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STRONGLY UNICHORD-FREE GRAPHS

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Abstract

Several recent papers have investigated unichord-free graphs—the graphs in which no cycle has a unique chord. This paper proposes a concept of strongly unichord-free graph, defined by being unichord-free with no cycle of length 5 or more having exactly two chords. In spite of its overly simplistic look, this can be regarded as a natural strengthening of unichord-free graphs—not just the next step in a sequence of strengthenings—and it has a variety of characterizations. For instance, a 2-connected graph is strongly unichord-free if and only if it is complete bipartite or complete or "minimally 2-connected" (defined as being 2-connected such that deleting arbitrary edges always leaves non-2-connected subgraphs).

Keywords: unichord-free graph, strongly chordal graph.

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1. Introduction

An edge ab is a *chord* of a cycle C if $a, b \in V(C)$ but $ab \notin E(C)$. A graph is *unichord-free* if no cycle has a unique chord. Such graphs were introduced (but not named) independently in [9, 14] and have since been studied (and named unichord-free) in additional papers, including [4, 5, 6, 7, 10]; in particular, [14] gives a constructive characterization. Examples of unichord-free graphs include the complete graphs and complete bipartite graphs, along with the Petersen and Heawood graphs (see [14]).

A chord ab of a cycle C has a *crossing chord* xy if their four endpoints occur in the order a, x, b, y around C, and the chords ab and xy are then said to *cross* each other. Define a chord of C to be *double-crossed* if it is crossed by at least two chords of C. For cycles C_1 and C_2 with $E(C_1) \cap E(C_2) = \{ab\}$, define the symmetric difference $C_1 \oplus C_2$ to be the cycle that has edge set $E(C_1) \cup E(C_2) - E(C_1) \cap E(C_2)$; thus ab is a chord of $C_1 \oplus C_2$. For a graph G with $S \subset V(G)$, let $G[S]$ denote the subgraph of G induced by S and, for convenience when C is a cycle of G and $v \in V(C)$, let $G[C] = G[V(C)]$ and let $C - v$ denote the path obtained by deleting v from C.

Lemma 1 [9]**.** *A graph is unichord-free if and only if every chord of every cycle has a crossing chord.*

Proof. First suppose G is a unichord-free graph and C is a minimum-length cycle of G such that C has a chord ab with no crossing chord (arguing by contradiction). Let $a'b'$ be a second chord of C (possibly with ab and $a'b'$ having one vertex in common). But then the cycle formed by the edge $a'b'$ and the a'-to-b' subpath of C through a and b would also have no crossing chord for ab, contradicting the assumed minimality of C.

The converse is immediate since a unique chord of a cycle is necessarily uncrossed. ∎

Define a ${}^{\geq}k$*-cycle* to be a cycle of length at least k, and define a graph to be *strongly unichord-free* if it is unichord-free and no ${}^{\geq}5$-cycle has exactly two chords. Induced subgraphs of strongly unichord-free are also strongly unichord-free, and it is simple to check that complete graphs and complete bipartite graphs are strongly unichord-free. Figure 1 shows the Petersen graph—a unichord-free graph that is fundamental to the constructive approach of [14]—that is not strongly unichord-free, since the 8-cycle with vertices in numerical order $1, 2, \ldots, 8$ has exactly two chords (26 and 48).

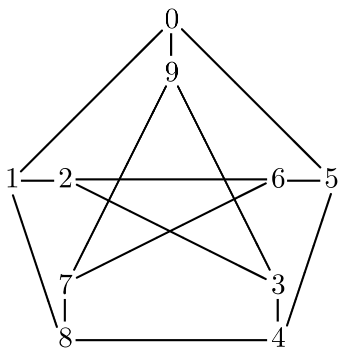

Figure 1. A unichord-free graph that is not strongly unichord-free.

The following deceptively simple observation will be surprisingly useful.

Theorem 2. *A unichord-free graph is strongly unichord-free if and only if every chord of every* ${}^{\geq}5$*-cycle is double-crossed.*

Proof. First suppose G is strongly unichord-free and xy is a chord of a minimum-length ${}^{\geq}5$-cycle C such that xy is not double-crossed (arguing by contradiction). By Lemma 1, xy is crossed by some chord uv of C. Since C cannot have exactly two chords, C has a third chord wz that, since it does not cross xy, we can assume is a chord of the x-to-y subpath π of $C-v$. The assumed minimality of C ensures that wz is not a chord of either the x-to-u or the u-to-y subpath of π, and so xy is a chord of the cycle C' formed by the edge wz and the w-to-z subpath of $C-u$, with xy having no crossing chord in C' (contradicting Lemma 1).

Conversely, suppose G is unichord-free and every chord of every ${}^{\geq}5$-cycle is double-crossed. Therefore, no ${}^{\geq}5$-cycle has exactly two chords. ■

Sections 2 and 3 will present additional characterizations of strongly unichord-free graphs that involve, respectively, chords of cycles and graph connectivity. In particular, Corollary 5 will somewhat justify the choice of the descriptor "strongly" with a statement that involves a parameter k for which the $k = 1$ case characterizes being unichord-free and the $k = 1$ and $k = 2$ cases together characterize being strongly unichord-free—as also do all of the $k \geq 1$ cases taken together. (This parallels how strongly chordal graphs, see [1], are characterized in [12] by a statement for which the $k = 1$ case characterizes chordal graphs and the $k = 1$ and $k = 2$ cases together characterize strongly chordal graphs—as do all of the $k \geq 1$ cases holding—showing that chordal graphs and strongly chordal graphs are not simply the first two of a sequence of increasingly stronger graph classes.)

2. CHARACTERIZATIONS INVOLVING CHORDS OF CYCLES

Lemma 3. *Suppose G is a strongly unichord-free graph that contains 2-connected subgraphs G_1 and G_2 that share at least one edge. If G_1 and G_2 are both complete (or both complete bipartite), then so is $G[V(G_1) \cup V(G_2)]$.*

Proof. Suppose G is strongly unichord-free and contains 2-connected subgraphs G_1 and G_2 with $ab \in E(G_1) \cap E(G_2)$, and let $H = G[V(G_1) \cup V(G_2)]$. We can assume that $G_1 \neq H \neq G_2$ to avoid a trivial conclusion.

First suppose G_1 and G_2 are both complete. Suppose (say) $x_1 \in V(G_1) \setminus V(G_2)$ and $x_2 \in V(G_2) \setminus V(G_1)$. Thus H has a 4-cycle C with vertices in the order a, x_1, b, x_2 around C and with chord ab. By Lemma 1, ab has the crossing chord x_1x_2 of C, and so x_1 and x_2 are adjacent. By the arbitrariness of the choice of x_1 and x_2, the graph H is complete.

Now suppose G_1 and G_2 are both complete bipartite. Suppose each G_i has partite sets S_i and S'_i where, without loss of generality, $a \in S_1 \cap S_2$ and (so) $b \in S'_1 \cap S'_2$. An edge x_1x_2 of H cannot have $x_1 \in S_1 \setminus \{a\}$ and $x_2 \in S_2 \setminus \{a\}$;

otherwise, the 2-connected subgraph G_1 would contain $z \in S'_1 \setminus \{b\}$ and the five edges $x_1x_2, x_2b, ba, az, zx_1$ would form a cycle of H with exactly one or two chords (bx_1, and maybe x_2z), contradicting that G is strongly unichord-free. Thus and similarly, every edge of H has one endpoint in $S_1 \cup S_2$ and the other in $S'_1 \cup S'_2$, and so the subgraph H is bipartite with partite sets $S_1 \cup S_2$ and $S'_1 \cup S'_2$. Suppose (say) $c \in S_1 \setminus S_2$ and $c' \in S'_2 \setminus S'_1$, so each G_i being 2-connected ensures there exist $d' \in S'_1 \setminus \{b\}$ and $d \in S_2 \setminus \{a\}$. Thus H has a 6-cycle C that consists of the path a, d', c, b in G_1 and the path b, d, c', a in G_2, with ab a chord of C. Since G is strongly unichord-free and cc' and dd' are the only other possible chords C can have in the bipartite subgraph H, cycle C has all three chords ab, cc', and dd', so $H[C] \cong K_{3,3}$ and c is adjacent to c'. By the arbitrariness of the choice of c and c' (and similarly starting from arbitrary $c \in S_2 \setminus S_1$ and $c' \in S'_1 \setminus S'_2$), the graph H is complete bipartite. ■

Theorem 4. *The following are equivalent for every graph.*

(1) *The graph is strongly unichord-free.*

(2) *The vertex set of every cycle induces a chordless cycle, a complete bipartite subgraph, or a complete subgraph.*

(3) *Every n-cycle has exactly zero, $n(n-4)/4$, or $n(n-3)/2$ chords.*

Proof. First, suppose G satisfies condition (1), and so every induced subgraph of G is strongly unichord-free, toward showing that (2) holds. Argue by induction on $n \geq 3$ that, for every n-cycle C of G, the subgraph $H = G[C]$ has either $H \cong C_n$ or $H \cong K_{h,h}$ with $h = n/2$ (using that all the vertices of a bipartite graph $H = H[C]$ must alternate around C between the two partite sets) or $H \cong K_n$. Let $x \sim y$ and $x \not\sim y$ denote, respectively, that vertices x and y are or are not adjacent.

If $n = 3$, then H must be $C_3 \cong K_3$.

If $n = 4$, then the unichord-free graph H must be either $C_4 \cong K_{2,2}$ or K_4.

Suppose $n \geq 5$ and the n-cycle C is not chordless. Thus C has a chord ab that, combined with the two a-to-b subpaths of C forms two cycles C' and C'' that have $C = C' \oplus C''$. We can assume that ab is chosen to make C' as small as possible, thus making C' chordless. Since the chord ab must have a crossing chord (say) $a'b''$ with $a' \in V(C')$ and $b'' \in V(C'')$ and since $a'b''$ must, in turn, have a crossing chord different from ab by Theorem 2 (and since C' is chordless), one of the two following alternatives must hold:

(i) *$a'b''$ is crossed by a chord cd of C''*; or

(ii) *$a'b''$ is crossed by a chord $a''b'$ of C where $a'' \in V(C'') \setminus \{a, b, b''\}$ and $b' \in V(C') \setminus \{a, a', b\}$.*

Figure 2 illustrates the notation that will be used in these two alternatives.

First suppose alternative (i) holds, say with the vertices in the order a, b, d, b'', c around C'' (possibly with $a = c$ or $b = d$). Say $C'' = C''_1 \oplus C''_2$ where $ab \in$

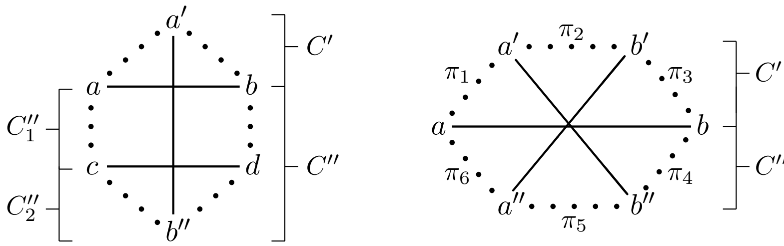

Figure 2. Illustrating alternatives (i) and (ii) in the proof of Theorem 4, with subpaths of the cycle C shown as dotted lines.

$E(C_1'') \setminus E(C_2'')$ and $cd \in E(C_1'') \cap E(C_2'')$. We can assume that cd is chosen to make C_1'' as small as possible, thus making C_1'' chordless.

Suppose for the moment that C_1'' is an odd cycle, so $G[C'']$ is neither a chordless cycle nor complete bipartite, and so $G[C'']$ is complete by the induction hypothesis. Thus, ab'' and bb'' are edges that combine with the a-to-a'-to-b subpath of C' to form a cycle $\widehat{C}$ such that $G[\widehat{C}]$ is neither a chordless cycle nor complete bipartite, so $G[\widehat{C}]$ is complete by the induction hypothesis. Since the complete subgraphs $G[C'']$ and $G[\widehat{C}]$ both contain ab, the subgraph $G[C]$ is complete by Lemma 3.

By the preceding paragraph, we can assume that C_1'' is a chordless even cycle. Since $G[C' \oplus C_1'']$ is neither a chordless cycle nor complete, $G[C' \oplus C_1'']$ is complete bipartite by the induction hypothesis. Since C'' is neither chordless nor complete, $G[C'']$ is complete bipartite by the induction hypothesis. Since the complete bipartite subgraphs $G[C' \oplus C_1'']$ and $G[C'']$ both contain edge ab, the subgraph $G[C]$ is complete bipartite by Lemma 3.

Now suppose instead that alternative (ii) holds, say with the six vertices a, a', b', b, b'', a'' in that order around C, with each of the chords ab, $a'b''$, and $a''b'$ crossing the other two. Partition C into subpaths $\pi_1, \ldots, \pi_6$ as shown in Figure 2. Just as the chord ab was chosen so that C' (the cycle formed by the edge ab and the path $\pi_1 \cup \pi_2 \cup \pi_3$) is chordless, we can assume that chord $a'b''$ was then chosen to form a chordless cycle with $\pi_5 \cup \pi_6 \cup \pi_1$, and that chord $a''b'$ was finally chosen to make a chordless cycle with $\pi_3 \cup \pi_4 \cup \pi_5$. Thus, $a \not\sim b'$, $a' \not\sim b$, $a \not\sim b''$, $a' \not\sim a''$, $b' \not\sim b''$, and $a'' \not\sim b$.

We can assume that at least one of the six paths π_i has length greater than 1 (otherwise, $H \cong K_{3,3}$ is already complete bipartite). Indeed, we can assume that π_1 has length greater than 1 (since the other five paths π_i behave the same as π_1 in the remainder of this proof), and so $a \not\sim a'$. Let C_1 be the cycle formed from the paths and edges in the set $\{\pi_2, \pi_3, ab, \pi_6, \pi_5, a'b''\}$, noting that $G[C_1]$ is a proper subgraph of G since $G[C_1]$ does not contain the internal vertices of π_1. Thus $a''b'$ is a chord of C_1, so $G[C_1]$ is neither a chordless cycle nor complete (since, for instance, $a \not\sim a'$), and so $G[C_1]$ is complete bipartite by the induction hypothesis.

If $b \sim b'$, let C_2 be formed from $\{\pi_2, a''b', \pi_6, ab, \pi_4, a'b''\}$, and if $b \not\sim b'$, let C_2 be formed from $\{\pi_1, \pi_2, a''b', \pi_5, \pi_4, ab\}$, noting that $G[C_2]$ is a proper subgraph of G either way (since $G[C_2]$ does not contain the internal vertices of π_1 when $b \sim b'$ or of π_3 when $b \not\sim b'$). Thus bb' (when $b \sim b'$) or $a'b''$ (when $b \not\sim b'$) is a chord of C_2, so $G[C_2]$ is neither a chordless cycle nor complete, and so $G[C_2]$ is complete bipartite by the induction hypothesis.

Since the complete bipartite subgraphs $G[C_1]$ and $G[C_2]$ both contain edge ab, the subgraph $G[C]$ is complete bipartite by Lemma 3. Therefore, G satisfies condition (2).

Next, suppose G satisfies (2), C is an n-cycle of G, and $H = G[C]$. If $H \cong C_n$, then C has zero chords; otherwise, by (2), H is complete bipartite or complete. If H is complete bipartite, then its vertices alternate around C between the two partite sets, so $H \cong K_{h,h}$ with $h = n/2$, and so C has exactly $n(n-4)/4$ chords. If $H \cong K_n$, then C has exactly $n(n-3)/2$ chords. Therefore, G satisfies (3).

Finally, suppose G satisfies (3). Since $n \geq 4$ implies $n(n-4)/4 \neq 1$ and $n(n-3)/2 \neq 1$, no ${}^{\geq}4$-cycle can have a unique chord, and so G is unichord-free. Since $n \geq 5$ implies $n(n-4)/4 \neq 2$ and $n(n-3)/2 \neq 2$, no ${}^{\geq}5$-cycle can have exactly two chords. Therefore G satisfies (1). ■

Corollary 5. *A graph is strongly unichord-free if and only if, for all $k \geq 1$, every chord of every ${}^{\geq}(k+3)$-cycle is crossed by at least k chords when $k = 1$ or k is even, and is crossed by at least $k-1$ chords when $k \geq 3$ is odd.*

Proof. First suppose G is strongly unichord-free, C is an n-cycle with a chord ab (so $n \geq 4$), and $H = G[C]$. Let $N^{\times}_{ab}$ denote the number of chords of C that cross ab. Theorem 4 ensures that H is complete bipartite or complete, and Lemma 1 ensures $N^{\times}_{ab} \geq 1$ when $k = 1$. Hence, we can assume $k \geq 2$ and $n \geq k+3 \geq 5$.

If H is complete bipartite, then n is even and $H \cong K_{h,h}$ with $h = n/2$, so $N^{\times}_{ab} \geq n-4$ (with equality when a and b are distance-3 apart along C). If H is complete, then $H \cong K_n$, so $N^{\times}_{ab} \geq n-3$ (with equality when a and b are distance-2 apart along C).

Therefore, if $k \geq 2$ is even, then $n \geq k+3$ ensures that *either* H is complete bipartite, $n \geq k+4$ is even, and $N^{\times}_{ab} \geq (k+4)-4$ *or* H is complete and $N^{\times}_{ab} \geq (k+3)-3$; thus, either way, $N^{\times}_{ab} \geq k$. Similarly, if $k \geq 3$ is odd, then $n \geq k+3$ ensures that *either* H is complete bipartite, $n \geq k+3$ is even, and $N^{\times}_{ab} \geq (k+3)-4$ *or* H is complete and $N^{\times}_{ab} \geq (k+3)-3$; thus, either way, $N^{\times}_{ab} \geq k-1$.

The converse follows from Lemma 1 (by taking $k = 1$ in the statement of the corollary) and from Theorem 2 (by taking $k = 2$). ■

3. Characterizations Involving Connectivity

A graph is *minimally k-connected* if it is k-connected and, for every edge ab, deleting ab leaves a graph that is not k-connected; see [8] for details.

Corollary 6. *A graph is strongly unichord-free if and only if the vertex set of every cycle induces a minimally k-connected subgraph for some $k \geq 2$.*

Proof. First suppose G is a strongly unichord-free graph with an n-cycle C, and let $H = G[C]$. By Theorem 4, either H is a chordless cycle (so H is minimally 2-connected) or H is complete bipartite with its vertices alternating around C between the two partite sets (so $H \cong K_{h,h}$ with $h = n/2$ and H is minimally $(n/2)$-connected) or H is complete (so $H \cong K_n$ with $n \geq 3$ and H is minimally $(n-1)$-connected).

Conversely, suppose G is not strongly unichord-free. Thus, G contains a subgraph $H = G[C]$ such that either (i) C is a $^{\geq}4$-cycle with a unique chord or (ii) C is a $^{\geq}5$-cycle with exactly two chords. In either case, H contains a degree-2 vertex, so H is 2-connected but not 3-connected, and deleting a chord of C leaves a 2-connected subgraph of H. Therefore, H is not minimally k-connected for any $k \geq 2$. ■

Minimally 2-connected graphs were introduced in [2, 13]. They appeared independently in [3], characterized as the 2-connected *chordless graphs*—defined by no cycle having a chord—along with a $\mathcal{O}(n^2m)$ recognition algorithm (on n vertices and m edges); also see [5]. Since being 2-connected, complete bipartite, and complete can each be recognized in linear time, Theorem 7 will guarantee a $\mathcal{O}(n^2m)$ recognition algorithm for strongly unichord-free graphs (in contrast to a $\mathcal{O}(nm)$ recognition algorithm for unichord-free graphs in [14]).

Theorem 7. *A 2-connected graph is strongly unichord-free if and only if the graph is either minimally 2-connected or complete bipartite or complete.*

Proof. First suppose G is a 2-connected, strongly unichord-free graph that is not minimally 2-connected. Thus G has an edge ab that can be deleted without losing 2-connectedness, so ab is a chord of some cycle C and so $G[C]$ is not a chordless cycle. Thus, $G[C]$ is either complete or complete bipartite by Theorem 4. For every path π, let π° denote the set of internal vertices of π.

Suppose for the moment that $G[C]$ is complete, and assume that H is an inclusion-maximal complete subgraph of G that contains C where $H \neq G$ (arguing by contradiction). Thus, the 2-connected, non-complete graph G has a minimum-length x-to-y path π for some $x, y \in V(H)$ where $\pi^\circ \neq \emptyset = \pi^\circ \cap V(H)$ and the cycle formed from π and the edge xy is chordless. Pick any triangle xyz of H. Since the cycle C' formed from π and the edges xz and yz cannot have the unique chord xy, there must be a vertex in π° that is adjacent to z, so the assumed minimality of π ensures that C' is a 4-cycle, and so $G[C'] \cong K_4$ is complete. Since the complete subgraphs H and $G[C']$ both contain xy, the subgraph $G[V(H) \cup V(C')]$ is complete by Lemma 3 (contradicting the assumed maximality of H).

By the preceding paragraph, we can assume that $G[C]$ is complete bipartite. Assume H is an inclusion-maximal complete bipartite subgraph of G that contains C, with $V(H)$ partitioned into bipartite sets S_1 and S_2. Since ab is a chord of a cycle of H, the complete bipartite graph $H \ncong K_{2,n}$ for any n, and so $|S_1| \geq 3$ and $|S_2| \geq 3$. Assume $H \neq G$ (arguing by contradiction). Thus, the 2-connected graph G has a minimum-length x-to-y path π for some $x, y \in V(H)$ where $\pi^\circ \neq \emptyset = \pi^\circ \cap V(H)$ and xy is the only possible chord that the path π can have.

Suppose for the moment that $x \in S_1$ and $y \in S_2$; thus x and y are adjacent and there are vertices $x' \in V(H) \cap S_1 \setminus \{x\}$ and $y' \in V(H) \cap S_2 \setminus \{y\}$ such that $G[\{x, x', y, y'\}] \cong K_{2,2}$. Thus G contains a ${}^{\geq}5$-cycle C' made from π and the path x, y', x', y. Since C' has the chord xy and $G[C']$ is not complete, $G[C']$ is complete bipartite by Theorem 4. Since the complete bipartite subgraphs H and $G[C']$ both contain xy, the subgraph $G[V(H) \cup V(C')]$ is complete bipartite by Lemma 3 (contradicting the assumed maximality of H).

By the preceding paragraph, we can assume that x and y are in the same partite set; say $x, y \in S_1$ (and so x and y are not adjacent) and, since $|S_1| \geq 3$ and $|S_2| \geq 3$) there are vertices $z \in S_1 \subset V(H)$ and $u, v \in S_2 \subset V(H)$ such that $G[\{u, v, x, y, z\}] \cong K_{2,3}$. Let C' be the ${}^{\geq}6$-cycle made from π and the path x, u, z, v, y of H, so C' has exactly two chords between vertices of H (namely, vx and uy) together with possible chords between vertices in π° and vertices in $\{u, v, z\}$. Since C' has the chord vx and $G[C']$ is not complete, $G[C']$ is complete bipartite by Theorem 4. Since the complete bipartite subgraphs H and $G[C']$ both contain vx, Lemma 3 ensures that $G[V(H) \cup V(C')]$ is complete bipartite (contradicting the assumed maximality of H).

The converse follows from every cycle of each minimally 2-connected graph being chordless implying that G is strongly unichord-free, together with every complete bipartite graph and every complete graph being strongly unichord-free. ■

Corollary 8. *A* 2*-connected graph with minimum degree at least* 3 *is strongly unichord-free if and only if the graph is either complete bipartite or complete.*

Proof. This follows from Theorem 7 since every minimally 2-connected graph has a vertex of degree 2; see [2, 3, 13]. ■

From a different point of view, [11] characterizes the graphs in which *some* chord of every ${}^{\geq}5$-cycle is double-crossed. The final section of [11] introduces—admittedly without any real motivation—those graphs in which *every* chord of every ${}^{\geq}5$-cycle is double-crossed, as in Theorem 2. Some of that discussion in [11] is translated into our current terminology in the following corollary (which would also follow using Theorem 7).

Corollary 9 [11]**.** *The following are equivalent for every unichord-free graph G that is* 2*-connected with no induced* ${}^{\geq}5$*-cycle.*

(1) *G is strongly unichord-free.*

(2) *G is complete bipartite or complete.*

(3) *Every* ${}^{\geq}5$*-cycle C of G has $G[C]$ nonplanar.*

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