Discussiones Mathematicae
Graph Theory 37 (2017) 1039–1053
doi:10.7151/dmgt.1984

SIGNED TOTAL ROMAN EDGE DOMINATION IN GRAPHS

Leila Asgharsharghi

and

Seyed Mahmoud Sheikholeslami

Department of Mathematics
Azarbaijan Shahid Madani University
Tabriz, I.R. Iran

e-mail: {l.sharghi;s.m.sheikholeslami}@azaruniv.edu

Abstract

Let $G = (V, E)$ be a simple graph with vertex set V and edge set E. A signed total Roman edge dominating function of G is a function $f : E \to \{-1, 1, 2\}$ satisfying the conditions that (i) $\sum_{e' \in N(e)} f(e') \geq 1$ for each $e \in E$, where $N(e)$ is the open neighborhood of e, and (ii) every edge e for which $f(e) = -1$ is adjacent to at least one edge e' for which $f(e') = 2$. The weight of a signed total Roman edge dominating function f is $\omega(f) = \sum_{e \in E} f(e)$. The signed total Roman edge domination number $\gamma'_{stR}(G)$ of G is the minimum weight of a signed total Roman edge dominating function of G. In this paper, we first prove that for every tree T of order $n \geq 4$, $\gamma'_{stR}(T) \geq \frac{17-2n}{5}$ and we characterize all extreme trees, and then we present some sharp bounds for the signed total Roman edge domination number. We also determine this parameter for some classes of graphs.

Keywords: signed total Roman dominating function, signed total Roman domination number, signed total Roman edge dominating function, signed total Roman edge domination number.

2010 Mathematics Subject Classification: 05C69.

1. Introduction

For terminology and notation on graph theory not defined here, the reader is referred to [2, 3, 8]. Let G be a simple graph with vertex set $V = V(G)$ and edge set $E = E(G)$. The order $|V|$ of G is denoted by $n = n(G)$ and size $|E|$ of G is denoted by $m = m(G)$. For every vertex $v \in V$, the *open neighborhood* of v is the set $N(v) = \{u \in V \mid uv \in E\}$ and the *closed neighborhood* of v is the set $N[v] = N(v) \cup \{v\}$. The *degree* of a vertex $v \in V$ is $\deg_G(v) = \deg(v) = |N(v)|$. The *minimum* and *maximum degree* of a graph G are denoted by $\delta = \delta(G)$ and $\Delta = \Delta(G)$, respectively. Two edges e_1, e_2 of G are called *adjacent* if they are distinct and have a common end-vertex. For every edge $e \in E$, the *open neighborhood* $N_G(e) = N(e)$ is the set of all edges adjacent to e and its *closed neighborhood* is $N_G[e] = N[e] = N(e) \cup \{e\}$. The *edge-degree* of an edge $e \in E$ is $\deg_G(e) = \deg(e) = |N(e)|$. Let $\Delta_e = \Delta_e(G)$ and $\delta_e = \delta_e(G)$ denote the *maximum edge-degree* and *minimum edge-degree* of G, respectively. The *complement* $\overline{G}$ of G is the simple graph with vertex set $V(G)$ defined by $uv \in E(\overline{G})$ if and only if $uv \notin E(G)$. We write K_n for the complete graph of order n, C_n for a cycle of length n and P_n for a path of length $n-1$. For a subset $S \subseteq E$ of edges of a graph G and a function $f : E \to \mathbb{R}$, we define $f(S) = \sum_{x \in S} f(x)$.

A subset $F \subseteq E$ is an *edge total dominating* set if every edge $e \in E$ is adjacent to at least one edge in F. The cardinality of a smallest edge total dominating set in a graph G is called the *edge total domination number* of G and is denoted by $\gamma'_t(G)$. The edge total domination number was introduced by Kulli and Patwari [5] and has been studied by several authors [6].

A *signed total edge dominating function* of G is a function $f : E \longrightarrow \{-1, 1\}$ such that $\sum_{e' \in N(e)} f(e') \geq 1$ for every $e \in E$. The *weight* of a signed total edge dominating function f is the sum of its function values over all edges. The *signed total edge domination number* $\gamma'_{st}(G)$ of G is the minimum weight of a signed total edge dominating function on G. The signed edge total domination was introduced in [9] and has been studied by several authors [4, 10].

A function $f : E \longrightarrow \{-1, 1, 2\}$ is called a *signed Roman edge dominating function* (SREDF) of G, if $f(N[e]) = \sum_{e' \in N[e]} f(e') \geq 1$ for each edge e of G and every edge e for which $f(e) = -1$ is adjacent to at least one edge e' for which $f(e') = 2$. The minimum of the values $f(E)$, taken over all signed Roman edge dominating functions f of G, is called the *signed Roman edge domination number* of G and is denoted by $\gamma'_{sR}(G)$. In [1] Ahangar *et al.* introduced this concept.

A *signed total Roman dominating function* (STRDF) on a graph $G = (V, E)$ is a function $f : V \to \{-1, 1, 2\}$ satisfying the conditions that (i) the sum of its function values over any open neighborhood is at least one, and (ii) every vertex u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$. The *weight* of an STRDF is the sum of its function values over all vertices. The *signed total Roman domination number* of G, denoted by $\gamma_{stR}(G)$, is the minimum weight of an STRDF in G. The signed total Roman domination number was introduced by Volkmann [7].

A *signed total Roman edge dominating function* (STREDF) on a graph G is a function $f\colon E \to \{-1, 1, 2\}$ satisfying the conditions that (i) $\sum_{e' \in N(e)} f(e') \geq 1$ for each edge $e \in E$, and (ii) every edge $e \in E$ for which $f(e) = -1$ is adjacent to at least one edge $e' \in E$ for which $f(e') = 2$. The *weight* of an STREDF is the sum of its function values over all edges. The *signed total Roman edge domination number* of G, denoted by $\gamma'_{stR}(G)$, is the minimum weight of an STREDF in G. For an STREDF f, let $E_i = E_i(f) = \{e \in E \mid f(e) = i\}$ for $i = -1, 1, 2$.

The aim of this paper is to initiate the study of the signed total Roman edge domination number. We first prove that for every tree T of order $n \geq 4$, $\gamma'_{stR}(T) \geq \frac{17-2n}{5}$ and we characterize all extreme trees, and then we present some sharp bounds for the signed total Roman edge domination number. We also determine this parameter for some classes of graphs.

We make use of the following results in this paper.

Observation 1. *Let G be a connected graph of order $n \geq 3$. If $f = (E_{-1}, E_1, E_2)$ is an STREDF on G, then*

(a) $m = |E_{-1}| + |E_1| + |E_2|$.

(b) $\omega(f) = 2|E_2| + |E_1| - |E_{-1}|$.

(c) $E_1 \cup E_2$ *is an edge total dominating set of G.*

Proof. Since (a) and (b) are immediate, we only prove (c). By definition, every edge of E_{-1} is adjacent to an edge of E_2 and so E_2 dominates E_{-1}. On the other hand, for every edge $e \in E_1 \cup E_2$, it follows from $f(N(e)) \geq 1$ that $|N(e) \cap (E_1 \cup E_2)| \geq 1$. Hence $E_1 \cup E_2$ is an edge total dominating set of G. ■

Proposition 2 [7]**.** *Let C_n be a cycle of order $n \geq 3$. Then*

$$\gamma_{stR}(C_n) = \begin{cases} \frac{n}{2} & n \equiv 0 \pmod 4, \\ \frac{n+3}{2} & n \equiv 1, 3 \pmod 4, \\ \frac{n+6}{2} & n \equiv 2 \pmod 4. \end{cases}$$

Proposition 3 [7]**.** *Let P_n be a path of order $n \geq 3$. Then $\gamma_{stR}(P_n) = \frac{n}{2}$ when $n \equiv 0 \pmod 4$, and $\gamma_{stR}(P_n) = \lceil \frac{n+3}{2} \rceil$ otherwise.*

Proposition 4 [7]**.** *If $n \geq 3$ is an integer, then $\gamma_{stR}(K_n) = 3$.*

The *line graph* of a graph G, written $L(G)$, is the graph whose vertices are the edges of G, with $ef \in E(L(G))$ when $e = uv$ and $f = vw$ in G. It is easy to see that $L(K_{1,n}) = K_n$, $L(C_n) = C_n$ and $L(P_n) = P_{n-1}$. The proof of the following result is straightforward and therefore omitted.

Observation 5. *For any connected graph G of order $n \geq 3$, $\gamma'_{stR}(G) = \gamma_{stR}(L(G))$.*

Using Observation 5 and Propositions 2, 3, and 4, we obtain the next results.

Corollary 6. *For* $n \geq 2$, $\gamma'_{stR}(K_{1,n}) = 2$ *when* $n = 2$, *and* $\gamma'_{stR}(K_{1,n}) = 3$ *otherwise.*

Corollary 7. *For* $n \geq 4$, $\gamma'_{stR}(P_n) = \frac{n-1}{2}$ *when* $n \equiv 1 \pmod 4$, *and* $\gamma'_{stR}(P_n) = \lceil \frac{n+2}{2} \rceil$ *otherwise.*

Corollary 8. *For* $n \geq 3$, $\gamma'_{stR}(C_n) = \frac{n}{2}$ *when* $n \equiv 0 \pmod 4$, $\gamma'_{stR}(C_n) = \frac{n+3}{2}$ *when* $n \equiv 1, 3 \pmod 4$, *and* $\gamma'_{stR}(C_n) = \frac{n+6}{2}$ *when* $n \equiv 2 \pmod 4$.

If G is a graph and f is an STREDF of G, then an edge e is said to be a $+1$ edge if $f(e) = 1$, a 2 edge if $f(e) = 2$ and a -1 edge if $f(e) = -1$. For each vertex $v \in V$ we also define $f(v) = \sum_{e \in E(v)} f(e)$, where $E(v)$ is the set of all edges at vertex v.

2. Trees

In this section we present a lower bound on the signed total Roman edge domination number for trees and we characterize all extreme trees.

To begin with, we need to introduce some terminology and notation. A vertex of degree one is called a *leaf*, and its neighbor is called a *support vertex.* If v is a support vertex, then L_v will denote the set of all leaves adjacent to v. A support vertex v is called a *strong support vertex* if $|L_v| > 1$. For a vertex v in a rooted tree T, let $C(v)$ denote the set of children of v, $D(v)$ denote the set of descendants of v and $D[v] = D(v) \cup \{v\}$. Also, the *depth* of v, depth(v), is the largest distance from v to a vertex in $D(v)$. The *maximal subtree* at v is the subtree of T induced by $D(v) \cup \{v\}$, and is denoted by T_v.

For $r, s \geq 1$, a *double star* $S(r, s)$ is a tree with exactly two vertices that are not leaves, with one adjacent to r leaves and the other to s leaves.

Proposition 9. *For* $r \geq s \geq 1$,

$$\gamma'_{stR}(S(r,s)) = \begin{cases} 4 & r, s \text{ are odd and } r, s \geq 3, \\ 3 & \text{otherwise.} \end{cases}$$

Proof. Let $S(r, s)$ be a double star whose central vertices are x, y with r pendant edges xx_i and s pendant edges yy_i. Since $S(1, 1) = P_4$, we have $\gamma'_{stR}(P_4) = 3$ by Corollary 7. Henceforth, we assume $r \geq 2$. Let $f = (E_{-1}, E_1, E_2)$ be a $\gamma'_{stR}(S(r, s))$-function, $f(xx_j) = \max_i f(xx_i)$ and $f(yy_k) = \max_i f(yy_i)$. Since $\sum_{e \in N(xx_j)} f(e) \geq 1$ and $\sum_{e \in N(yy_k)} f(e) \geq 1$, we have

$$(1) \quad \sum_{i=1}^{r} f(xx_i) \geq f(xx_j) + 1 - f(xy) \quad \text{and} \quad \sum_{i=1}^{s} f(yy_i) \geq f(yy_k) + 1 - f(xy).$$

Summing them up to get

$$(2)\qquad \omega(f) = \sum_{i=1}^{r} f(xx_i) + \sum_{i=1}^{s} f(yy_i) + f(xy) \geq f(xx_j) + f(yy_k) + 2 - f(xy).$$

If $E_{-1} = \emptyset$, then in fact $f(e) = 1$ for all edges e and so $\omega(f) = r + s + 1$ implying $\omega(f) \geq 4$. So now assume that $E_{-1} \neq \emptyset$. If $f(xy) \leq 1$, we may assume $f(xx_j) = 2$ and $f(yy_k) \geq 1$ which leads to $\omega(f) \geq 4$ by (2). If $f(xy) = 2$, then $\omega(f) = \sum_{e\in N(xy)} f(e)+f(xy) \geq 1+2 = 3$. Suppose to the contrary that $\omega(f) = 3$, but r, s are odd and $r, s \geq 3$. This is possible only when $\sum_{i=1}^{r} f(xx_i) + \sum_{i=1}^{s} f(yy_i) = 1$. By symmetry, we may assume that $\sum_{i=1}^{r} f(xx_i) \leq 0$. By (1), $f(xx_j) \leq 1$. Since r is odd, we have $\sum_{i=1}^{r} f(xx_i) \leq -1$. This yields that

$$f(xx_j) = \sum_{i=1}^{r} f(xx_i) - \sum_{e\in N(xx_j)} f(e) + f(xy) \leq -1 - 1 + 2 = 0,$$

and so $f(xx_j) = -1$. Therefore, $f(xx_i) = -1$ for $1 \leq i \leq r$ and $\sum_{e\in N(xx_j)} f(e) = 3 - r \leq 0$, a contradiction.

To see the upper bound, define an STREDF g by $g(xy) = 2$, $g(xx_i) = (-1)^{i-1}$ for $1 \leq i \leq r$ and $g(yy_j) = (-1)^{j-1}$ for $1 \leq j \leq s$ by a modification in the following two cases: (i) both r and s are even, in which modify $g(yy_1) = 2$, (ii) $s = 1$ and r is odd, in which modify $g(xx_1) = -1$ and $g(yy_1) = 2$. ■

Let r be a positive integer and T_r be the tree obtained from the star $K_{1,3r+1}$ with central vertex x and leaves $x_1, \ldots, x_{3r+1}$ by adding two pendant edges at x_i such as x_iy_i, x_iz_i, for each $1 \leq i \leq r + 2$ (Figure 1). Suppose $\mathcal{F} = \{T_r \mid r \geq 1\}$.

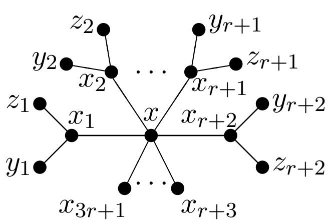

Figure 1. Family $\mathcal{F}$.

Lemma 10. *If $T \in \mathcal{F}$, then* $\gamma'_{stR}(T) = \frac{17-2|V(T)|}{5}$.

Proof. Let $T \in \mathcal{F}$. Then $T = T_r$ for some positive integer r. To show that $\gamma'_{stR}(T) \leq \frac{17-2|V(T)|}{5}$, define $f : E(T) \to \{-1, 1, 2\}$ by $f(xx_i) = 2$ for each $1 \leq i \leq r + 2$ and $f(e) = -1$ otherwise. Clearly, f is an STREDF of T of weight

$\frac{17-2|V(T)|}{5}$ and so $\gamma'_{stR}(T) \leq \frac{17-2|V(T)|}{5}$. Now, we show that $\gamma'_{stR}(T) \geq \frac{17-2|V(T)}{5}$. Let f be a $\gamma'_{stR}(T)$-function. By definition, $f(N(x_iy_i)) = f(xx_i) + f(x_iz_i) \geq 1$ for each $1 \leq i \leq r+2$. This implies that

$$\begin{aligned}\gamma'_{stR}(T) = \omega(f) &= \sum_{i=1}^{r+2} f(N(x_iy_i)) + \sum_{i=1}^{r+2} f(x_iy_i) + \sum_{i=r+3}^{3r+1} f(xx_i)\\ &\geq -2r+1 = \frac{17-2|V(T)|}{5}.\end{aligned}$$

Thus $\gamma'_{stR}(T) = \frac{17-2|V(T)|}{5}$ and the proof is complete. ■

Next result is an immediate consequence of Lemma 10.

Corollary 11. *For every integer $r \geq 1$, there exists a connected graph G such that $\gamma'_{stR}(G) = 1-2r$.*

Theorem 12. *Let T be a tree of order $n \geq 4$. Then*

$$\gamma'_{stR}(T) \geq \frac{17-2n}{5},$$

with equality if and only if $T \in \mathcal{F}$.

Proof. The proof is by induction on n. If $\text{diam}(T) \leq 3$, then T is a star or a double star and we have $\gamma'_{stR}(T) > \frac{17-2n}{5}$ by Corollary 6 and Proposition 9. Hence the statement holds for all trees T with $\text{diam}(T) \leq 3$ as well as all trees of order $n = 4$. Assume T is an arbitrary tree of order $n \geq 5$ and $\text{diam}(T) \geq 4$. Let f be a $\gamma'_{stR}(T)$-function. We proceed further with a series of claims that we may assume satisfied by the tree T and the STREDF f.

Claim 1. *T has no non-pendant edge e with $f(e) = -1$.*

Proof. Assume $e = u_1u_2 \in E(T)$ is a non-pendant edge in T with $f(e) = -1$. Let $T - e = T_{u_1} \cup T_{u_2}$, where T_{u_i} is the component of $T - e$ containing u_i for $i = 1, 2$. Obviously, $\gamma'_{stR}(T) = f(E(T_{u_1})) - 1 + f(E(T_{u_2}))$ and the function f, restricted to T_{u_i}, is an STREDF and hence $\gamma'_{stR}(T_{u_i}) \leq f(E(T_{u_i}))$ for $i = 1, 2$. Clearly, $|V(T_{u_i})| \geq 3$ for each $i = 1, 2$. If $|V(T_{u_1})| = |V(T_{u_2})| = 3$, then $T_{u_1} = T_{u_2} = K_{1,2}$ and it is easy to verify that $\gamma'_{stR}(T) > \frac{17-2n}{5}$. Let $|V(T_{u_1})| \geq 4$. If $|V(T_{u_2})| = 3$, then $T_{u_2} = K_{1,2}$ and $f(E(T_{u_2})) \geq 2$. It follows from the induction hypothesis that $\gamma'_{stR}(T) \geq 2 + \frac{17-2(n-3)}{5} - 1 > \frac{17-2n}{5}$. Suppose $|V(T_{u_2})| \geq 4$. By the induction hypothesis we obtain

$$\gamma'_{stR}(T) \geq \gamma'_{stR}(T_{u_1}) + \gamma'_{stR}(T_{u_2}) - 1 \geq \frac{29-2n}{5} > \frac{17-2n}{5}.$$

□

By Claim 1 and the fact that f is an STREDF of T, we conclude that $f(v) \geq 0$ for each support vertex v and $f(v) \geq 2$ for each vertex v which is not a leaf or a support vertex.

Claim 2. *T has no two pendant edges vu_1 and vu_2 with $f(vu_1) = 1$ and $f(vu_2) = -1$.*

Proof. Let vu_1 and vu_2 be two pendant edges in T such that $f(vu_1) = 1$ and $f(vu_2) = -1$. Assume $T' = T - \{u_1, u_2\}$. If $|V(T')| \leq 3$, then it is easy to see that $\gamma'_{stR}(T) > \frac{17-2n}{5}$. Suppose $|V(T')| \geq 4$. Clearly, the function f, restricted to T', is an STREDF on T' and by the induction hypothesis we have

$$\gamma'_{stR}(T) \geq \gamma'_{stR}(T') \geq \frac{17 - 2(n-2)}{5} > \frac{17 - 2n}{5}. \qquad \square$$

Claim 3. *T has no two pendant edges vu_1 and vu_2 with $f(vu_1) = 2$ and $f(vu_2) = -1$.*

Proof. Let T have two pendant edges vu_1 and vu_2 with $f(vu_1) = 2$ and $f(vu_2) = -1$. Since T is not a star, we deduce from Claim 1 that there is a non-pendant edge vv' such that $f(vv') \geq 1$. If $f(vv') = 2$, then assume that $T' = T - \{u_1\}$ and define $g : E(T') \to \{-1, 1, 2\}$ by $g(vu_2) = 1$ and $g(e) = f(e)$ for $e \in E(T') - \{vu_2\}$. Obviously, g is an STREDF on T' of weight $\gamma'_{stR}(T)$ and by the induction hypothesis we have $\gamma'_{stR}(T) \geq \gamma'_{stR}(T') \geq \frac{17-2(n-1)}{5} > \frac{17-2n}{5}$. If $f(vv') = 1$, then assume that $T' = T - \{u_1, u_2\}$ and define $g : E(T') \to \{-1, 1, 2\}$ by $g(vv') = 2$ and $g(e) = f(e)$ for $e \in E(T') - \{vv'\}$. Obviously, g is an STREDF on T' of weight $\gamma'_{stR}(T)$ and by the induction hypothesis we have

$$\gamma'_{stR}(T) \geq \gamma'_{stR}(T') \geq \frac{17 - 2(n-2)}{5} > \frac{17 - 2n}{5}. \qquad \square$$

Claim 4. *T has no two pendant edges vu_1 and vu_2 with $f(vu_1) = f(vu_2) = 1$.*

Proof. Let vu_1 and vu_2 be two pendant edges in T such that $f(vu_1) = f(vu_2) = 1$. It follows from Claims 1 and 2 that there is no -1 edge at v. Assume $T' = T - \{u_1\}$ and define $g : E(T') \to \{-1, 1, 2\}$ by $g(vu_2) = 2$ and $g(e) = f(e)$ for $e \in E(T') - \{vu_2\}$. Clearly, g is an STREDF on T' of weight $\gamma'_{stR}(T)$ and by the induction hypothesis we have $\gamma'_{stR}(T) \geq \gamma'_{stR}(T') \geq \frac{17-2(n-1)}{5} > \frac{17-2n}{5}$. $\square$

We conclude from Claims 2, 3 and 4 that all pendant edges at a vertex are either -1 edges or positive edges. Choose a diametral path $v_1v_2 \cdots v_d$ in T to maximize $\deg_T(v_2)$ and root T at v_d. For $2 \leq i \leq d-1$, let $v_iu_i^1, v_iu_i^2, \ldots, v_iu_i^{r_i}$ be all pendant edges at v_i and $f(v_iu_i^1) \leq f(v_iu_i^2) \leq \cdots \leq f(v_iu_i^{r_i})$ and let s_i be the largest index such that $f(v_iu_i^{s_i}) = -1$. Then either $s_i = r_i$ or $s_i = 0$ for each i. We consider two cases.

Case 1. $s_2 = 0$. We consider three subcases as follows.

Subcase 1.1. $f(v_2u_2^1) = 2$ and $s_3 \geq 1$. Let $T' = T - \{u_2^1\}$ and define function f' by $f'(v_3u_3^1) = 1$ and $f'(x) = f(x)$ where $x \in E(T') - \{v_3u_3^1\}$. Then f' is an STREDF of T' with fewer vertices, and $\omega(f) = \omega(f') > \frac{17-2n}{5}$.

Subcase 1.2. $f(v_2u_2^1) = 1$ and $s_3 \geq 1$. Let $T' = T - \{u_2^1, u_3^1\}$ and $f' = f|_{T'}$. Then f' is an STREDF of T' with fewer vertices, and $\omega(f) = \omega(f') > \frac{17-2n}{5}$.

Subcase 1.3. $f(v_2u_2^1) \geq 1$ and $s_3 = 0$. Let $T' = T - \{u_2^1\}$ and $f' = f|_{T'}$. Then f' is an STREDF of T' with fewer vertices, and $\omega(f) = \omega(f') + f(v_2u_2^1) > \frac{17-2n}{5}$.

Case 2. $s_2 = r_2$. Since f is a $\gamma'_{stR}(T)$-function, we have $f(v) = \sum_{e\in E(v)} f(e) \geq 0$ for every support vertex v and so the case $s_2 = r_2 \geq 3$ is impossible. We consider four subcases.

Subcase 2.1. $s_2 = r_2 = 2$ and $s_3 \geq 2$. If $s_3 = 2$ or $f(v_3x) = 2$ for some $x \in N(v_3) - \{v_2\}$, then let $T' = T - \{u_2^1, u_2^2, v_2, u_3^1, u_3^2\}$. If $|V(T')| = 3$, then clearly $\gamma'_{stR}(T) > \frac{17-2n}{5}$. If $|V(T')| \geq 4$, then the function f, restricted to T' is an STREDF of T' of weight $\omega(f) + 2$ and by the induction hypothesis we have

$$\gamma'_{stR}(T) = \omega(f) \geq \omega(f|_{T'}) - 2 \geq \frac{17 - 2(n-5)}{5} - 2 = \frac{17 - 2n}{5}. \tag{3}$$

Let $s_3 \geq 3$ and $f(v_3x) \leq 1$ for each $x \in N(v_3) - \{v_2\}$. It follows from $f(N(v_2v_3)) \geq 1$ that $|N(v_3) - (\{u_3^i \mid 1 \leq i \leq s_3\} \cup \{v_2\})| \geq s_3 + 3$. Assume $x \in N(v_3) - (\{u_3^i \mid 1 \leq i \leq s_3\} \cup \{v_2, v_4\})$. Then $f(v_3x) = 1$ and x is a support vertex of degree 2 by Claim 4. Let x' be the leaf adjacent to x and let $T' = T - \{u_2^1, u_2^2, v_2, u_3^1, u_3^2, x'\}$. Define $h : E(T') \to \{-1, 1, 2\}$ by $h(v_3v_4) = 2$ and $h(e) = f(e)$ for $e \in E(T') - \{v_3v_4\}$. Obviously, h is an STREDF on T' of weight at most $\omega(f) + 2$ and it follows from the induction hypothesis that

$$\gamma'_{stR}(T) = \omega(f) \geq \omega(f|_{T'}) - 2 \geq \frac{17 - 2(n-6)}{5} - 2 > \frac{17 - 2n}{5}. \tag{4}$$

Subcase 2.2. $s_2 = r_2 = 1$ and $s_3 \geq 2$. If $s_3 = 2$ or $f(v_3x) = 2$ for some $x \in N(v_3) - \{v_2\}$, then let $T' = T - \{u_2^1, v_2, u_3^1, u_3^2\}$ and $f' = f|_{T'}$. Clearly, f' is an STREDF of T' of weight $\omega(f) + 1$ and we conclude from the induction hypothesis that $\omega(f) = \omega(f') - 1 > \frac{17-2n}{5}$. If $s_3 \geq 3$ and $f(v_3x) \leq 1$ for each $x \in N(v_3) - \{v_2\}$, then by similar argument as subcase 2.1, we obtain $\omega(f) > \frac{17-2n}{5}$.

Subcase 2.3. $s_2 = r_2 \leq 2$ and $s_3 = 1$. Since $f(N(v_2v_3)) \geq 1$, we must have $\deg(v_3) \geq 4$. It follows from Claims 2 and 3 that all neighbors of v_3, with exception of v_4 and u_3^1, are support vertices. By changing the value of f if necessary, we may assume, without loss of generality, that f assigns 2 to all edges at v_3 with exception $v_3u_3^1, v_3v_4$. Note that $f(v_4) \geq 0$ if $s_4 \geq 1$, and $f(v_4) \geq 2$ if

$s_4 = 0$. If $\deg(v_3) \geq 5$, then the function f, restricted to $T' = T - (T_{v_2} \cup \{u_3^1\})$, is an STREDF of T' of weight at most $\omega(f) + 1$ and by the induction hypothesis we have

$$\gamma'_{stR}(T) \geq \frac{17 - 2(n-4)}{5} - 1 > \frac{17 - 2n}{5}.$$

Let $\deg(v_3) = 4$. If $\operatorname{diam}(T) = 4$, then it is easy to verify that $\gamma'_{stR}(T) \geq \frac{17-2n}{5}$ with equality if and only if $T = T_1$ and so $T \in \mathcal{F}$. Let $\operatorname{diam}(T) \geq 5$. Assume $w \notin \{v_2, v_4\}$ is the support vertex adjacent to v_3. If $s_4 = 0$, then the function f, restricted to $T' = T - (T_{v_2} \cup T_w \cup \{u_3^1\})$, is an STREDF of T' of weight at most $\omega(f) + 1$ and by the induction hypothesis we have

$$\gamma'_{stR}(T) \geq \frac{17 - 2(n-7)}{5} - 1 > \frac{17 - 2n}{5}.$$

If $s_4 = 1$, then the function f, restricted to $T' = T - (T_{v_2} \cup T_w \cup \{u_3^1, u_4^1\})$, is an STREDF of T' of weight at most $\omega(f) + 2$ and as above we have $\gamma'_{stR}(T) > \frac{17-2n}{5}$. If $s_4 = 2$, then the function f, restricted to $T' = T - (T_{v_2} \cup T_w \cup \{u_3^1, u_4^1, u_4^2\})$, is an STREDF on T' of weight at most $\omega(f) + 3$ and by the induction hypothesis we have

$$\gamma'_{stR}(T) \geq \frac{17 - 2(n-9)}{5} - 3 > \frac{17 - 2n}{5}.$$

If $s_4 \geq 3$, then assume $T' = T - (T_{v_3} \cup \{u_4^1, u_4^2, u_4^3\})$. If $|V(T')| = 3$, then it is not hard to see that $\gamma'_{stR}(T) > \frac{17-2n}{5}$. If $|V(T')| \geq 4$, then define $h : E(T') \to \{-1, 1, 2\}$ by $h(v_4v_5) = 2$ and $h(e) = f(e)$ for $e \in E(T') - \{v_4v_5\}$. Obviously, h is an STREDF on T' of weight at most $\omega(f) + 2$ and it follows from the induction hypothesis that

$$\gamma'_{stR}(T) \geq \omega(h) - 2 \geq \frac{17 - 2(n-11)}{5} - 2 > \frac{17 - 2n}{5}.$$

Subcase 2.4. $s_2 = r_2 \leq 2$ and $s_3 = 0$. If $s_4 = 0$ and $\deg(v_3) \geq 4$, then the function f, restricted to $T' = T - T_{v_2}$, is an STREDF on T' of weight at most $\omega(f)$ and so $\gamma'_{stR}(T) > \frac{17-2n}{5}$. If $s_4 = 0$, $\deg(v_3) = 3$ and $w \in N(v_3) - \{v_2, v_4\}$, then the function f, restricted to $T' = T - (T_{v_2} \cup T_w)$, is an STREDF of T' of weight at most $\omega(f)$ and by the induction hypothesis we obtain $\gamma'_{stR}(T) > \frac{17-2n}{5}$.

Suppose that $s_4 = 1$. Let $T' = T - T_{v_3}$. If $|V(T')| = 3$, then it is easy to see that $\gamma'_{stR}(T) > \frac{17-2n}{5}$. If $|V(T')| \geq 4$, then define h on T' by $h(v_4u_4^1) = 1$ and $h(e) = f(e)$ for each $e \in E(T')$. It is easy to verify that h is an STREDF of T' of weight at most $\omega(f) + 1$ and by the induction hypothesis we have

$$\gamma'_{stR}(T) \geq \frac{17 - 2(n-7)}{5} - 1 > \frac{17 - 2n}{5}.$$

Now, assume $s_4 \geq 2$. First let $\deg(v_3) \geq 4$. By changing the values of f if necessary, we may assume, without loss of generality, that f assigns 2 to all non-pendant edges at v_3 with exception v_3v_4. If $\deg(v_3) \geq 5$ or $\deg(v_3) \geq 4$ and $f(v_3v_4) = 2$, then the function f, restricted to $T' = T - (T_{v_2} \cup \{u_4^1\})$, is an STREDF of T' of weight at most $\omega(f) + 1$ and by the induction hypothesis we have $\gamma'_{stR}(T) > \frac{17-2n}{5}$. Assume $\deg(v_3) = 4$ and $f(v_3v_4) = 1$. Then let $T' = T - T_{v_2}$ and define $h : E(T') \to \{-1, 1, 2\}$ by $f(v_3v_4) = 2$ and $h(e) = f(e)$ for $e \in E(T') - \{v_3v_4\}$. Clearly, h is an STREDF of T' of weight $\omega(f) + 1$ and it follows from the induction hypothesis that $\gamma'_{stR}(T) > \frac{17-2n}{5}$.

Let now $\deg(v_3) = 3$. If $\operatorname{diam}(T) = 4$, then it is not hard to verify that $\gamma'_{stR}(T) > \frac{17-2n}{5}$. Suppose $\operatorname{diam}(T) \geq 5$ and $T' = T - (T_{v_3} \cup \{u_4^1, u_4^2\})$. Then $|V(T')| \leq n - 7$. If $|V(T')| = 3$, then $T' = P_3$ and clearly $\gamma'_{stR}(T) > \frac{17-2n}{5}$. Assume $|V(T')| \geq 4$. Then the function f, restricted to T', is an STREDF of T' of weight at most $\omega(f) + 2$ and it follows from the induction hypothesis that $\gamma'_{stR}(T) = \omega(f) \geq \omega(f|_{T'}) - 2 \geq \frac{17-2(n-7)}{5} - 2 > \frac{17-2n}{5}$.

If $T \in \mathcal{F}$, then by Lemma 10 we have $\gamma'_{stR}(T) = \frac{17-2n}{5}$. Conversely, let $\gamma'_{stR}(T) = \frac{17-2n}{5}$. Regarding the proof, $T = T_1$ or T satisfies Subcase 2.1. It follows from (4) that $\gamma'_{stR}(T') = \frac{17-2(n-5)}{5}$ and $f|_{T'}$ is a $\gamma'_{stR}(T')$-function. By the induction hypothesis we deduce that $T' \in \mathcal{F}$ and so $T' = T_r$ for some positive integer r. If v_3 is not the central vertex of T', then $\sum_{e \in N(v_2v_3)} f(e) \leq 0$ which is a contradiction. Thus v_3 is the central vertex of T' which implies that $T = T_{r+1} \in \mathcal{F}$. This completes the proof. ■

3. General Bounds

In this section we present basic properties of $\gamma'_{stR}(G)$ and sharp bounds on the signed total Roman edge domination number of a graph.

Theorem 13. *If G is a graph of size m, maximum degree Δ and minimum degree δ, then*

$$\gamma'_{stR}(G) \geq \frac{m(2\delta - 1)}{2(\Delta - 1)} - m.$$

Proof. Let f be a $\gamma'_{stR}(G)$-function and define $g : E \to \{0, 2, 3\}$ by $g(e) = f(e)+1$ for each $e \in E$. We have

$$\begin{aligned}\sum_{e \in E} g(N(e)) &\geq \sum_{e=xy \in E} (f(N(e)) + \deg(x) + \deg(y) - 2) \\ &\geq 2m\delta + \sum_{e=xy \in E} (f(N(e)) - 2) \geq 2m\delta - m = m(2\delta - 1)\,.\end{aligned}$$

On the other hand,

$$\begin{aligned}\sum_{e\in E} g(N(e)) &= \sum_{e=xy\in E} (\deg(x)+\deg(y)-2)g(e)\\ &\le \sum_{e\in E}(2\Delta-2)g(e) = (2\Delta-2)g(E)\,.\end{aligned}$$

Thus $g(E) \ge \frac{m(2\delta-1)}{2\Delta-2}$. Since $f(E) = g(E) - m$, we have

$$\gamma'_{stR}(G) \ge \frac{m(2\delta-1)}{2(\Delta-1)} - m.$$

■

Corollary 14. *If G is an r-regular graph with $r \ge 2$ of order n, then $\gamma'_{stR}(G) \ge \frac{rn}{4(r-1)}$.*

The cycle C_{4t} demonstrates that Theorem 13 and Corollary 14 are sharp.

Example 15. Consider the complete graph K_4 with vertex set $\{v_1, v_2, v_3, v_4\}$. By Corollary 14, we have $\gamma'_{stR}(K_4) \ge 2$. Define the function $f : E(K_4) \to \{-1, 1, 2\}$ by $f(v_1v_2) = f(v_1v_3) = f(v_1v_4) = -1, f(v_2v_3) = 1$ and $f(v_2v_4) = f(v_3v_4) = 2$. Clearly, f is a signed total Roman edge dominating function of K_4 of weight 2 and so $\gamma'_{stR}(K_4) = 2$.

Applying Corollary 14, we present a so called Nordhaus-Gaddum type inequality for the signed total Roman edge domination number of regular graphs.

Theorem 16. *If G is an r-regular graph with $r \ge 2$ of order $n \ge 3$ such that G and $\overline{G}$ are connected and $r \le \frac{n-1}{2}$, then*

$$\gamma'_{stR}(G) + \gamma'_{stR}(\overline{G}) \ge \frac{rn}{n-3}\,.$$

If n is even, then

$$\gamma'_{stR}(G) + \gamma'_{stR}(\overline{G}) \ge \frac{rn}{n-2}\,.$$

Proof. Since G is r-regular, the complement $\overline{G}$ is $(n-r-1)$-regular. It follows from Corollary 14 that

$$\gamma'_{stR}(G) + \gamma'_{stR}(\overline{G}) \ge \frac{n}{4}\left(\frac{r}{r-1} + \frac{n-r-1}{n-r-2}\right).$$

Since $r \le \frac{n-1}{2}$, we have

$$\gamma'_{stR}(G) + \gamma'_{stR}(\overline{G}) \ge \frac{rn}{4}\left(\frac{1}{r-1} + \frac{1}{n-r-2}\right).$$

Since the function $f(x) = \frac{1}{x-1} + \frac{1}{n-x-2}$ gets its minimum at $x = \frac{n-1}{2}$ when $2 \leq x \leq n-3$, we obtain

$$\gamma'_{stR}(G) + \gamma'_{stR}(\overline{G}) \geq \frac{rn}{4}\left(\frac{1}{r-1} + \frac{1}{n-r-2}\right) \geq \frac{rn}{4}\left(\frac{2}{n-3} + \frac{2}{n-3}\right) = \frac{rn}{n-3},$$

as desired. If n is even, then the function f gets its minimum at $r = x = \frac{n-2}{2}$ or $r = x = \frac{n}{2}$, since r is an integer. Thus

$$\begin{aligned}\gamma'_{stR}(G) + \gamma'_{stR}(\overline{G}) &\geq \frac{rn}{4}\left(\frac{1}{r-1} + \frac{1}{n-r-2}\right) \geq \frac{rn}{4}\left(\frac{2}{n-4} + \frac{2}{n-2}\right) \\ &\geq \frac{rn}{4}\left(\frac{2}{n-2} + \frac{2}{n-2}\right) = \frac{rn}{n-2},\end{aligned}$$

and the proof is complete. ■

Theorem 17. *Let G be a graph of size m and minimum degree $\delta \geq 3$. Then*

$$\gamma'_{stR}(G) \leq m - 2\delta + 5.$$

Proof. Let $v \in V$ be a vertex, $t = \delta - 1$ and $u_1, u_2, \ldots, u_t \in N(v)$. Define $f : E \to \{-1, 1, 2\}$ by $f(vu_i) = -1$ for $1 \leq i \leq t-1$, $f(vu_t) = 2$ and $f(x) = 1$ otherwise. Then $f(vw) = -(t-1) + 2 + (\deg(v) - (t+1)) + \deg(w) - 1 \geq 2\delta - 2t + 1 > 1$ for $w \in N(v) - \{u_t\}$ and $f(vu_t) = -(t-1) + (\deg(v) - t) + \deg(u_t) - 1 \geq 2\delta - 2t > 1$. Let $e = wz$ such that $w, z \neq v$. If $\delta = 3$, then clearly $f(wz) \geq 2$. If $\delta \geq 4$, then $f(wz) \geq \deg(w) + \deg(z) - 6 \geq 2\delta - 6 > 1$. Therefore, f is an STREDF on G of weight $m - 2t + 3$ and so $\gamma'_{stR}(G) \leq m - 2t + 3 = m - 2\delta + 5$. ■

Theorem 18. *If G is a connected graph of size $m \geq 2$, then*

$$\gamma'_{stR}(G) \leq \min\left\{m, \frac{m + \gamma'_{st}(G)}{2}\right\}.$$

Proof. Obviously, $\gamma'_{stR}(G) \leq m$. Now let f be a $\gamma'_{st}(G)$-function, and let $P = \{e \in E \mid f(e) = 1\}$ and $M = \{e \in E \mid f(e) = -1\} = \{e_1, e_2, \ldots, e_{|M|}\}$. Suppose $e'_i \in P$ is an edge adjacent to e_i for each i. Define $g : E \to \{-1, 1, 2\}$ by $g(e'_i) = 2$ for $1 \leq i \leq |M|$ and $g(e) = f(e)$ otherwise. It is easy to see that g is an STREDF on G of weight at most $\gamma'_{st}(G) + |M|$. Moreover, since $\gamma'_{st}(G) = |P| - |M|$ and $m = |P| + |M|$, we have $|P| = \frac{m + \gamma'_{st}(G)}{2}$. Thus

$$\gamma'_{stR}(G) \leq \omega(g) \leq \gamma'_{st}(G) + |M| = |P| = \frac{m + \gamma'_{st}(G)}{2}.$$

■

Theorem 19. *Let $G \neq C_6$ be a graph of order $n \geq 5$. Then*

$$\gamma'_{stR}(G) \leq m - 1.$$

Proof. If $\delta(G) \geq 3$, then the result is immediate by Theorem 17. Henceforth, we assume $\delta(G) \leq 2$. Consider two cases.

Case 1. $\delta = 2$. If $\Delta = \delta = 2$, then $G = C_n$ and since $G \neq C_6$, we are done by Corollary 8. Let $\Delta \geq 3$ and u_0 be a vertex of maximum degree and let $P = u_0u_1 \cdots u_k$ be a longest path in G beginning at u_0. Let $w \in N(u_0) - \{u_1\}$. If either $\deg(u_1) \geq 3$ or $\deg(u_2) \geq 3$, then define $f : E \to \{-1, 1, 2\}$ by $f(u_0u_1) = -1$, $f(u_0w) = 2$ and $f(e) = 1$ otherwise. Let $\deg(u_1) = \deg(u_2) = 2$. If $k = 2$, then clearly $u_3 = u_0$ and define $f : E \to \{-1, 1, 2\}$ by $f(u_0u_1) = -1$, $f(u_0u_2) = 2$ and $f(e) = 1$ otherwise. If $k = 3$, then $u_4 = u_0$ and define $f : E \to \{-1, 1, 2\}$ by $f(u_0u_3) = f(u_3u_2) = 2$, $f(u_1u_2) = f(u_0u_1) = -1$ and $f(e) = 1$ otherwise. If $u_0 \neq u_3, u_4$, then define $f : E \to \{-1, 1, 2\}$ by $f(u_0u_1) = f(u_3u_4) = f(u_4u_5) = 2$, $f(u_1u_2) = f(u_2u_3) = -1$ and $f(e) = 1$ otherwise. It is easy to verify that, in all cases, f is an STREDF of G with weight at most $m - 1$.

Case 2. $\delta = 1$. Let $v_0 \in V$ be a vertex of minimum degree and $P = v_0v_1 \cdots v_k$ be a longest path in G beginning at v_0. If G is a path, then we are done by Corollary 7. Let $\deg(v_i) \geq 3$ for some i. If either $i = 1$ or $i = 2$, then define $f : E \to \{-1, 1, 2\}$ by $f(v_1v_0) = -1$, $f(v_2v_1) = 2$ and $f(e) = 1$ otherwise. It is easy to see that f is an STREDF of G with weight $m - 1$. Let $\deg(v_1) = \deg(v_2) = 2$. Note that since $n \geq 5$, we have $\deg(v_3) \geq 2$. Consider the following subcases.

Subcases 2.1. $\deg(v_4) \geq 3$. If $\deg(v_5) \geq 2$, then define $f : E \to \{-1, 1, 2\}$ by $f(v_0v_1) = f(v_3v_4) = -1$, $f(v_1v_2) = f(v_2v_3) = f(v_4v_5) = 2$ and $f(e) = 1$ otherwise. If $\deg(v_5) = 1$, then define $f : E \to \{-1, 1, 2\}$ by $f(v_0v_1) = f(v_4v_5) = -1$, $f(v_1v_2) = f(v_2v_3) = f(v_3v_4) = 2$ and $f(e) = 1$ otherwise. Obviously, in both cases, f is an STREDF of G with weight $m - 1$.

Subcase 2.2. $\deg(v_4) = 2$. If $\deg(v_5) \geq 2$, then define $f : E \to \{-1, 1, 2\}$ by $f(v_0v_1) = f(v_3v_4) = -1$, $f(v_1v_2) = f(v_2v_3) = f(v_6v_5) = 2$ and $f(e) = 1$ otherwise. If $\deg(v_5) = 1$, then define $f : E \to \{-1, 1, 2\}$ by $f(v_0v_1) = f(v_4v_5) = -1$, $f(v_1v_2) = f(v_2v_3) = f(v_4v_5) = 2$ and $f(e) = 1$ otherwise. Then f is an STREDF of G with weight $m - 1$.

Subcase 2.3. $\deg(v_4) = 1$. Then the function f defined by $f(v_0v_1) = f(v_3v_4) = -1$, $f(v_1v_2) = f(v_2v_3) = 2$ and $f(e) = 1$ otherwise, is an STREDF of G with weight $m - 1$. This completes the proof. ■

Theorem 20. *Let G be a connected graph of order $n \geq 3$ and size m. Then $\gamma_{stR}(G) = m$ if and only if $G \cong P_3, P_4, C_3, C_4, C_6$, or $K_{1,3}$.*

Proof. Let G be a connected graph of size $m \geq 2$ and let $\gamma_{stR}(G) = m$. By Theorem 19, either $n \leq 4$ or $G = C_6$ and by Theorem 17, $\delta \leq 2$. The case $G = C_6$ is obvious by Corollary 8. Let $n \leq 4$ and $\delta \leq 2$. If $\delta = 2$, we must have $G = C_3, C_4$ and $C_4 + e$. If $G = C_3, C_4$, we are done by Corollary 8. Let $G = C_4 + e$

and $V(C_4+e) = \{v_1, v_2, v_3, v_4\}$, where $e = v_1v_3$. Define $f : E(C_4+e) \to \{-1, 1, 2\}$ by $f(v_1v_3) = -1$, $f(v_2v_3) = 2$ and $f(x) = 1$ otherwise. Clearly, f is an STREDF of $C_4 + e$ with weight 4. Thus $G \neq C_4 + e$. Let $\delta = 1$. It is easy to see that the only graphs satisfying the conditions are P_3, P_4 or $K_{1,3}$. This completes the proof. ■

Theorem 21. *If G is a graph of size m, maximum edge-degree Δ_e and minimum edge-degree δ_e, then*

$$\gamma'_{stR}(G) \geq \frac{2 - \delta_e + \sqrt{(\delta_e - 2)^2 + 12m(\delta_e + 1)}}{3} - m.$$

Proof. Let $f = (E_{-1}, E_1, E_2)$ be a $\gamma'_{stR}(G)$-function. Define $P = E_1 \cup E_2$ and $|P| = p$. Then $\gamma'_{stR}(G) \geq 2p - m$. For any edge $e \in E$, by the definition of the signed total Roman edge domination number, we can easily verify the following inequality:

$$|N(e) \cap P| \geq \left\lceil \frac{\deg(e) + 1}{3} \right\rceil,$$

and hence

$$\sum_{e \in E_{-1}} |N(e) \cap P| \geq \frac{\deg(e) + 1}{3}(m - p) \geq \frac{\delta_e + 1}{3}(m - p).$$

So there exists at least one edge $e \in P$ such that e is adjacent to $\frac{(\delta_e+1)(m-p)}{3p}$ edges of E_{-1}. Hence

$$p - 1 \geq |N(e) \cap P| \geq 1 + \frac{(\delta_e + 1)(m - p)}{3p}.$$

By the above inequality, we deduce that

$$p \geq \frac{2 - \delta_e + \sqrt{(\delta_e - 2)^2 + 12m(\delta_e + 1)}}{6},$$

and so

$$\gamma'_{stR}(G) \geq 2p - m \geq \frac{2 - \delta_e + \sqrt{(\delta_e - 2)^2 + 12m(\delta_e + 1)}}{3} - m.$$ ■

4. Conclusion

In this paper, we introduce a new variant of the Roman domination problem, called the signed total Roman edge domination problem, on graphs. We show that for any tree T of order $n \geq 4$, $\gamma'_{stR}(G) \geq \frac{17-2n}{5}$ and classify all extreme trees. Moreover, we present some lower bounds for general graphs. As a further study, it is interesting to establish sharp upper bounds for this parameter and to determine the value of this parameter for some well-known classes of graphs. We conclude this paper with an open problem.

Problem. Prove or disprove: For any tree of order $n \geq 3$, $\gamma'_{stR}(T) \leq \lceil \frac{n+2}{2} \rceil$.

References

[1] H.A. Ahangar, J. Amjadi, S.M. Sheikholeslami, L. Volkmann and Y. Zhao, *Signed Roman edge domination numbers in graphs*, J. Comb. Optim. **31** (2016) 333–346. doi:10.1007/s10878-014-9747-8

[2] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of Domination in Graphs (Marcel Dekker, New York, 1998).

[3] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: Advanced Topics (Marcel Dekker, New York, 1998).

[4] H. Karami, A. Khodkar and S.M. Sheikholeslami, *Lower bounds on signed edge total domination numbers in graphs*, Czechoslovak Math. J. **58** (2008) 595–603. doi:10.1007/s10587-008-0038-3

[5] V.R. Kulli and D.K. Patwari, *On the edge domination number of a graph*, in: Proc. of the Symp. on Graph Theory and Combinatorics (Cochin, 1991), Centre Math. Sci. (Trivandrum, 1991) 75–81.

[6] W. Song, L. Miao, H. Wang and Y. Zhao, *Maximal matching and edge domination in complete multipartite graphs*, Int. J. Comput. Math. **91** (2014) 857–862. doi:10.1080/00207160.2013.818668

[7] L. Volkmann, *Signed total Roman domination in graphs*, J. Comb. Optim. **32** (2016) 855–871. doi:10.1007/s10878-015-9906-6

[8] D.B. West, Introduction to Graph Theory (Second Edition) (Prentice Hall, USA, 2001).

[9] B. Xu, *On signed edge total domination numbers of graphs*, J. East China Jiaotong Univ. **23** (2006) 129–131.

[10] J.F. Zhao and B.G. Xu, *On signed edge total domination numbers of graphs*, J. Math. Research & Exposition **31** (2011) 209–214.

Received 25 April 2016
Revised 27 August 2016
Accepted 27 August 2016