Discussiones Mathematicae
Graph Theory 42 (2022) 315–316
https://doi.org/10.7151/dmgt.2253

CORRIGENDUM TO: BOUNDS ON THE NUMBER OF EDGES OF EDGE-MINIMAL, EDGE-MAXIMAL AND l-HYPERTREES [DISCUSSIONES MATHEMATICAE GRAPH THEORY 36 (2016) 259–278]

Péter G.N. Szabó

Department of Computer Science and Information Theory
Budapest University of Technology and Economics
3-9., Műegyetem rkp., H-1111 Budapest, Hungary

e-mail: szape@cs.bme.hu

Abstract

In this corrigendum, we correct the proof of Theorem 10 from our paper titled ,,Bounds on the number of edges of edge-minimal, edge-maximal and l-hypertrees".

Keywords: hypertree, chain in hypergraph, edge-minimal hypertree, edge-maximal hypertree, 2-hypertree, Steiner system.

2010 Mathematics Subject Classification: 05C65, 05D99.

1. The Corrected Proof of Theorem 10

In the original proof of Theorem 10, we stated that

$$\frac{1}{k-1}\binom{n}{k-1}-\frac{1}{(k-1)(n-k+1)}|\mathcal{E}|$$
$$\leq \frac{1}{k-1}\binom{n}{k-1}-\frac{1}{(k-1)^2(n-k+1)}\binom{n}{k-1}.$$

This can only be true if $|\mathcal{E}| \geq \frac{1}{k-1}\binom{n}{k-1}$, but in reality, the exact opposite is true, i.e., $|\mathcal{E}| \leq \frac{1}{k-1}\binom{n}{k-1}$ (see Theorem 9).

Below, we present the corrected proof of Theorem 10.

Theorem 10. *If $\mathcal{H} = (V, \mathcal{E})$ is a k-uniform 2-hypertree, then* $|\mathcal{E}| \leq \frac{1}{k-1}\binom{n}{k-1} - \frac{1}{(k-1)^3}\binom{n}{k-2}$.

Proof. We use the simple fact that $\sum_{i=1}^{n-k+1} C_i \geq \frac{1}{n-k+1}|\mathcal{E}|$, which follows from $|\mathcal{E}| = \sum_{i=1}^{n-k+1} iC_i \leq (n-k+1)\sum_{i=1}^{n-k+1} C_i$.

Comparing it to the Star-equation (Theorem 9), we get

$$\begin{aligned} |\mathcal{E}| &\leq \frac{1}{k-1}\binom{n}{k-1} - \frac{1}{(k-1)(n-k+1)}|\mathcal{E}| - \frac{1}{k-1}l \\ &\leq \frac{1}{k-1}\binom{n}{k-1} - \frac{1}{(k-1)(n-k+1)}|\mathcal{E}|, \end{aligned}$$

which implies, that

$$\begin{aligned} |\mathcal{E}| &\leq \left((k-1) + \frac{1}{(n-k+1)}\right)^{-1}\binom{n}{k-1} \\ &= \left(\frac{1}{k-1} - \frac{1}{(k-1)^2(n-k+1)+(k-1)}\right)\binom{n}{k-1} \\ &\leq \left(\frac{1}{k-1} - \frac{1}{(k-1)^2(n-k+2)}\right)\binom{n}{k-1} \\ &= \frac{1}{k-1}\binom{n}{k-1} - \frac{1}{(k-1)^3}\binom{n}{k-2}. \end{aligned}$$

■

Received 22 May 2019
Revised 19 September 2019
Accepted 23 September 2019