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HAMILTONICITIES OF DOUBLE DOMINATION CRITICAL AND STABLE CLAW-FREE GRAPHS

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Abstract

A graph G with the double domination number $\gamma_{\times 2}(G) = k$ is said to be k-$\gamma_{\times 2}$-critical if $\gamma_{\times 2}(G + uv) < k$ for any $uv \notin E(G)$. On the other hand, a graph G with $\gamma_{\times 2}(G) = k$ is said to be k-$\gamma^{+}_{\times 2}$-stable if $\gamma_{\times 2}(G + uv) = k$ for any $uv \notin E(G)$ and is said to be k-$\gamma^{-}_{\times 2}$-stable if $\gamma_{\times 2}(G - uv) = k$ for any $uv \in E(G)$. The problem of interest is to determine whether or not 2-connected k-$\gamma_{\times 2}$-critical graphs are Hamiltonian. In this paper, for $k \geq 4$, we provide a 2-connected k-$\gamma_{\times 2}$-critical graph which is non-Hamiltonian. We prove that all 2-connected k-$\gamma_{\times 2}$-critical claw-free graphs are Hamiltonian when $2 \leq k \leq 5$. We show that the condition claw-free when $k = 4$ is best possible. We further show that every 3-connected k-$\gamma_{\times 2}$-critical claw-free graph is Hamiltonian when $2 \leq k \leq 7$. We also investigate Hamiltonian properties of k-$\gamma^{+}_{\times 2}$-stable graphs and k-$\gamma^{-}_{\times 2}$-stable graphs.

Keywords: double domination, critical, stable, Hamiltonian.

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1. Introduction

All graphs in this paper are connected and simple (i.e., no loops or multiple edges). We let G denote a finite graph with vertex set $V(G)$ and edge set $E(G)$. For a vertex subset S of G, $\langle S \rangle$ denotes the subgraph of G induced by S. The *neighborhood* $N_G(x)$ of a vertex x in G is the set of vertices of G which are adjacent to x. The *degree* $\deg_G(v)$ of a vertex v in G is $|N_G(v)|$. For a vertex subset X and a vertex y of G, we let $N_X[y] = (N_G(y) \cap X) \cup \{y\}$. For a graph G, $\omega(G)$ denotes the number of components of G. A *cut set* S is a vertex subset for which $\omega(G-S) > \omega(G)$. The *connectivity* κ is the minimum cardinality of a cut set. A graph G is *l-connected* if $\kappa \geq l$. An *independent set* is a set of pairwise non-adjacent vertices. A graph G is *bipartite* if there exists a bipartition X and Y of $V(G)$ such that X and Y are independent sets. A *complete bipartite graph* $K_{m,n}$ is a bipartite graph with the partite sets X and Y such that $|X| = m$ and $|Y| = n$ containing all edges joining the vertices between X and Y. A *star* $K_{1,n}$ is a complete bipartite graph when $m = 1$, in particular if $n = 3$, a star $K_{1,3}$ is called a *claw*. For integers $s_1, s_2, s_3 \geq 1$, let $u_1, u_2, \ldots, u_{s_1+1}$; $v_1, v_2, \ldots, v_{s_2+1}$ and $w_1, w_2, \ldots, w_{s_3+1}$ be three disjoint paths of length s_1, s_2 and s_3, respectively. A *net* N_{s_1,s_2,s_3} is constructed by adding edges $u_{s_1+1}v_{s_2+1}, v_{s_2+1}w_{s_3+1}$ and $w_{s_3+1}u_{s_1+1}$. For a family of graphs $\mathcal{F}$, a graph G is said to be *$\mathcal{F}$-free* if there is no induced subgraph of G isomorphic to H for all $H \in \mathcal{F}$.

For vertex subsets X and Y of G, we say that X *doubly dominates* Y if $|N_X[y]| \geq 2$ for all $y \in Y$. We write $X \succ_{\times 2} Y$ if X doubly dominates Y. Moreover, if $Y = V(G)$, then X is a *double dominating set* of G. A smallest double dominating set of G is called a *$\gamma_{\times 2}$-set* of G. The *double domination number* of G is the cardinality of a $\gamma_{\times 2}$-set of G and is denoted by $\gamma_{\times 2}(G)$. A graph G is said to be *k double domination critical*, or *k-$\gamma_{\times 2}$-critical*, if $\gamma_{\times 2}(G) = k$ and $\gamma(G + uv) < k$ for all $uv \notin E(G)$. On the other hand, a graph G is said to be *double domination edge addition stable*, or *k-$\gamma^+_{\times 2}$-stable*, if $\gamma_{\times 2}(G) = k$ and $\gamma(G + uv) = k$ for all $uv \notin E(G)$ and a graph G is said to be *double domination edge removal stable*, or *k-$\gamma^-_{\times 2}$-stable*, if $\gamma_{\times 2}(G) = k$ and $\gamma(G - uv) = k$ for all $uv \in E(G)$. A graph which is either k-$\gamma^+_{\times 2}$-stable or k-$\gamma^-_{\times 2}$-stable is called *double domination stable*.

This paper focuses on the Hamiltonicity of double domination critical graphs and double domination stable graphs. It is worth noting that there are some results concerning Hamiltonicities of critical graph with respect to other types of domination numbers. For example, see [1,5,6,8–11,13,17,19]. For related results in k-$\gamma_{\times 2}$-critical graphs, Thacker [12] first studied these graphs. He characterized 3-$\gamma_{\times 2}$-critical graphs and 4-$\gamma_{\times 2}$-critical graphs with maximum diameter. It is easy to see that 2-$\gamma_{\times 2}$-critical graphs are complete graphs of order at least two. When $k = 4$, Wang and Kang [14] showed that G is factor-critical if G is a connected 4-$\gamma_{\times 2}$-critical $K_{1,4}$-free graph of odd order with minimum degree two. Wang and Shan [15] showed further that if the order is even and at least six then the connected 4-$\gamma_{\times 2}$-critical $K_{1,4}$-free graph has a perfect matching except one family of graphs. Moreover, if G is a 2-connected 4-$\gamma_{\times 2}$-critical claw-free of even order with minimum degree three or G is a 3-connected 4-$\gamma_{\times 2}$-critical $K_{1,4}$-free of even order with minimum degree four, then G is bi-critical. Recently, Wang *et al.* [16] established that if a graph G is a 3-connected 4-$\gamma_{\times 2}$-critical claw-free graph of odd order with minimum degree at least four, then G is 3-factor-critical except one family of graphs. All the related results have not been done when $k \geq 5$. In double domination stable graphs, we introduce a new concept in k-$\gamma^+_{\times 2}$-stable graphs and investigate their Hamiltonian property in this paper. For k-$\gamma^-_{\times 2}$-stable graphs, Chellali and Haynes [4] established fundamental properties of these graphs.

In this paper, we proceed as follows. In Section 2, we provide some results that we use in our proofs. In Section 3, for $k \geq 4$, we give a construction of a 2-connected k-$\gamma_{\times 2}$-critical graph which is non-Hamiltonian. We prove that 2-connected k-$\gamma_{\times 2}$-critical claw-free graphs are Hamiltonian when $2 \leq k \leq 5$. By the construction, we have that the condition claw-free is sharp when $k = 4$. We show further that every 3-connected k-$\gamma_{\times 2}$-critical claw-free graph is Hamiltonian when $2 \leq k \leq 7$. In Section 4, for $k \geq 2$, we give constructions of a class of k-$\gamma^+_{\times 2}$-stable non-Hamiltonian graphs and a class of k-$\gamma^-_{\times 2}$-stable non-Hamiltonian graphs. We prove that 2-connected k-$\gamma^+_{\times 2}$-stable claw-free graphs are Hamiltonian when $2 \leq k \leq 3$. We also prove that 3-connected k-$\gamma^+_{\times 2}$-stable claw-free graphs are Hamiltonian when $2 \leq k \leq 5$. For k-$\gamma^-_{\times 2}$-stable graphs, we prove that 2-connected k-$\gamma^-_{\times 2}$-stable claw-free graphs are Hamiltonian when $2 \leq k \leq 4$. We also prove that 3-connected k-$\gamma^-_{\times 2}$-stable claw-free graphs are Hamiltonian when $2 \leq k \leq 6$.

2. Preliminaries

In this section, we state a number of results from the literature that we make use of in our work. We begin with a result of Chvátal [3] which is a well known property of a Hamiltonian graph.

Proposition 1 [3]**.** *If G is a Hamiltonian graph, then $\frac{|S|}{\omega(G-S)} \geq 1$ for every cut set $S \subseteq V(G)$.*

In the following, we introduce the technique in Ryjáček [7] so called *local completion* to study Hamiltonian properties of claw-free graphs. Let G be a claw-free graph. A vertex x in G is *eligible* if $\langle N_G(x) \rangle$ is connected and non-complete. Further, let G_x be the graph such that $V(G_x) = V(G)$ and $E(G_x) = E(G) \cup \{uv :$ for a pair of non-adjacent vertices $u, v \in N_G(x)\}$. Then, we repeat this process until there is no eligible vertex in the graph. That is, we will have a finite sequence of graphs $G_0, G_1, \ldots, G_{n_0}$ such that $G = G_0$ and, for $1 \leq i \leq n_0$, we have $G_i = (G_{i-1})_y$ where y is an eligible vertex of G_{i-1}. The process finishes at G_{n_0} which contains no eligible vertex. Here G_{n_0} is the *closure* of G and is denoted by $cl(G)$. Brousek *et al.* [2] use this operation to establish the Hamiltonicities of $\{K_{1,3}, N_{s_1,s_2,s_3}\}$-free graphs. Before we state this theorem, we need to provide some classes of graphs from [2].

The Class $\mathcal{H}_1$. Let $Z_1, \ldots, Z_5$ be complete graphs of order at least three. For $1 \leq i \leq 3$, let q_i, z_i be two different vertices of Z_i. Moreover, let q'_1, q'_2, q'_3 be three different vertices of Z_4 and z'_1, z'_2, z'_3 be three different vertices of Z_5. A graph in this class is constructed from $Z_1, \ldots, Z_5$ by identifying q'_i with q_i and z'_i with z_i for $1 \leq i \leq 3$. A graph in this class is given in Figure 1(a).

The Class $\mathcal{H}_2$. Let c_1, c_2, c_3, c_1 and d_1, d_2, d_3, d_1 be two disjoint triangles. We also let T_1 and T_2 be two complete graphs of order at least three and T_3 a complete graph of order at least two. Let c'_i, d'_i be two different vertices of T_i for $1 \leq i \leq 2$ and let c'_3, r be two different vertices of T_3. A graph in this class is obtained by identifying c'_i with c_i and d'_i with d_i for $1 \leq i \leq 2$ and identifying c'_3 with c_3 and adding an edge rd_3. A graph in this class is illustrated by Figure 1(b).

The Class $\mathcal{H}_3$. Let $h_1, h_2, \ldots, h_6, h_1$ be a cycle of six vertices and K a complete graph of order at least three. Let s and s' be two different vertices of K. We define a graph G in the class $\mathcal{H}_3$ by adding edges $sh_1, sh_6, s'h_3, s'h_4$. A graph in this class is illustrated by Figure 1(c).

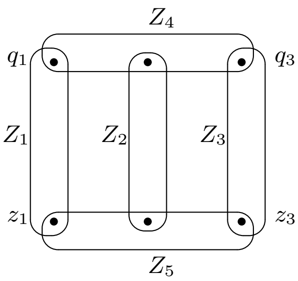

Figure 1(a). The Class $\mathcal{H}_1$.

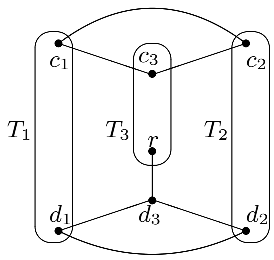

Figure 1(b). The Class $\mathcal{H}_2$.

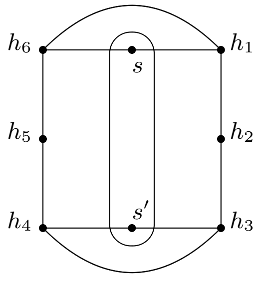

Figure 1(c). The Class $\mathcal{H}_3$.

Let $P = p_1, p_2, p_3$, $P' = p'_1, p'_2, p'_3$ and $P'' = p''_1, p''_2, p''_3$ be three paths of length two. The graph $P_{3,3,3}$ is constructed from P, P' and P'' by adding edges so that

$\{p_1, p_1', p_1''\}$ and $\{p_3, p_3', p_3''\}$ form two complete graphs of order three. Brousek *et al.* [2] proved the following.

Theorem 2 [2]**.** *Let G be a 2-connected $\{K_{1,3}, N_{1,2,2}, N_{1,1,3}\}$-free graph. Then either G is Hamiltonian, or G is isomorphic to $P_{3,3,3}$ or $cl(G) \in \mathcal{H}_1 \cup \mathcal{H}_2 \cup \mathcal{H}_3$.*

Recently, Xiong *et al.* [18] established the following theorem.

Theorem 3 [18]**.** *Let G be a 3-connected $\{K_{1,3}, N_{s_1,s_2,s_3}\}$-free graph. If $s_1 + s_2 + s_3 \leq 9$ and $s_i \geq 1$, then G is Hamiltonian.*

We conclude this section by giving some results on double domination. Thacker [12] established some observations of this parameter.

Observation 4 [12]**.** *For $k \geq 2$, let G be a k-$\gamma_{\times 2}$-critical graph. Moreover, for a pair of non-adjacent vertices u and v of G, we let D_{uv} be a $\gamma_{\times 2}$-set of $G + uv$. Then $D_{uv} \cap \{u, v\} \neq \emptyset$.*

The following proposition is a special case of a result of Thacker [12] by restricting the original result to connected graphs.

Proposition 5 [12]**.** *For any connected graph G, let u and v be a pair of non-adjacent vertices of G. Then*

$$\gamma_{\times 2}(G) - 2 \leq \gamma_{\times 2}(G + uv) \leq \gamma_{\times 2}(G).$$

The following result, from [4], gives the double domination number of a graph when any edge is removed.

Observation 6 [12]**.** *For a graph G and edge $uv \in E(G)$ such that $G - uv$ have no isolated vertex, $\gamma_{\times 2}(G) \leq \gamma_{\times 2}(G - uv)$.*

3. Double Domination Critical Graphs

In this section, we use the claw-free property to determine when 2-connected k-$\gamma_{\times 2}$-critical claw-free graphs are Hamiltonian. First of all, we give a construction of k-$\gamma_{\times 2}$-critical graphs when $k \geq 4$ which are non-Hamiltonian.

The class $\mathcal{D}(k)$. For $k \geq 4$, let $A = \{a_i b_i : 1 \leq i \leq k - 1\}$ be a set of $k - 1$ independent edges and let x be an isolated vertex. A graph G in the class $\mathcal{D}(k)$ is constructed by:

- joining x to every vertex in $V(A)$, and
- adding edges so that $b_1, b_2, \ldots, b_{k-1}$ form a clique.

A graph G in this class is illustrated by Figure 2.

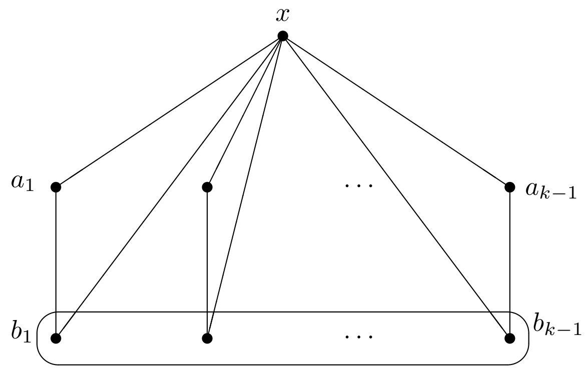

Figure 2. A graph in the class $\mathcal{D}(k)$.

Lemma 7. *For an integer $k \geq 4$, if $G \in \mathcal{D}(k)$, then G is a 2-connected k-$\gamma_{\times 2}$-critical non-Hamiltonian graph.*

Proof. We first show that $\gamma_{\times 2}(G) = k$. Obviously, $\{x, a_1, a_2, \ldots, a_{k-1}\} \succ_{\times 2} G$. By the minimality of $\gamma_{\times 2}(G)$, we have $\gamma_{\times 2}(G) \leq k$. It remains to show that $\gamma_{\times 2}(G) \geq k$. Let D be a $\gamma_{\times 2}$-set of G. To doubly dominate $\{a_1\}$, we must have $|\{x, a_1, b_1\} \cap D| \geq 2$. Similarly, to doubly dominate $\{a_2, a_3, \ldots, a_{k-1}\}$, we have $|D \cap \{a_i, b_i\}| \geq 1$ for all $2 \leq i \leq k-1$. Thus $|D| \geq k$. This implies that $\gamma_{\times 2}(G) = k$.

We next establish the criticality. Let u and v be a pair of non-adjacent vertices of G. As x is adjacent to every vertex, we must have that $x \notin \{u, v\}$. Thus $\{u, v\} \subseteq \{a_1, b_1, a_2, b_2, \ldots, a_{k-1}, b_{k-1}\}$. By the construction, at least one of u and v is not in $\{b_1, b_2, \ldots, b_{k-1}\}$. Without loss of generality let $u = a_1$ and $v \in \{a_2, b_2\}$. Clearly, $\{x, v, a_3, a_4, \ldots, a_{k-2}, b_{k-1}\} \succ_{\times 2} G + uv$. That is $\gamma_{\times 2}(G + uv) \leq k - 1 < \gamma_{\times 2}(G)$. This establishes the criticality and hence, G is a k-$\gamma_{\times 2}$-critical graph.

We finally show that G is non-Hamiltonian. Suppose to the contrary that G is Hamiltonian. Observe that $N_G(a_1) = \{x, b_1\}$. Thus, G is Hamiltonian if and only if $G - a_1$ has a Hamiltonian path P from x to b_1. Since $N_{G-a_1}(a_2) = \{x, b_2\}$, it follows that the path x, a_2, b_2 is a subgraph of P. Similarly, as $N_{G-a_1}(a_3) = \{x, b_3\}$, we must have that the path x, a_3, b_3 is a subgraph of P. Thus b_3, a_3, x, a_2, b_2 is a subgraph of P. This contradicts x is one of the two end vertices of P. Therefore, G is non-Hamiltonian. This completes the proof. ■

In the following, we recall the classes $\mathcal{H}_1, \mathcal{H}_2$ and $\mathcal{H}_3$ and the graph $P_{3,3,3}$ from the previous section. We give an observation for the lower bound of the double domination numbers of graphs in these classes.

Observation 8. *Let G be a 2-connected claw-free graph. If $cl(G) \in \mathcal{H}_1 \cup \mathcal{H}_2 \cup \mathcal{H}_3$ or G is isomorphic to $P_{3,3,3}$, then $\gamma_{\times 2}(G) \geq 6$.*

Proof. We first consider the case when $cl(G) \in \mathcal{H}_1 \cup \mathcal{H}_2 \cup \mathcal{H}_3$. Let D be a $\gamma_{\times 2}$-set of $cl(G)$. In view of Proposition 5, it suffices to show that $\gamma_{\times 2}(cl(G)) \geq 6$. Suppose first that $cl(G) \in \mathcal{H}_1$. Since $|V(Z_i)| \geq 3$, there exist vertices $s_i \in V(Z_i) - \{q_i, z_i\}$ for all $i \in \{1, 2, 3\}$. To doubly dominate $\{s_1, s_2, s_3\}$, we have that $|D \cap V(Z_i)| \geq 2$. Thus $\gamma_{\times 2}(cl(G)) = |D| \geq 6$.

We now suppose that $cl(G) \in \mathcal{H}_2$. Because $|V(T_i)| \geq 3$, there exist vertices $r_i \in V(T_i) - \{c_i, d_i\}$ for all $i \in \{1, 2\}$. To doubly dominate $\{r, r_1, r_2\}$, we have that $|D \cap V(T_i)| \geq 2$ and $|D \cap (V(T_3) \cup \{d_3\})| \geq 2$. Thus $\gamma_{\times 2}(cl(G)) = |D| \geq 6$.

We now suppose that $cl(G) \in \mathcal{H}_3$. Because $|V(K)| \geq 3$, there exists a vertex $s'' \in V(K) - \{s, s'\}$. To doubly dominate $\{s'', h_2, h_5\}$, we have that $|D \cap V(K)| \geq 2, |D \cap \{h_1, h_2, h_3\}| \geq 2$ and $|D \cap \{h_4, h_5, h_6\}| \geq 2$. Thus $\gamma_{\times 2}(cl(G)) = |D| \geq 6$.

We finally consider the case when G is isomorphic to $P_{3,3,3}$. Let D' be a $\gamma_{\times 2}$-set of G. To doubly dominate $\{p_2, p'_2, p''_2\}$, we have that $|D' \cap V(P)| \geq 2, |D' \cap V(P')| \geq 2$ and $|D' \cap V(P'')| \geq 2$. Thus $\gamma_{\times 2}(G) = |D'| \geq 6$. This completes the proof. ■

We next establish the following lemma concerning the minimum number of vertices of a double dominating set when some independent set is given.

Lemma 9. *Let G be a claw-free graph, I be an independent set and X be a set of vertices such that $X \succ_{\times 2} I$. If there exists a vertex in $X - I$ adjacent to at most one vertex in I, then $|I| + 1 \leq |X|$.*

Proof. Let w be a vertex in $X - I$ which is adjacent to at most one vertex in I. Moreover, we let $I_1 = X \cap I, I_2 = I - I_1$ and $X' = X - (I_1 \cup \{w\})$. Clearly, $|X| = |X'| + |I_1| + 1$ and $|I| = |I_1| + |I_2|$. Let H be a subgraph of G such that $V(H) = X' \cup \{w\} \cup I$ and $E(H) = \{uv \in E(G) : u \in X' \cup \{w\}$ and $v \in I\}$. Clearly, H is bipartite with the bipartition sets $X' \cup \{w\}$ and I. Since $X \succ_{\times 2} I$, every vertex in I_1 is adjacent to at least one vertex in $X' \cup \{w\}$. Moreover, every vertex in I_2 is adjacent to at least two vertices in $X' \cup \{w\}$. Thus, $\deg_H(v) \geq 1$ for all $v \in I_1$ and $\deg_H(v) \geq 2$ for all $v \in I_2$. This gives the degree sum of vertices in I as the following

(1) $$|I_1| + 2|I_2| \leq \Sigma_{v \in I} \deg_H(v).$$

Because G is claw-free, every vertex in X' is adjacent to at most two vertices in I. Therefore $\deg_H(u) \leq 2$ for all $u \in X'$. Since w is adjacent to at most one vertex in I, it follows that

(2) $$\Sigma_{u \in X' \cup \{w\}} \deg_H(u) \leq 2\,|X'| + 1.$$

Because H is bipartite, $\Sigma_{v\in I}\deg_H(v) = \Sigma_{u\in X'\cup\{w\}}\deg_H(u)$. By (1) and (2), we have $|I_1| + 2|I_2| \leq 2|X'| + 1$. Hence,

$$\begin{aligned}|I| = & \ |I_1| + |I_2| \leq \left(\frac{|I_1|}{2} + |I_2|\right) + \frac{|I_1|}{2} \leq \left(|X'| + \frac{1}{2}\right) + \frac{|I_1|}{2} \\ < & \ |X'| + |I_1| + 1 = |X|.\end{aligned}$$

This completes the proof. ■

We are now ready to establish our main theorems. We recall a net N_{s_1,s_2,s_3} from the first section.

Theorem 10. *Let G be a 2-connected k-$\gamma_{\times 2}$-critical claw-free graph. If $2 \leq k \leq 5$, then G is Hamiltonian.*

Proof. We first show that G is $\{K_{1,3}, N_{1,2,2}, N_{1,1,3}\}$-free. Suppose to the contrary that G contains $N_{1,2,2}$ or $N_{1,1,3}$ as an induced subgraph. We first consider the case when G contains $N_{1,2,2}$ as an induced subgraph. Consider $G + v_1w_1$. By Observation 4, $|D_{v_1w_1} \cap \{v_1, w_1\}| \geq 1$. Suppose that $|D_{v_1w_1} \cap \{v_1, w_1\}| = 1$. By symmetry, we let $w_1 \in D_{v_1w_1}$. Because $D_{v_1w_1}$ is a double dominating set, w_1 is adjacent to a vertex in $D_{v_1w_1}$, w say. Clearly, $\{u_1, v_2, w_1, w_3\}$ is an independent set. By claw-freeness, w is adjacent to at most one vertex in $\{u_1, v_2, w_3\}$. Let $X = D_{v_1w_1} - \{w_1\}$. Clearly, $X \succ_{\times 2} \{u_1, v_2, w_3\}$ and X contains w. In view of Lemma 9, $|X| \geq 4$. This implies that $|D_{v_1w_1}| \geq 5$ contradicting the criticality of G. Suppose that $\{v_1, w_1\} \subseteq D_{v_1w_1}$. Let $X' = D_{v_1w_1} - \{w_1\}$. Thus $X' \succ_{\times 2} \{u_1, v_2, w_3\}$ and X' contains v_1 which is adjacent to at most one vertex in $\{u_1, v_2, w_3\}$. This implies by Lemma 9 that $|X'| \geq 4$. Hence, $|D_{v_1w_1}| \geq 5$ contradicting the criticality of G. Therefore, G does not contains $N_{1,2,2}$ as an induced subgraph.

We now consider the case when G contains $N_{1,1,3}$ as an induced subgraph. Consider $G + u_1v_1$. By Observation 4, $|D_{u_1v_1} \cap \{u_1, v_1\}| \geq 1$. Suppose that $|D_{u_1v_1} \cap \{u_1, v_1\}| = 1$. By symmetry, we let $u_1 \in D_{u_1v_1}$. As $D_{u_1v_1}$ is a double dominating set, we must have that u_1 is adjacent to a vertex u in $D_{u_1v_1}$. Clearly, $\{u_1, v_2, w_1, w_3\}$ is an independent set. Since G is claw-free, u is adjacent to at most one vertex in $\{v_2, w_1, w_3\}$. Let $Y = D_{u_1v_1} - \{u_1\}$. Clearly, $Y \succ_{\times 2} \{v_2, w_1, w_3\}$ and Y contains u. In view of Lemma 9, $|Y| \geq 4$. Thus $|D_{u_1v_1}| \geq 5$ contradicting the criticality of G. We then suppose that $\{u_1, v_1\} \subseteq D_{u_1v_1}$. Let $Y' = D_{u_1v_1} - \{u_1\}$. Thus $Y' \succ_{\times 2} \{v_2, w_1, w_3\}$ and Y' contains v_1 which is adjacent to at most one vertex in $\{v_2, w_1, w_3\}$. This implies by Lemma 9 that $|Y'| \geq 4$. Hence, $|D_{u_1v_1}| \geq 5$ contradicting the criticality of G. Therefore, G does not contain $N_{1,1,3}$ as an induced subgraph. Hence, G is $\{K_{1,3}, N_{1,2,2}, N_{1,1,3}\}$-free.

Because $\gamma_{\times 2}(G) \leq 5$, by Observation 8, G is not isomorphic to $P_{3,3,3}$ and $cl(G) \notin \mathcal{H}_1 \cup \mathcal{H}_2 \cup \mathcal{H}_3$. In view of Theorem 2, G is Hamiltonian. This completes the proof. ■

We can see that a graph G in the class $\mathcal{D}(k)$ when $4 \leq k \leq 5$ is non-Hamiltonian. Thus the condition claw-free in Theorem 10 is necessary. Moreover, when $k = 4$, the graph in the class $\mathcal{D}(k)$ is $K_{1,4}$-free. Hence, the condition claw-free is best possible for $k = 4$. We conclude this section with the following theorem which shows that a graph k-$\gamma_{\times 2}$-critical when $6 \leq k \leq 7$ is Hamiltonian if it is 3-connected and claw-free.

Theorem 11. *For an integer $2 \leq k \leq 7$, let G be a 3-connected k-$\gamma_{\times 2}$-critical claw-free graph. Then G is Hamiltonian.*

Proof. We will show that G is $N_{3,3,3}$-free. Suppose to the contrary that G contains $N_{3,3,3}$ as an induced subgraph. Consider $G + u_1v_1$. Observation 4 yields that $|D_{u_1v_1} \cap \{u_1, v_1\}| \geq 1$. Suppose first that $|D_{u_1v_1} \cap \{u_1, v_1\}| = 1$. By symmetry, we may let $u_1 \in D_{u_1v_1}$. It is easy to see that $\{u_1, u_3, v_2, v_4, w_1, w_3\}$ is an independent set. As $D_{u_1v_1}$ is a double dominating set, we must have that u_1 is adjacent to a vertex u in $D_{u_1v_1}$. Since G is claw-free, u is adjacent to at most one vertex in $\{u_3, v_2, v_4, w_1, w_3\}$. Let $X = D_{u_1v_1} - \{u_1\}$. Clearly, $X \succ_{\times 2} \{u_3, v_2, v_4, w_1, w_3\}$ and X contains u. Lemma 9 implies that $|X| \geq 6$. Thus $|D_{u_1v_1}| \geq 7$ contradicting the criticality of G. We then suppose that $\{u_1, v_1\} \subseteq D_{u_1v_1}$. Let $X' = D_{u_1v_1} - \{u_1\}$. Thus $X' \succ_{\times 2} \{u_3, v_2, v_4, w_1, w_3\}$ and X' contains v_1 which is adjacent to at most one vertex in $\{u_3, v_2, v_4, w_1, w_3\}$. This implies by Lemma 9 that $|X'| \geq 6$. Hence, $|D_{u_1v_1}| \geq 7$ contradicting the criticality of G. Therefore, G does not contain $N_{3,3,3}$ as an induced subgraph. Theorem 3 implies that G is Hamiltonian. This completes the proof. ■

We see that the graphs in the class $\mathcal{D}(k)$ when $6 \leq k \leq 7$ are non-Hamiltonian. Thus, the condition claw-free together with 3-connected is necessary in Theorem 11.

4. Double Domination Stable Graphs

In this section, we use the claw-free property to determine when 2-connected k-$\gamma^+_{\times 2}$-stable claw-free graphs and 2-connected k-$\gamma^-_{\times 2}$-stable claw-free graphs are Hamiltonian. We first establish the following lemma concerning the minimum number of vertices of a double dominating set when some independent set is given. The proof of which is similar to Lemma 9. For completeness, we provide the proof.

Lemma 12. *Let G be a claw-free graph, I be an independent set and X be a set of vertices such that $X \succ_{\times 2} I$. Then $|I| \leq |X|$.*

Proof. Let $I_1 = X \cap I, I_2 = I - I_1$ and $X' = X - I_1$. Clearly, $|X| = |X'| + |I_1|$ and $|I| = |I_1| + |I_2|$. Let H be a subgraph of G such that $V(H) = X' \cup I$ and $E(H) = \{uv \in E(G) : u \in X'$ and $v \in I\}$. Clearly, H is bipartite with the bipartition sets X' and I. Since $X \succ_{\times 2} I$, every vertex in I_1 is adjacent to at least one vertex in X' and, every vertex in I_2 is adjacent to at least two vertices in X'. Thus, $\deg_H(v) \geq 1$ for all $v \in I_1$ and $\deg_H(v) \geq 2$ for all $v \in I_2$. This gives the degree sum of vertices in I as the following

$$|I_1| + 2|I_2| \leq \Sigma_{v \in I} \deg_H(v). \tag{3}$$

Because G is claw-free, every vertex in X' is adjacent to at most two vertices in I. Therefore $\deg_H(u) \leq 2$ for all $u \in X'$. Thus,

$$\Sigma_{u \in X'} \deg_H(u) \leq 2|X'| = 2|X| - 2|I_1|. \tag{4}$$

Because H is bipartite, $\Sigma_{v \in I} \deg_H(v) = \Sigma_{u \in X'} \deg_H(u)$. By (3) and (4), we have $|I_1| + 2|I_2| \leq 2|X| - 2|I_1|$. Hence,

$$|I| = |I_1| + |I_2| \leq 3|I_1|/2 + |I_2| \leq |X|.$$

This completes the proof. ■

By Lemma 12 and Theorems 2 and 3, we easily establish the following corollaries.

Corollary 13. *Let G be a 2-connected claw-free graph with $\gamma_{\times 2}(G) \leq 3$. Then G is Hamiltonian.*

Proof. By Observation 8, $cl(G) \notin \mathcal{H}_1 \cup \mathcal{H}_2 \cup \mathcal{H}_3$ and G is not isomorphic to $P_{3,3,3}$. Thus, by Theorem 2, it suffices to show that G is $\{N_{1,2,2}, N_{1,1,3}\}$-free. Suppose to the contrary that G contains $N_{1,2,2}$ as an induced subgraph. Clearly, $\{u_1, v_1, v_3, w_2\}$ is an independent set of four vertices. Lemma 12 yields that $4 \leq \gamma_{\times 2}(G) \leq 3$, a contradiction. Thus, G is $N_{1,2,2}$-free. We can prove that G is $N_{1,1,3}$-free by the same arguments. Thus, by Theorem 2, G is Hamiltonian. ■

Corollary 14. *Let G be a 3-connected claw-free graph with $\gamma_{\times 2}(G) \leq 5$. Then G is Hamiltonian.*

Proof. By Theorem 3, it suffices to show that G is $N_{3,3,3}$-free. Suppose to the contrary that G contains $N_{3,3,3}$ as an induced subgraph. Clearly, $\{u_1, u_3, v_1, v_3, w_1, w_3\}$ is an independent set of six vertices. Lemma 12 gives that $6 \leq \gamma_{\times 2}(G) \leq 5$, a contradiction. Thus, G is $N_{3,3,3}$-free. By Theorem 3, G is Hamiltonian. ■

4.1. k-$\gamma^{+}_{\times 2}$-stable claw-free graphs

In this subsection, we study Hamiltonian property of k-$\gamma^{+}_{\times 2}$-stable claw-free graphs. Although all 2-$\gamma_{\times 2}$-critical graphs of order at least three are Hamiltonian because they are complete graphs, this is not always true for 2-$\gamma^+_{\times 2}$-stable graphs. That is there exist 2-$\gamma^+_{\times 2}$-stable graphs which are non-Hamiltonian. We first give a construction of k-$\gamma^+_{\times 2}$-stable graphs when $k \geq 2$ which are non-Hamiltonian.

The class $\mathcal{S}^+(k)$. For $k \geq 2$, let K_k be a complete graph of order k with the vertices $x_1, x_2, \ldots, x_k$ and for $1 \leq i \neq j \leq k$, we let $a_{\{i,j\}}, b_{\{i,j\}}, c_{\{i,j\}}$ be $3 \cdot \binom{k}{2}$ isolated vertices. The graph G in this class is obtained from K_k and all the $3 \cdot \binom{k}{2}$ isolated vertices by adding the edges $a_{\{i,j\}}x_p, b_{\{i,j\}}x_p, c_{\{i,j\}}x_p$ for all $1 \leq i \neq j \leq k$ and for all $p \in \{i,j\}$. The following lemma establishes the properties of the graphs in the class $\mathcal{S}^+(k)$.

Lemma 15. *For an integer $k \geq 2$, if $G \in \mathcal{S}^+(k)$, then G is a 2-connected k-$\gamma^+_{\times 2}$-stable non-Hamiltonian graph.*

Proof. Clearly, G is 2-connected. Moreover, it is easy to see that G is k-$\gamma^+_{\times 2}$-stable when $k = 2$. Hence, we assume that $k \geq 3$. We first show that $\gamma_{\times 2}(G) = k$. Obviously, $V(K_k) \succ_{\times 2} G$. By the minimality of $\gamma_{\times 2}(G)$, we have $\gamma_{\times 2}(G) \leq k$. Let D be a $\gamma_{\times 2}$-set of G. So, $|D| \leq k$. We will show that $V(K_k) \subseteq D$. Suppose to the contrary that $\{x_1, x_2, \ldots, x_k\} \not\subseteq D$. Without loss of generality, let $x_1 \notin D$. To doubly dominate $A_1 = \{a_{\{1,j\}}, b_{\{1,j\}}, c_{\{1,j\}} : 1 < j \leq k\}$, we must have that $A_1 \subseteq D$. Since $k \geq 3$, it follows that $k \geq |D| \geq |A_1| = 3k-3 > k$, a contradiction. Thus $V(K_k) \subseteq D$ and $|D| \geq k$. This implies that $\gamma_{\times 2}(G) = k$.

We next establish the stability. Let u and v be a pair of non-adjacent vertices of G and let D_{uv} be a $\gamma_{\times 2}$-set of $G + uv$. Because $V(K_k) \succ_{\gamma_{\times 2}} G + uv$, it follows that $|D_{uv}| \leq |V(K_k)| = k$. It suffice to show that $|D_{uv}| \geq k$. By the construction, $|\{u,v\} \cap V(K_k)| \leq 1$. We first consider the case when $|\{u,v\} \cap V(K_k)| = 1$. Without loss of generality let $u = x_1$ and $v = a_{\{2,3\}}$. Suppose that there exists $x_i \notin D_{uv}$. If $i \neq \{1,2,3\}$, then, to doubly dominate $A_i = \{a_{\{i,j\}}, b_{\{i,j\}}, c_{\{i,j\}} : 1 \leq j \leq k \text{ and } j \neq i\}$, we must have that $A_i \subseteq D_{uv}$. This implies that $|D_{uv}| \geq 3k - 3 > k$ contradicting $|D_{uv}| \leq k$. Thus, $i \in \{1,2,3\}$. To doubly dominate $A_i - \{a_{\{2,3\}}\}$, we must have that $(A_i - \{a_{\{2,3\}}\}) \subseteq D_{uv}$. This implies that $k \geq |D_{uv}| \geq 3k - 4 > k$, a contradiction. Thus, $V(K_k) \subseteq D_{uv}$. This implies that $|D_{uv}| \geq k$.

We now consider the case when $|\{u,v\} \cap V(K_k)| = 0$. Similarly, suppose that there exists $x_i \notin D_{uv}$. To doubly dominate $A_i - \{u,v\}$, we must have that $(A_i - \{u,v\}) \subseteq D_{uv}$. This implies that $k \geq |D_{uv}| \geq (3k-3) - 2 > k$, a contradiction. Thus, $V(K_k) \subseteq D_{uv}$. This implies that $|D_{uv}| \geq k$. This establishes the stability and hence, G is a k-$\gamma^+_{\times 2}$-stable graph.

We finally show that G is non-Hamiltonian. Clearly, $V(K_k)$ is a cut set of G such that $G - V(K_k)$ has $3 \cdot \binom{k}{2}$ isolated vertices as the components. Thus, $\frac{|V(K_k)|}{\omega(G - V(K_k))} < 1$. By Proposition 1, G is non-Hamiltonian. This completes the proof. ∎

By Lemma 15, there exist 2-connected k-$\gamma^+_{\times 2}$-stable graphs which are non-Hamiltonian for all $k \geq 2$. However, by using Corollaries 13 and 14, we easily obtain that all 2-connected k-$\gamma^+_{\times 2}$-stable graphs are Hamiltonian when k is small. The proofs are omitted as the class of 2-connected k-$\gamma^+_{\times 2}$-stable graphs is a subclass of graphs with $\gamma_{\times 2}(G) = k$.

Corollary 16. *Let G be a 2-connected k-$\gamma^+_{\times 2}$-stable claw-free graph. If $2 \leq k \leq 3$, then G is Hamiltonian.*

Corollary 17. *For an integer $2 \leq k \leq 5$, let G be a 3-connected k-$\gamma^+_{\times 2}$-stable claw-free graph. Then G is Hamiltonian.*

Observe that a graph $G \in \mathcal{S}^+(2)$ is $K_{1,4}$-free. Hence, the condition claw-free in Corollaries 13 and 16 is best possible when $k = 2$. For a graph $G \in \mathcal{S}^+(3)$, it is easy to see that the graph $G' = G - c_{\{1,2\}} - b_{\{1,3\}} - c_{\{1,3\}} - b_{\{2,3\}} - c_{\{2,3\}}$ is 3-$\gamma^+_{\times 2}$-stable $K_{1,4}$-free graph. Hence, the condition claw-free in Corollaries 13 and 16 is best possible when $k = 3$.

4.2. k-$\gamma^-_{\times 2}$-stable claw-free graphs

We first give a construction of k-$\gamma^-_{\times 2}$-stable graphs when $k \geq 2$ which are non-Hamiltonian.

The class $\mathcal{S}^-(k)$. For $k \geq 2$, we let K_{2k} be a complete graph of order $2k$ with the vertices $x_1, y_1, x_2, y_2, \ldots, x_k, y_k$ and for $1 \leq i \neq j \leq k$, we let $a_{\{i,j\}}, b_{\{i,j\}}, c_{\{i,j\}}, d_{\{i,j\}}, e_{\{i,j\}}$ be $5 \cdot \binom{k}{2}$ isolated vertices. The graph G in this class is obtained from K_{2k} and all the $5 \cdot \binom{k}{2}$ isolated vertices by joining the vertices x_p and y_p to $a_{\{i,j\}}, b_{\{i,j\}}, c_{\{i,j\}}, d_{\{i,j\}}, e_{\{i,j\}}$ for all $1 \leq i \neq j \leq k$ and for all $p \in \{i, j\}$. The following lemma establishes the properties of the graphs in the class $\mathcal{S}^-(k)$.

Lemma 18. *For an integer $k \geq 2$, if $G \in \mathcal{S}^-(k)$, then G is a 2-connected k-$\gamma^-_{\times 2}$-stable non-Hamiltonian graph.*

Proof. Clearly, G is 2-connected. Moreover, it is easy to see that G is k-$\gamma^-_{\times 2}$-stable when $k = 2$. Hence, we assume that $k \geq 3$. We first show that $\gamma_{\times 2}(G) = k$. Since $\{x_1, x_2, \ldots, x_k\} \succ_{\times 2} G$, it follows that $\gamma_{\times 2}(G) \leq k$. It remains to show that $\gamma_{\times 2}(G) \geq k$. Let D be a $\gamma_{\times 2}$-set of G. By the minimality of D, we have $|D| \leq k$. If $|\{x_i, y_i\} \cap D| \geq 1$ for all $1 \leq i \leq k$, then $|D| \geq k$ as required. We may suppose that there exists $i \in \{1, 2, \ldots, k\}$ such that $\{x_i, y_i\} \cap D = \emptyset$. To doubly dominate $A_i = \big\{a_{\{i,j\}}, b_{\{i,j\}}, c_{\{i,j\}}, d_{\{i,j\}}, e_{\{i,j\}} : 1 \leq j \leq k \text{ and } j \neq i\big\}$, we must have that $\big|D \cap \big\{a_{\{i,j\}}, b_{\{i,j\}}, c_{\{i,j\}}, d_{\{i,j\}}, e_{\{i,j\}}, x_j, y_j\big\}\big| \geq 2$ for all $1 \leq j \leq k$ and $j \neq i$. This implies that $k \geq |D| \geq 2(k-1) > k$, a contradiction. Thus, $\gamma_{\times 2}(G) = |D| \geq k$. Therefore $\gamma_{\times 2}(G) = k$.

We next establish the stability. Let u and v be a pair of adjacent vertices of G. By Observation 6, we have $\gamma_{\times 2}(G-uv) \geq k$. Hence, it suffices to show that there exists a $\gamma_{\times 2}$-set of $G - uv$ containing k vertices. Clearly, $|\{u, v\} \cap V(K_{2k})| \geq 1$. We first suppose that $|\{u, v\} \cap V(K_{2k})| = 1$. Without loss of generality let $u \in V(K_{2k})$. If $u \in \{x_1, x_2, \ldots, x_k\}$, then $\{y_1, y_2, \ldots, y_k\} \succ_{\times 2} G - uv$. If $u \in \{y_1, y_2, \ldots, y_k\}$, then $\{x_1, x_2, \ldots, x_k\} \succ_{\times 2} G - uv$. Hence, we now suppose that $u, v \in V(K_{2k})$. We consider the case when $\{u, v\} = \{x_i, y_j\}$ for some $i, j \in \{1, 2, \ldots, k\}$. Clearly $\{x_1, x_2, \ldots, x_k\} \succ_{\times 2} G - uv$. We now consider the case when $\{u, v\} = \{x_i, x_j\}$. Thus, $\{y_1, y_2, \ldots, y_k\} \succ_{\times 2} G - uv$. Similarly, $\{x_1, x_2, \ldots, x_k\} \succ_{\times 2} G - uv$ when $\{u, v\} = \{y_i, y_j\}$. Therefore, G is k-$\gamma^-_{\times 2}$-stable graph.

We finally show that G is non-Hamiltonian. Clearly, $V(K_{2k})$ is a cut set of G such that $G - V(K_{2k})$ has $5 \cdot \binom{k}{2}$ isolated vertices as the components. Thus, $\frac{|V(K_{2k})|}{\omega(G-V(K_k))} < 1$. By Proposition 1, G is non-Hamiltonian. This completes the proof. ■

We next establish the following theorems.

Theorem 19. *Let G be a 2-connected k-$\gamma^-_{\times 2}$-stable claw-free graph. If $2 \leq k \leq 4$, then G is Hamiltonian.*

Proof. We first show that G is $\{N_{1,2,2}, N_{1,1,3}\}$-free. Suppose to the contrary that G contains $N_{1,2,2}$ or $N_{1,1,3}$ as an induced subgraph. We first consider the case when G contains $N_{1,2,2}$ as an induced subgraph. Consider $G - v_1v_2$. By stability of G, $\gamma_{\times 2}(G - v_1v_2) = k$. Clearly, $\{u_1, v_1, v_2, w_1, w_3\}$ is an independent set of $G - v_1v_2$ containing 5 vertices. Lemma 12 implies that $4 \geq \gamma_{\times 2}(G - v_1v_2) \geq 5$, a contradiction. Thus, G is $N_{1,2,2}$-free.

We now consider the case when G contains $N_{1,1,3}$ as an induced subgraph. Consider $G - u_1u_2$. By stability of G, $\gamma_{\times 2}(G - u_1u_2) = k$. Clearly, $\{u_1, u_2, v_1, w_1, w_3\}$ is an independent set of $G - u_1u_2$ containing 5 vertices. Lemma 12 implies that $4 \geq \gamma_{\times 2}(G - v_1v_2) \geq 5$, a contradiction. Thus, G is $N_{1,1,3}$-free. Hence, G is $\{K_{1,3}, N_{1,2,2}, N_{1,1,3}\}$-free.

Because $\gamma_{\times 2}(G) \leq 4$, by Observation 8, G is not isomorphic to $P_{3,3,3}$ and $cl(G) \notin \mathcal{H}_1 \cup \mathcal{H}_2 \cup \mathcal{H}_3$. In view of Theorem 2, G is Hamiltonian. This completes the proof. ■

Theorem 20. *Let G be a 3-connected k-$\gamma^-_{\times 2}$-stable claw-free graph. If $2 \leq k \leq 6$, then G is Hamiltonian.*

Proof. We will show that G is $N_{3,3,3}$-free. Suppose to the contrary that G contains $N_{3,3,3}$ as an induced subgraph. Consider $G - u_1u_2$. By stability of G, $\gamma_{\times 2}(G - u_1u_2) = k$. We see that $\{u_1, u_2, u_4, v_1, v_3, w_1, w_3\}$ is an independent set of $G - u_1u_2$ containing 7 vertices. By Lemma 12, $6 \geq \gamma_{\times 2}(G - u_1u_2) \geq 7$, a contradiction. Therefore, G does not contain $N_{3,3,3}$ as an induced subgraph. Theorem 3 implies that G is Hamiltonian. This completes the proof. ■

5. Discussion

On double domination critical graphs. For $6 \leq k \leq 7$, we have seen neither 2-connected k-$\gamma_{\times 2}$-critical claw-free graphs which are non-Hamiltonian nor 3-connected k-$\gamma_{\times 2}$-critical graphs which are non-Hamiltonian. Hence, the questions that arise are, for an integer $6 \leq k \leq 7$, is every 2-connected k-$\gamma_{\times 2}$-critical claw-free graph Hamiltonian? and is every 3-connected k-$\gamma_{\times 2}$-critical graph Hamiltonian?

On double domination stable graphs. We have seen neither 2-connected k-$\gamma^{+}_{\times 2}$-stable claw-free graphs which are non-Hamiltonian for $4 \leq k \leq 5$ nor 2-connected k-$\gamma^{-}_{\times 2}$-stable graphs which are non-Hamiltonian for $5 \leq k \leq 6$. Hence, the questions that arise are, for $4 \leq k \leq 5$, is every 2-connected k-$\gamma^{+}_{\times 2}$-stable claw-free graph Hamiltonian? and, for $5 \leq k \leq 6$, is every 2-connected k-$\gamma^{-}_{\times 2}$-stable claw-free graph Hamiltonian?

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