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CORES, JOINS AND THE FANO-FLOW CONJECTURES

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Abstract

The Fan-Raspaud Conjecture states that every bridgeless cubic graph has three 1-factors with empty intersection. A weaker one than this conjecture is that every bridgeless cubic graph has two 1-factors and one join with empty intersection. Both of these two conjectures can be related to conjectures on Fano-flows. In this paper, we show that these two conjectures are equivalent to some statements on cores and weak cores of a bridgeless cubic graph. In particular, we prove that the Fan-Raspaud Conjecture is equivalent to a conjecture proposed in [E. Steffen, 1*-factor and cycle covers of cubic graphs*, J. Graph Theory **78** (2015) 195–206]. Furthermore, we disprove a conjecture proposed in [G. Mazzuoccolo, *New conjectures on perfect matchings in cubic graphs*, Electron. Notes Discrete Math. **40** (2013) 235–238] and we propose a new version of it under a stronger connectivity assumption. The weak oddness of a cubic graph G is the minimum number of odd components (i.e., with an odd number of vertices) in the complement of a join of G. We obtain an upper bound of weak oddness in terms of weak cores, and thus an upper bound of oddness in terms of cores as a by-product.

Keywords: cubic graphs, Fan-Raspaud Conjecture, cores, weak-cores.

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1. Introduction

We study 1-factors (i.e., perfect matchings) in cubic graphs. If G is a graph, then $V(G)$ and $E(G)$ denote its vertex set and edge set, respectively. In 1994, the following statement was conjectured to be true by Fan and Raspaud.

Conjecture 1 [1]**.** *Every bridgeless cubic graph has three* 1*-factors* M_1, M_2, M_3 *such that* $M_1 \cap M_2 \cap M_3 = \emptyset$.

We remark that Conjecture 1 is implied by the celebrated Berge-Fulkerson Conjecture [2], which states that every bridgeless cubic graph has six 1-factors such that each edge is contained in precisely two of them.

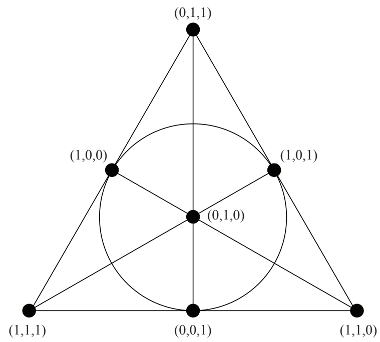

Figure 1. Fano plane $\mathcal{F}_7$.

The study of Conjecture 1 leads to a deep analysis of Fano-flows on graphs. Consider the Fano plane $\mathcal{F}_7$ with the points labeled with the seven non-zero elements of $\mathbb{Z}_2^3$ as drawn in Figure 1. Clearly, if a cubic graph has a nowhere-zero $\mathbb{Z}_2^3$-flow, then for every vertex the flow values on its incident edges are pairwise different and they lie on a line of the Fano plane. Thus, every bridgeless cubic graph has a nowhere-zero Fano-flow by Jaeger's 8-flow Theorem [3]. However, it is possible that not all combinations of three elements of $\mathbb{Z}_2^3$ appear at a vertex of G. For $k \leq 7$, a *k-line Fano-flow* is a Fano-flow of G, where at most k lines of $\mathcal{F}_7$ appear as flow values at the vertices of G. Clearly, a 3-edge-colorable cubic graph has a 1-line Fano-flow. Máčajová and Škoviera [8] proved that each Fano-flow of a bridgeless cubic class 2 graph needs all seven points and at least four lines of the Fano plane. Furthermore, they proved that every bridgeless cubic graph has a 6-line Fano-flow, and conjectured that 4 lines are sufficient.

Conjecture 2 [8]**.** *Every bridgeless cubic graph has a* 4*-line Fano-flow.*

A natural relaxation of Conjecture 2 is the following conjecture.

Conjecture 3 [8]**.** *Every bridgeless cubic graph has a* 5*-line Fano-flow.*

Let H be a graph. If either $X \subseteq V(H)$ or $X \subseteq E(H)$, then $H[X]$ denotes the subgraph of H induced by X. A *join* of H is a set J of edges such that the degree of every vertex have the same parity in H and $H[J]$. If no confusion can arise, we use J instead of $H[J]$.

Conjectures 2 and 3 have surprisingly counterparts in terms of 1-factors. Máčajová and Škoviera [8] proved that Conjecture 2 is equivalent to Conjecture 1. Analogously, one can easily obtain the equivalence between Conjecture 3 and Conjecture 4, and the one between 6-line Fano-flow theorem and Proposition 5.

Conjecture 4. *Every bridgeless cubic graph has two* 1*-factors* M_1, M_2 *and a join* J *such that* $M_1 \cap M_2 \cap J = \emptyset$.

Proposition 5. *Every bridgeless cubic graph has a* 1*-factor* M *and two joins* J_1 *and* J_2 *such that* $M \cap J_1 \cap J_2 = \emptyset$.

Let G be a bridgeless cubic graph. The *oddness* $\omega(G)$ of G is the minimum number of odd circuits of a 2-factor of G. We define the *weak oddness* $\omega'(G)$ of G as the minimum number of odd components, with respect to its order, of the complement of a join. Clearly, $\omega'(G) \leq \omega(G)$. There was a long standing discussion on the question whether $\omega(G) = \omega'(G)$ for all bridgeless cubic graphs G, but, recently, Lukot'ka and Mazák [7] provide a negative answer by constructing an example of a cubic graph having $\omega'(G) = 12$ and $\omega(G) = 14$.

Máčajová and Škoviera [9] proved Conjecture 1 for cubic graphs with oddness at most 2. This implies the truth of Conjecture 4 for these graphs as well. A proof of this particular result is given in [5] by Kaiser and Raspaud. However, it is easy to see that $\omega'(G) = 2$ if and only if $\omega(G) = 2$, for each bridgeless cubic graph G. Hence, the result of [9] is even true for graphs with weak oddness at most 2.

Let J be a join of a cubic graph G. Thus every vertex has degree either 1 or 3 in J. A *J-vertex* is a vertex of degree 3 in J. Let $n(J)$ denote the number of J-vertices.

Let G be a cubic graph and S be a set of three joins J_1, J_2 and J_3 of G. For each $i \in \{0, \ldots, 3\}$, let $E_i(S)$ be the set of edges that are contained in precisely i elements of S. When no confusion can arise, we write E_i instead of $E_i(S)$. The *weak core* of G with respect to S (or to J_1, J_2 and J_3) is a subgraph G_c induced by the union of sets E_0, E_2 and E_3, that is, $G_c = G[E_0 \cup E_2 \cup E_3]$. The weak core G_c is further called an *l-weak k-core*, where precisely l elements of S are not 1-factors and $k = |E_0| + \frac{3}{2} \sum_{i=1}^{3} n(J_i)$. The parameter k is the generalization of the analogous definition for cores; our particular choice will be more clear in the proof of Theorem 15. We define $\mu_3'(G) = \min\{k \colon G \text{ has a weak } k\text{-core}\}$. A 0-weak core is called a *core* as well. Define $\mu_3(G) = \min\{k \colon G \text{ has a } k\text{-core}\}$. Clearly, $\mu_3'(G) \leq \mu_3(G)$. It is easy to see that a bridgeless cubic graph G is 3-edge-colorable if and only if $\mu_3'(G) = 0$.

The core of a cubic graph was introduced by Steffen [11] working on perfect matching covers, and the parameter $\mu_3(G)$ was taken as a measurement on the edge-uncolorability of class 2 cubic graphs. Weak cores are a natural generalization of the definition of cores for covers with three joins. In this paper, we study both cores and weak cores of cubic graphs.

A join J of G is *simple* if the subgraph induced by all the J-vertices contains no circuit. Clearly, every 1-factor of G is a simple join, and every join of G contains a simple join as a subset. A weak core G_c of G is *simple* if all the joins with respect to G_c are simple. A weak core is cyclic if it is a cycle.

Conjecture 1 can be easily formulated as a conjecture on cores in bridgeless cubic graphs.

Conjecture 6 [11]**.** *Every bridgeless cubic graph has a cyclic core.*

Steffen proposed the following seemingly weaker conjecture.

Conjecture 7 [11]**.** *Every bridgeless cubic graph has a bipartite core.*

It is clear that Conjecture 6 implies Conjecture 7 because all circuits in a cyclic core are of even length. Here, we show that the converse implication is also true, that is, Conjectures 6 and 7 are equivalent. Hence, our result furnishes a new equivalent formulation for Fan-Raspaud Conjecture. We also show that the condition on the cores can be further relaxed. We even show that the following conjectures are equivalent to Fan-Raspaud Conjecture.

Conjecture 8. *Every bridgeless cubic graph has a triangle-free core.*

Conjecture 9. *Every bridgeless cubic graph has three* 1*-factors such that the complement of their union is an acyclic graph.*

Analogously, we formulate Conjecture 4 as a conjecture on 1-weak core.

Conjecture 10. *Every bridgeless cubic graph has a cyclic* 1*-weak core.*

We prove the equivalence between this conjecture and the statement that every bridgeless cubic graph has a triangle-free simple 1-weak core.

In general, Fano-flows can be related to cyclic weak cores. Instead of the k-line Fano-flow problem, we ask the following equivalent question.

Problem 11. What is the minimum k such that every bridgeless cubic graph has a cyclic k-weak core?

As above, it was proved that $k < 2$ and conjectured that either $k = 0$ or $k = 1$.

We summarize all announced implications in Figure 2.

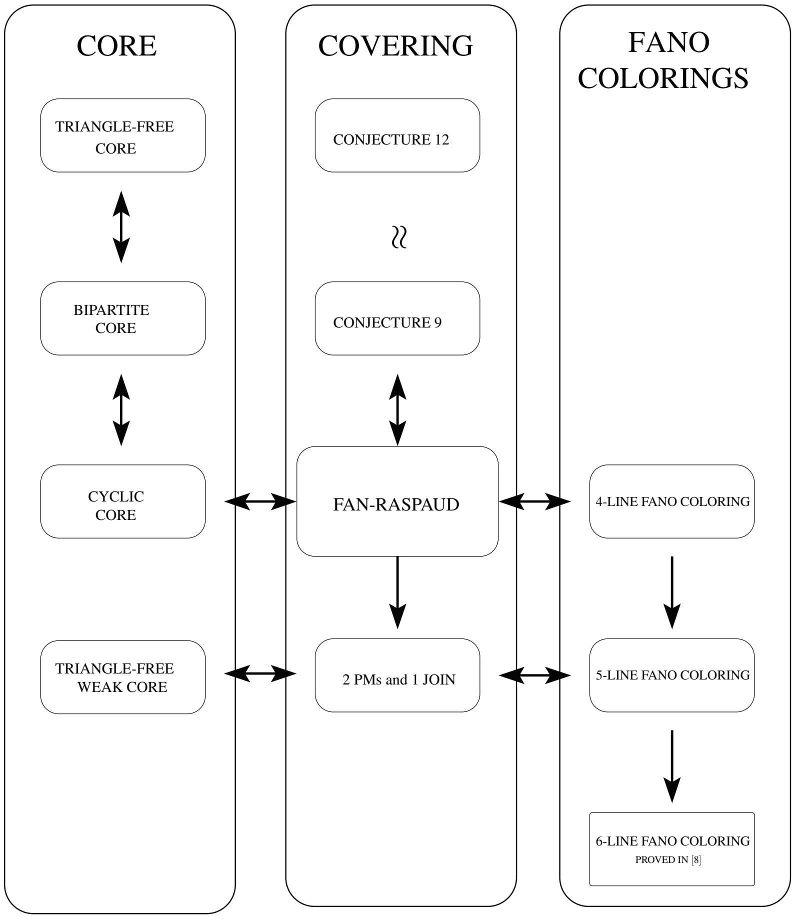

Figure 2. Conjectures related to Fan-Raspaud Conjecture.

Section 2 studies properties of weak cores, and it shows that the weak oddness of a bridgeless cubic graph is bounded in terms of its weak cores.

Finally, in the last section, we disprove the following stronger version of Conjecture 9.

Conjecture 12 [10]**.** *Every bridgeless cubic graph has two* 1*-factors such that the complement of their union is an acyclic graph.*

Even if we prove that previous conjecture is false in that general form, we believe that it could be still true under stronger connectivity assumptions. In particular, we recall that it was verified true for all snarks, hence cyclically 4-edge-connected cubic graphs, of order at most 34 (see [10]).

More precisely, we wonder if every 3-connected (cyclically 4-edge-connected) cubic graph has two 1-factors such that the complement of their union is an acyclic graph.

2. The Weak Core of a Cubic Graph

Let J_1, J_2 and J_3 be three joins of a cubic graph G. We say that a vertex v of G has type (x, y, z) if the three edges incident with v are covered x, y and z times by $\{J_1, J_2, J_3\}$, respectively. We denote by a, b, c, d, e, f, g the number of vertices of type $(3,3,3), (3,2,2), (3,1,1), (2,2,1), (1,1,1), (2,1,0), (3,0,0)$, respectively (see also Figure 3). Clearly, every vertex has precisely one type. Note that vertices of type $(3,3,3)$, $(3,2,2)$, $(3,1,1)$ and $(2,2,1)$ are J_i-vertices for some i.

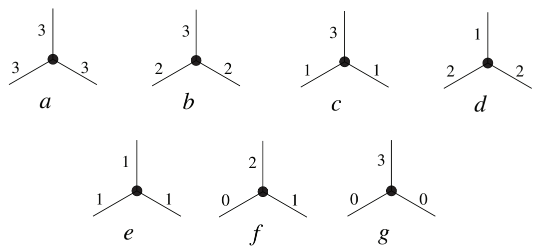

Figure 3. Vertex types.

Proposition 13. *Let G be a cubic graph, and J_1, J_2 and let J_3 be three joins of G. Then*

$$|E_0| + \sum_{i=1}^{3} n(J_i) = |E_2| + 2|E_3|.$$

Proof. By type definitions, we have $\sum_i n(J_i) = 3a + 2b + c + d$, $|E_0| = \frac{f}{2} + g$, $|E_2| = b + d + \frac{f}{2}$ and $|E_3| = \frac{3a}{2} + \frac{b}{2} + \frac{c}{2} + \frac{g}{2}$. Hence, $\sum_i n(J_i) + |E_0| = 3a + 2b + c + d + \frac{f}{2} + g = |E_2| + 2|E_3|$ holds. ■

Proposition 14. *If G_c is a weak core of a cubic graph G, then $G[E_0 \cup E_2]$ is either an empty graph or a cycle.*

Proof. By type definitions, it is easy to see that every vertex is incident with either none or precisely two edges of $E_0 \cup E_2$. Therefore, $G[E_0 \cup E_2]$ is either an empty graph or a cycle. ■

Let H be a graph. We denote by $|H|_{odd}$ the number of odd components of H, that is components with an odd number of vertices. If J is a join of H, then $\overline{J}$ denotes the complement of J.

Theorem 15. *Let G be a bridgeless cubic graph and let G_c be a weak k-core with respect to three joins J_1, J_2 and J_3. Then $\sum_{i=1}^{3} |\overline{J_i}|_{odd} \leq 2k$.*

Proof. Each component of the complement of J_i is either an isolated vertex or a circuit. Any odd circuit of $\overline{J_i}$ contains either one edge from E_0 or a J_k-vertex with $k \neq i$. We call an odd circuit of $\overline{J_i}$ *bad* if it has no J_k-vertex for $k \neq i$. In what follows, we distinguish elements of E_0 according to their behavior with respect to bad circuits. We define

$X_i = \{e\colon e$ is the unique edge in $C \cap E_0$ and C is a bad circuit of $\overline{J_i}\}$, for $i = 1, 2, 3$;

$Y_i = \{e\colon e \in E_0 \setminus X_i$ and $e \in C \cap E_0$ and C is a bad circuit of $\overline{J_i}\}$, for $i = 1, 2, 3$.

Set $x = |X_1| + |X_2| + |X_3|$ and $y = |Y_1| + |Y_2| + |Y_3|$.

Since $X_i \cap Y_i = \emptyset$, it follows that

$$x + y \leq 3|E_0|. \tag{1}$$

Moreover, if $e \in X_i$, then $e \notin X_j$, and $e \notin X_k$ for $j, k \neq i$, that is

$$x \leq |E_0|. \tag{2}$$

Combining equations (1) and (2) implies

$$x + \frac{y}{2} \leq 2|E_0|. \tag{3}$$

Now, we are in position to prove our assertion. Since in an odd circuit of $\overline{J_i}$ there is either a J_k-vertex ($k \neq i$) or an edge of X_i or two edges of Y_i, the following relation holds:

$$|\overline{J_i}|_{odd} \leq |X_i| + \frac{|Y_i|}{2} + \sum_{i=1}^{3} n(J_i).$$

Therefore, by summing up for all three joins we deduce

$$\sum_{i=1}^{3} |\overline{J_i}|_{odd} \leq x + \frac{y}{2} + 3\sum_{i=1}^{3} n(J_i) \leq 2|E_0| + 3\sum_{i=1}^{3} n(J_i) = 2k,$$

where the last inequality directly follows from (3). ∎

Corollary 16. *If G is a bridgeless cubic graph, then $\omega'(G) \leq \frac{2}{3}\mu_3'(G)$.*

Proof. Let G_c be a weak $\mu_3'(G)$-core of G with respect to three joins J_1, J_2 and J_3. By Theorem 15, we have $|\overline{J_1}|_{odd} + |\overline{J_2}|_{odd} + |\overline{J_3}|_{odd} \leq 2\mu_3'(G)$. By the minimality of the weak oddness $\omega'(G)$ it follows that $\omega'(G) \leq \frac{2}{3}\mu_3'(G)$. ∎

The following results were already obtained in [4], but now it turns out that they are just a particular case of our previous theorem. This shows that the definition of weak k-core is the right generation of k-core.

Theorem 17 [4]**.** *Let G be a bridgeless cubic graph and let G_c be a k-core with respect to three 1-factors M_1, M_2 and M_3. Then $\sum_{i=1}^{3} |\overline{M_i}|_{odd} \leq 2k$.*

Corollary 18 [4]**.** *If G is a bridgeless cubic graph, then $\omega(G) \leq \frac{2}{3}\mu_3(G)$.*

Proof. Let G_c be a $\mu_3(G)$-core of G with respect to three 1-factors M_1, M_2 and M_3. By Theorem 17, we have $|\overline{M_1}|_{odd} + |\overline{M_2}|_{odd} + |\overline{M_3}|_{odd} \leq 2\mu_3(G)$. By the minimality of $\omega(G)$, it follows that $\omega(G) \leq \frac{2}{3}\mu_3(G)$. ■

3. Equivalent Statements

Let G_1 and G_2 be two bridgeless graphs, e_1 and e_2 be two edges such that $e_1 = u_1v_1 \in E(G_1)$ and $e_2 = u_2v_2 \in E(G_2)$. The 2*-cut connection* on $\{e_1, e_2\}$ is a graph operation that consists of deleting edges e_1 and e_2 and adding two new edges u_1u_2 and v_1v_2. Clearly, the graph obtained from G_1 and G_2 by applying 2-cut connection is also bridgeless.

Theorem 19. *The following four statements are equivalent.*

(1) (*Conjecture* 3) *Every bridgeless cubic graph has a* 5*-line Fano-flow.*

(2) (*Conjecture* 4) *Every bridgeless cubic graph has a join J and two* 1*-factors M_1 and M_2 such that $J \cap M_1 \cap M_2 = \emptyset$.*

(3) *Every bridgeless cubic graph has a cyclic* 1*-weak core.*

(4) *Every bridgeless cubic graph has a triangle-free simple* 1*-weak core.*

Proof. The equivalence of statements (1) and (2) is proved in [6] (Theorem 3.1).

(2) $\to$ (3): By Proposition 14, the 1-weak core with respect to M_1, M_2 and J is cyclic. Therefore, statement (2) implies statement (3).

(3) $\to$ (4): Suppose to the contrary that there is a bridgeless cubic graph G that has no triangle-free simple 1-weak core. Let G_c be a cyclic 1-weak core of G with respect to two 1-factors M_1, M_2 and a join J such that $E(G_c)$ is minimum. We claim that G_c is simple. Otherwise, J is not simple, that is, G contains a circuit C such that each vertex of C is a J-vertex. Recall that G_c is cyclic, by type definitions according to M_1, M_2 and J, every vertex of C has type $(2, 2, 1)$. Let J_1 be the new join obtained from join J by removing all the edges of C. Thus J_1 is also a join of G. The 1-weak core with respect to M_1, M_2 and J_1 is cyclic and has fewer edges than G_c, a contradiction. This completes the proof of the claim.

By our supposition and the previous claim, G_c has a triangle $[xyz]$. It follows that two of vertices x, y and z have type $(2, 1, 0)$ and the last one has type $(2, 2, 1)$, which is the only possible case. Without loss of generality we assume that z is of type $(2,2,1)$. Set $J_2 = J \cup \{xy\} \setminus \{xz, yz\}$. Clearly, J_2 is a join of G. Now the 1-weak core with respect to M_1, M_2 and J_2 is cyclic and has fewer edges than G_c, a contradiction. Therefore, statement (3) implies statement (4).

(4) $\to$ (2): Let G be a bridgeless cubic graph with edge set $\{e_1, \ldots, e_m\}$. Take m copies $T_1, \ldots, T_m$ of the complete graph K_4. For each $i \in \{1, \ldots, m\}$, apply the 2-cut connection on e_i and an edge of T_i, and let e_i' and e_i'' be the two added new edges. The resulting graph G' is bridgeless and cubic. By (3), G' has a triangle-free simple 1-weak core H. Let H be with respect to two 1-factors M_1, M_2 and a simple join J. For every join F of G', since F contains either both of e_i' and e_i'' or none of them for each $i \in \{1, \ldots, m\}$, let $con(F) = \{e\colon e = e_i \in E(G)$, and $e_i', e_i'' \in F\}$. Clearly, $con(F)$ is a join of G and in particular, $con(F)$ is a 1-factor of G if F is a 1-factor of G'. We claim that $con(M_1) \cap con(M_2) \cap con(J) = \emptyset$ and hence, statement (1) holds. Suppose to the contrary that G has an edge e_1 contained in all of $con(M_1), con(M_2)$ and $con(J)$. It follows that $e_1', e_1'' \in M_1 \cap M_2 \cap J$, and hence one can easily deduce that in copy T_1, the 1-weak core H contains either a triangle or a circuit of length 4 whose vertices are J-vertices, a contradiction. Therefore, statement (3) implies statement (1). ∎

Theorem 20. *The following five statements are equivalent.*

(1) (*Conjecture* 2) *Every bridgeless cubic graph has a* 4*-line Fano-flow.*
(2) (*Conjecture* 1) *Every bridgeless cubic graph has three* 1*-factors* M_1, M_2, M_3 *such that* $M_1 \cap M_2 \cap M_3 = \emptyset$.
(3) (*Conjecture* 7) *Every bridgeless cubic graph has a bipartite core.*
(4) (*Conjecture* 8) *Every bridgeless cubic graph has a triangle-free core.*
(5) (*Conjecture* 9) *Every bridgeless cubic graph has three* 1*-factors such that the complement of their union is an acyclic graph.*

Proof. The equivalence of statements (1) and (2) is proved in [6] (Theorem 3.1).

If statement (2) holds, then by Proposition 14, the core G_c of a bridgeless cubic graph G with respect to M_1, M_2, M_3 is cyclic. More precisely, each circuit in G_c contains edges from E_0 and E_2 alternate in cyclic order. Hence, the core G_c is bipartite and triangle-free, and $G[E_0]$ is an acyclic graph. Hence statement (2) implies all of the statements (3), (4) and (5).

Let G be a bridgeless cubic graph with edge set $\{e_1, \ldots, e_m\}$. Take m copies $T_1, \ldots, T_m$ of the complete graph K_4. For each $i \in \{1, \ldots, m\}$, apply 2-cut connection on e_i and an edge of T_i, and let e_i' and e_i'' be the two added new edges. Let G' be the resulting graph, which is bridgeless and cubic. Let H be a core of G' with respect to three 1-factors M_1, M_2, M_3. For every 1-factor F of G', since F contains either both of e_i' and e_i'' or none of them for each $i \in \{1, \ldots, m\}$, we can let $con(F) = \{e\colon e = e_i \in E(G)$, and $e_i', e_i'' \in F\}$. Clearly, $con(F)$ is a 1-factor of G. We claim that if H is either bipartite or triangle-free or if the complement of the union of M_1, M_2, M_3 is acyclic, then $con(M_1), con(M_2)$ and $con(M_3)$ have empty intersection. This claim completes the proof. Suppose to the contrary that G has an edge e_1 such that $e_1 \in con(M_1) \cap con(M_2) \cap con(M_3)$. It follows that $e_1', e_1'' \in M_1 \cap M_2 \cap M_3$. Hence in copy T_1, core H contains triangles and $G[E_0]$ contains a circuit of length 4, a contradiction with the supposition of our claim. ■

4. Counterexample to Conjecture 12

If the Fan-Raspaud Conjecture is true, then every bridgeless cubic graph has two 1-factors, say M_1 and M_2, with no odd edge-cut in their intersection; in particular, the complement of $M_1 \cup M_2$ is a bipartite graph which is the union of paths and even circuits. One could asks if even circuits could be forbidden in such bipartite graph. It is verified to be true for all snarks of order at most 34 and proposed as a conjecture in [10].

Here, we disprove Conjecture 12 in its present formulation by using the same technique already used in the proof of Theorem 19.

Let P be the Petersen graph and let $\{e_1, \ldots, e_{15}\}$ be its edge-set. Take 15 copies $T_1, \ldots, T_{15}$ of the complete graph K_4. For each $i \in \{1, \ldots, 15\}$, apply a 2-cut connection on e_i and an arbitrary edge of T_i. Denote by G the obtained graph. Let M_1 and M_2 be two 1-factors of G, and let $con(M_1)$ and $con(M_2)$ be the two corresponding 1-factors of P, respectively. Since every pair of 1-factors of P has exactly an edge in common, without loss of generality we can assume $\{e_1\} = con(M_1) \cap con(M_2)$. Hence, T_1 has an edge covered twice and a 4-circuit uncovered, that is the complement of $M_1 \cup M_2$ is not acyclic.

We would like to stress that the graph G has a lot of 2-edge-cuts, so we wonder if an analogous version of Conjecture 12 could hold true for 3-connected or cyclically 4-edge-connected cubic graphs.

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