|S| = s$ and $|T| = t$ such that $E(S,T) = \emptyset$. The bipartite-hole-number $\widetilde{\alpha}(G)$ is the least integer r that can be written as $r = s+t-1$ for some positive integers s and t such that G does not contain an (s,t)-bipartite-hole.

As stated in [8], an equivalent definition of $\widetilde{\alpha}(G)$ is the maximum integer r such that G contains an (s,t)-bipartite-hole for every pair of nonnegative integers s and t with $s + t = r$.

In [8], the authors presented the following tight sufficient condition for Hamiltonicity in terms of the minimum degree and the bipartite hole number, improving Theorem 1.

Theorem 2 [8]**.** *A graph G with at least three vertices is Hamiltonian if $\delta(G) \geq \widetilde{\alpha}(G)$.*

As noted in [8], it is easy to check that a graph G with $\delta(G) \geq n/2$ has no $\left(1, \left\lfloor \frac{n}{2} \right\rfloor\right)$-bipartite-hole, so for such a graph $\delta(G) \geq n/2 \geq \widetilde{\alpha}(G)$. Motivated by this result, it is natural to consider possible counterparts of this result for other Hamiltonian properties.

In [8], the authors also presented the following result.

Theorem 3 [8]**.** *Let $r \geq 0$ be an integer and let G be a graph with at least three vertices such that $\delta(G) \geq (r+1)\widetilde{\alpha}(G) + 3r$. Then G contains $r+1$ edge-disjoint Hamilton cycles.*

The rest of this note is organized as follows. In Section 2, we will present our results, including the natural counterparts of Theorem 2 for traceable graphs and for Hamilton-connected graphs. In Section 3, we will present the proofs of our results.

2. MAIN RESULTS

We start with the following counterpart of Theorem 2 for traceable graphs.

Theorem 4. *A graph G on at least three vertices is traceable if $\delta(G) \geq \widetilde{\alpha}(G) - 1$.*

It is easy to come up with examples showing that the result is sharp. Consider, e.g., the nontraceable graph $G = K_{r,r+2}$, for which clearly $\delta(G) = r$ and $\widetilde{\alpha}(G) = r+2$. Our next result is a counterpart of Theorem 3, providing a sufficient condition for the existence of many edge-disjoint Hamilton paths.

Theorem 5. *Let $r \geq 0$ be an integer, and let G be a graph on at least three vertices with $\delta(G) \geq (r+1)\widetilde{\alpha}(G) + 3r - 1$. Then G contains $r+1$ edge-disjoint Hamilton paths, which have $2(r+1)$ distinct end vertices.*

We believe that the above result is only sharp for $r = 0$, but we were not able to relax the condition either, and leave it as an open problem. Next, we present the analogue of Theorem 4 for Hamilton-connected graphs.

Theorem 6. *A graph G on at least three vertices is Hamilton-connected if $\delta(G) \geq \widetilde{\alpha}(G) + 1$.*

This result is also sharp, in the sense that there exist non-Hamilton-connected graphs G with $\delta(G) = \widetilde{\alpha}(G) = r$ for any positive integer r. An obvious example is the graph $G = K_{r,r}$, satisfying $\delta(G) = \widetilde{\alpha}(G) = r$. An analogue of Dirac's Theorem for Hamilton-connected graphs states that a graph G of order n is Hamilton-connected if $\delta(G) \geq \frac{n+1}{2}$. It is not difficult to show that Theorem 6 improves this result. For a graph satisfying $\delta(G) \geq \frac{n+1}{2}$, it is easy to check that there is no $\left(1, \left\lfloor \frac{n-1}{2} \right\rfloor\right)$-bipartite-hole. Hence, for such a graph $\delta(G) \geq \frac{n+1}{2} \geq \widetilde{\alpha}(G) + 1$.

The following generalization of Dirac's Theorem for Hamilton-connected graphs is due to Chvátal and Erdős.

Theorem 7 [2]**.** *A graph G with at least three vertices is Hamilton-connected if $\kappa(G) \geq \alpha(G) + 1$.*

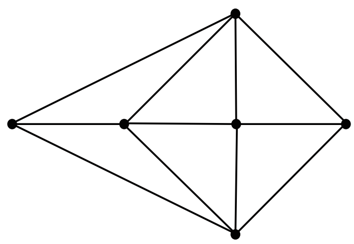

Figure 1. Graph G.

We observe that the condition of Theorem 6 is very similar to that of Theorem 7. But comparing these two theorems, neither condition implies the other. We first show an example of a graph G meeting the condition of Theorem 6 but not of Theorem 7. Let G be the graph on vertex set $V(G) = V(A) \cup V(B) \cup V(C)$, where $A = K_\ell$, $B = \overline{K_k}$, $C = K_k$, $\ell \leq k$, and all these subgraphs are mutually vertex-disjoint. Let the edge set of G be defined as $E(G) = E(A) \cup E(C) \cup \{ab \mid a \in V(A), b \in V(B)\} \cup \{bc \mid b \in V(B), c \in V(C)\}$. Obviously, we have $\kappa(G) = k = \alpha(G)$, and if we take $\ell \geq 3$ and $k \geq \ell + 3$, we get $\delta(G) = \ell - 1 + k \geq \min\{2\ell + 1, k + 1\} + 1 = \widetilde{\alpha}(G) + 1$. In the other direction, the graph G that is depicted in Figure 1 satisfies $\kappa(G) = 3$, $\alpha(G) = 2$ but $\delta(G) = \widetilde{\alpha}(G) = 3$.

In the next section, we will present the details of our proofs of the above theorems.

3. The Proofs

Our proof of Theorem 4 is an easy consequence of Theorem 2 and the following observation.

Lemma 8 (Exercise 18.1.6 on Page 474 of [1]). *Let G be a graph on at least two vertices. Then G is traceable if and only if $G \vee K_1$ is Hamiltonian.*

Proof of Theorem 4. Suppose $H = G \vee K_1$ with vertex set $V(H) = V(G) \cup \{v\}$ and edge set $E(H) = E(G) \cup \{vx \mid x \in V(G)\}$. By the definition of the bipartite-hole-number, we know that $\widetilde{\alpha}(H) = \widetilde{\alpha}(G)$. Then $\delta(H) = \delta(G) + 1 \geq \widetilde{\alpha}(G) - 1 + 1 = \widetilde{\alpha}(G) = \widetilde{\alpha}(H)$. Using Theorem 2, we obtain that H is Hamiltonian. Then by Lemma 8, G is traceable. ■

Our proof of Theorem 5 is also based on Lemma 8, and makes use of Theorem 3.

Proof of Theorem 5. Let $H = G \vee K_1$ be defined as above. Similarly as in the above proof, we get $\delta(H) \geq (r + 1)\widetilde{\alpha}(G) + 3r - 1 + 1 = (r + 1)\widetilde{\alpha}(H) + 3r$. By Theorem 3, H has $r + 1$ edge-disjoint Hamilton cycles. Using Lemma 8, we conclude that G has $r + 1$ edge-disjoint Hamilton paths, and that these paths have $2(r + 1)$ distinct end vertices. ■

At the end of this note, we present our proof of Theorem 6.

Proof of Theorem 6. If $\widetilde{\alpha}(G) = 1$, then G is complete, and so G is Hamilton-connected. Hence we may suppose that $\widetilde{\alpha}(G) \geq 2$ and G is not Hamilton-connected. Then there exist two vertices u and v such that there is no Hamilton path connecting them. By Theorem 2, we know G is Hamiltonian. Let C be a Hamilton cycle in G, and let $|V(C)| = n$. Label the vertices in $V(C)$ with $[n] = \{1, 2, \ldots, n\}$ in order according to the clockwise direction, where $u = n$ and $v = k$ for some $k \notin \{1, n - 1, n\}$. For a set $S \subseteq V(C)$, denote by S^+ the set of immediate successors x^+ on C of elements x in S, and denote by S^- the set of immediate predecessors x^-.

Let $1 \le s \le t$ be such that $\widetilde{\alpha}(G)+1 = s+t$ and G has no (s,t)-bipartite-hole. Since $\widetilde{\alpha}(G) \ge 2$, we have $1 \le s \le \frac{\widetilde{\alpha}(G)+1}{2} < \widetilde{\alpha}(G)$, and hence

$$|N(1) \cap \{1,2\})| = 1 \le s \le \delta(G) - 2 \le |N(1) \cap (2,n)| = d(1) - 2.$$

Therefore we can choose $\ell \in (1,n)$ such that $|N(1) \cap (1,\ell]| = s$. We choose the smallest ℓ with this property and note that this choice implies $1\ell \in E(G)$.

We know that 1 is not adjacent to $k+1$ since there is no Hamilton path from n to k. Hence, we have $\ell \in (1,k]$ or $\ell \in (k+1,n)$. Next, we consider these two cases.

Case 1. $\ell \in (1,k]$. We describe five situations (referring to Figure 2) in which there is a Hamilton path connecting n and k, denoted as an (n,k)-H-path in the remainder of the proof.

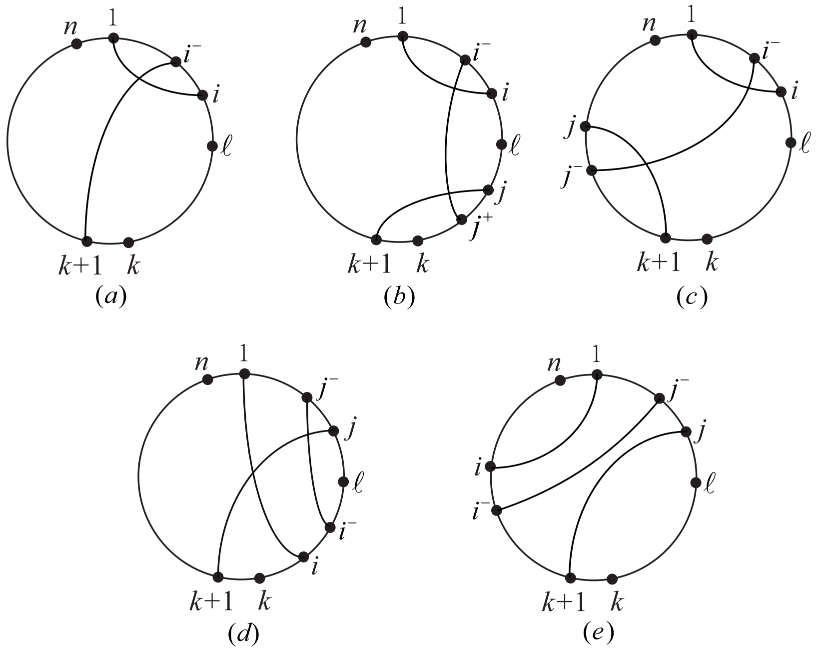

Figure 2. Situations (a)–(e).

(a) If for some $i \in (1,\ell]$ we have $i \in N(1)$ and $i^- \in N(k+1)$, then $\overleftarrow{C}[n,k+1]\overleftarrow{C}[i^-,1]\overrightarrow{C}[i,k]$ is an (n,k)-H-path.

(b) If for some $i \in (1,\ell]$ and $j \in (\ell,k]$ we have $i \in N(1)$, $j \in N(k+1)$ and $i^-j^+ \in E(G)$, then $\overleftarrow{C}[n,k+1]\overleftarrow{C}[j,i]\overrightarrow{C}[1,i^-]\overrightarrow{C}[j^+,k]$ is an (n,k)-H-path. In the particular case that $j = k$, then $\overleftarrow{C}[n,k+1]\overleftarrow{C}[i^-,1]\overrightarrow{C}[i,k]$ is an (n,k)-H-path.

(c) If for some $i \in (1, \ell]$ and $j \in (k+1, n]$ we have $i \in N(1)$, $j \in N(k+1)$ and $i^-j^- \in E(G)$, then $\overleftarrow{C}[n, j]\overrightarrow{C}[k+1, j^-]\overleftarrow{C}[i^-, 1]\overrightarrow{C}[i, k]$ is an (n, k)-H-path.

(d) If for some $i \in (\ell, k]$ and $j \in (1, \ell]$ we have $i \in N(1)$, $j \in N(k+1)$ and $i^-j^- \in E(G)$, then $\overleftarrow{C}[n, k+1]\overrightarrow{C}[j, i^-]\overleftarrow{C}[j^-, 1]\overrightarrow{C}[i, k]$ is an (n, k)-H-path.

(e) If for some $i \in (k+1, n]$ and $j \in (1, \ell]$ we have $i \in N(1)$, $j \in N(k+1)$ and $i^-j^- \in E(G)$, then $\overleftarrow{C}[n, i]\overrightarrow{C}[1, j^-]\overleftarrow{C}[i^-, k+1]\overrightarrow{C}[j, k]$ is an (n, k)-H-path.

We shall show that at least one of these situations must occur.

Suppose for a contradiction that this is not the case. Then for every $\ell \in (1, k]$

$$E[(N(1) \cap (1, \ell])^-, (N(k+1) \cap (\ell, k])^+ \cup (N(k+1) \cap (k+1, n])^-] = \emptyset, \tag{1}$$

since (a), (b) and (c) do not occur; and

$$E[(N(1) \cap (\ell, k])^- \cup (N(1) \cap (k+1, n])^-, (N(k+1) \cap (1, \ell])^-] = \emptyset, \tag{2}$$

since (d) and (e) do not occur.

Then equation (1) implies $|(N(k+1) \cap (\ell, k])^+ \cup (N(k+1) \cap (k+1, n])^-| < t$.

Since the two sets $(N(k+1) \cap (\ell, k])^+$ and $(N(k+1) \cap (k+1, n])^-$ both contain the vertex $k+1$, we have

$$|(N(k+1) \cap (\ell, k])^+ \cap (N(k+1) \cap (k+1, n])^-| = |\{k+1\}| = 1$$

when $\ell < k$, and

$$|(N(k+1) \cap (\ell, k])^+ \cap (N(k+1) \cap (k+1, n])^-| = |\emptyset| = 0$$

when $\ell = k$. Then

$$\begin{aligned}
d(k+1) &= |N(k+1) \cap (1, \ell]| + |N(k+1) \cap (\ell, k]| + |N(k+1) \cap (k+1, n]| \\
&= |N(k+1) \cap (1, \ell]| + |(N(k+1) \cap (\ell, k])^+| + |(N(k+1) \cap (k+1, n])^-| \\
&= |N(k+1) \cap (1, \ell]| + |(N(k+1) \cap (\ell, k])^+ \cup (N(k+1) \cap (k+1, n])^-| \\
&\quad + |(N(k+1) \cap (\ell, k])^+ \cap (N(k+1) \cap (k+1, n])^-| \\
&\geq \delta(G).
\end{aligned}$$

Now we have $|N(k+1) \cap (1, \ell]| > \delta(G) - t - 1 \geq \widetilde{\alpha}(G) + 1 - t - 1 = s - 1$, i.e., $|N(k+1) \cap (1, \ell]| \geq s$ when $\ell < k$, and $|N(k+1) \cap (1, \ell]| > \delta(G) - t \geq \widetilde{\alpha}(G) + 1 - t = s$, i.e., $|N(k+1) \cap (1, \ell]| \geq s + 1$ when $\ell = k$. No matter whether $\ell < k$ or $\ell = k$, equation (2) implies $|(N(1) \cap (\ell, k])^- \cup (N(1) \cap (k+1, n])^-| < t$. It is obvious that $(N(1) \cap (\ell, k])^-$ and $(N(1) \cap (k+1, n])^-$ are disjoint. Hence

$$\begin{aligned}
\delta(G) \leq d(1) &= |N(1) \cap (1, \ell]| + |N(1) \cap (\ell, k]| + |N(1) \cap (k+1, n]| \\
&= |N(1) \cap [1, \ell]| + |(N(1) \cap (\ell, k])^-| + |(N(1) \cap (k+1, n])^-| \\
&= |N(1) \cap [1, \ell]| + |(N(1) \cap (\ell, k])^- \cup (N(1) \cap (k+1, n])^-| \\
&< s + t = \widetilde{\alpha}(G) + 1 \leq \delta(G),
\end{aligned}$$

a contradiction.

Case 2. $\ell \in (k+1, n)$. Here, we describe four situations (referring to Figure 3) in which there is an (n, k)-H-path. Recall that $1\ell \in E(G)$.

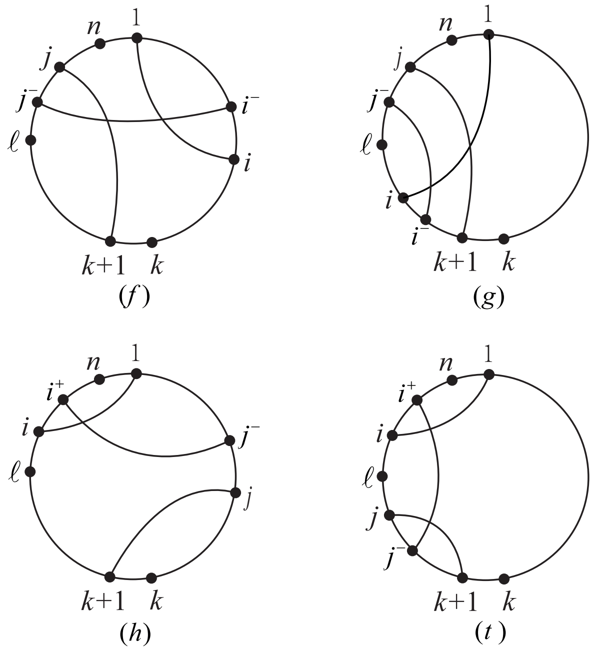

Figure 3. Situations (f)–(t).

(f) If for some $i \in (1, k]$ and $j \in (\ell, n]$ we have $i \in N(1)$, $j \in N(k+1)$ and $i^-j^- \in E(G)$, then $\overleftarrow{C}[n, j]\overrightarrow{C}[k+1, j^-]\overleftarrow{C}[i^-, 1]\overrightarrow{C}[i, k]$ is an (n, k)-H-path.

(g) If for some $i \in (k+1, \ell]$ and $j \in (\ell, n]$ we have $i \in N(1)$, $j \in N(k+1)$ and $i^-j^- \in E(G)$, then $\overleftarrow{C}[n, j]\overrightarrow{C}[k+1, i^-]\overleftarrow{C}[j^-, i]\overrightarrow{C}[1, k]$ is an (n, k)-H-path.

(h) If for some $i \in [\ell, n)$ and $j \in [1, k]$ we have $i \in N(1)$, $j \in N(k+1)$ and $i^+j^- \in E(G)$, then $\overleftarrow{C}[n, i^+]\overleftarrow{C}[j^-, 1]\overleftarrow{C}[i, k+1]\overrightarrow{C}[j, k]$ is an (n, k)-H-path.

(t) If for some $i \in [\ell, n)$ and $j \in (k+1, \ell]$ we have $i \in N(1)$, $j \in N(k+1)$ and $i^+j^- \in E(G)$, then $\overleftarrow{C}[n, i^+]\overleftarrow{C}[j^-, k+1]\overrightarrow{C}[j, i]\overrightarrow{C}[1, k]$ is an (n, k)-H-path.

We shall show that at least one of these situations must occur. Suppose for a contradiction that this is not the case. Then for every $\ell \in (1, k)$

$$E[(N(1) \cap (1, \ell])^-, (N(k+1) \cap (\ell, n])^-] = \emptyset, \tag{3}$$

since (f) and (g) do not occur; and

$$E[(N(1) \cap [\ell, n))^+, (N(k+1) \cap [1, \ell])^-] = \emptyset, \tag{4}$$

since (h) and (t) do not occur.

Then equation (3) implies $|N(k+1) \cap (\ell, n]| = |(N(k+1) \cap (\ell, n])^-| < t$. Then

$$\begin{aligned}|(N(k+1) \cap [1,\ell])^-| &= |(N(k+1) \cap [1,\ell])| \geq \delta(G) - |(N(k+1) \cap (\ell, n])| \\ &> \delta(G) - t \geq \widetilde{\alpha}(G) + 1 - t = s + t - t = s.\end{aligned}$$

Now we have $|(N(k+1) \cap [1,\ell])^-| \geq s+1$. Then equation (4) implies that $|N(1) \cap [\ell, n)| = |(N(1) \cap [\ell, n))^+| < t$. Therefore

$$\begin{aligned}\delta(G) \leq d(1) &= |N(1) \cap (1,\ell]| + |N(1) \cap [\ell, n)| + |\{n\}| - |\{\ell\}| \\ &\leq s + t - 1 = \widetilde{\alpha}(G) + 1 - 1 \leq \delta(G) - 1,\end{aligned}$$

a contradiction.

This completes the proof of Theorem 6. ■

References

[1] J.A. Bondy and U.S.R. Murty, Graph Theory, in: Grad. Texts in Math. **244** (Springer, New York, 2008).

[2] V. Chvátal and P. Erdős, *A note on Hamiltonian circuits*, Discrete Math. **2** (1972) 111–113.
https://doi.org/10.1016/0012-365X(72)90079-9

[3] G.A. Dirac, *Some theorems on abstract graphs*, Proc. Lond. Math. Soc. (3) **2** (1952) 69–81.
https://doi.org/10.1112/plms/s3-2.1.69

[4] G.-H. Fan, *New sufficient conditions for cycles in graphs*, J. Combin. Theory Ser. B **37** (1984) 221–227.
https://doi.org/10.1016/0095-8956(84)90054-6

[5] R.J. Faudree, R.J. Gould, M.S. Jacobson and R.H. Schelp, *Neighborhood unions and hamiltonian properties in graphs*, J. Combin. Theory Ser. B **47** (1989) 1–9.
https://doi.org/10.1016/0095-8956(89)90060-9

[6] R. Gould, *Advances on the Hamiltonian problem—A survey*, Graphs Combin. **19** (2003) 7–52.
https://doi.org/10.1007/s00373-002-0492-x

[7] H. Li, *Generalizations of Dirac's theorem in Hamiltonian graph theory—A survey*, Discrete Math. **313** (2013) 2034–2053.
https://doi.org/10.1016/j.disc.2012.11.025

[8] C. McDiarmid and N. Yolov, *Hamilton cycles, minimum degree, and bipartite holes*, J. Graph Theory **86** (2017) 277–285.
https://doi.org/10.1002/jgt.22114

[9] O. Ore, *Note on Hamilton circuits*, Amer. Math. Monthly **67** (1960) 55.
https://doi.org/10.2307/2308928

[10] R.H. Shi, 2-*neighborhoods and hamiltonian conditions*, J. Graph Theory **16** (1992) 267–271.
https://doi.org/10.1002/jgt.3190160310

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