Discussiones Mathematicae
Graph Theory 37 (2017) 1055–1065
doi:10.7151/dmgt.1974

THE EXISTENCE OF $P_{\geq 3}$-FACTOR COVERED GRAPHS

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Abstract

A spanning subgraph F of a graph G is called a $P_{\geq 3}$-factor of G if every component of F is a path of order at least 3. A graph G is called a $P_{\geq 3}$-factor covered graph if G has a $P_{\geq 3}$-factor including e for any $e \in E(G)$. In this paper, we obtain three sufficient conditions for graphs to be $P_{\geq 3}$-factor covered graphs. Furthermore, it is shown that the results are sharp.

Keywords: $P_{\geq 3}$-factor, $P_{\geq 3}$-factor covered graph, toughness, isolated toughness, regular graph.

2010 Mathematics Subject Classification: 05C70, 05C38.

1. Introduction

The graphs considered in this paper are finite, undirected and simple. We denote by $G = (V(G), E(G))$ a graph, where $V(G)$ and $E(G)$ denote its vertex set and edge set respectively. For $x \in V(G)$, the degree of x in G is denoted by $d_G(x)$. For $S \subseteq V(G)$, we use $G - S$ to denote the subgraph obtained from G by deleting vertices in S together with edges incident to vertices in S. A set $S \subseteq V(G)$ is said to be independent if no two vertices in S are adjacent to each other. The number of isolated vertices of a graph G is denoted by $i(G)$. We use $\omega(G)$ to denote the number of components of a graph G. Other basic graph-theoretic terminologies can be found in [4].

A factor of a graph is a spanning subgraph of the graph. Especially, a (g, f)-factor of a graph G is defined as a spanning subgraph F such that $g(x) \leq d_F(x) \leq f(x)$ for each $x \in V(G)$, where $g(x)$ and $f(x)$ are two nonnegative integer-valued functions defined on $V(G)$ with $g(x) \leq f(x)$ for any $x \in V(G)$. If $g(x) = f(x) = k$ for any $x \in V(G)$, then a (g, f)-factor of G is called a k-factor. A 1-factor is also called a perfect matching. Since all of these notions concern the degree of vertices, they are often defined as *degree factors*. Degree factors in graphs attract a great deal of attentions [2, 7, 11, 13, 15, 16, 17].

On the other hand, when we focus on components of a factor, we lead to the notion of *component factors*. For a set $\mathcal{H}$ of connected graphs, an $\mathcal{H}$-factor of a graph G is a spanning subgraph F of G if every component of F is isomorphic to an element of $\mathcal{H}$. Especially, if each component of F is a path, then F is said to be a path-factor. Apparently, a 1-factor is a P_2-factor. A $P_{\geq k}$-factor means a path-factor in which every component path has at least k vertices, where $k \geq 2$. A graph G is defined as a $P_{\geq k}$-factor covered graph if G admits a $P_{\geq k}$-factor including e for any $e \in E(G)$.

Egawa, Fujita and Ota [6] studied the existence of $K_{1,3}$-factors in graphs. Kano, Lu and Yu [10] presented a sufficient condition for graphs to have $\{K_{1,2}, K_{1,3}, K_5\}$-factors. Kano and Saito [12] obtained a result on the existence of a $\{K_{1,l} : m \leq l \leq 2m\}$-factor and conjectured that a graph G satisfying $i(G-S) \leq \frac{|S|}{m}$ for each $S \subseteq V(G)$ actually contains a $(\{K_{1,l} : m \leq l \leq 2m-1\} \cup \{K_{2m+1}\})$-factor, where $m \geq 2$ is an integer. Zhang, Yan and Kano [18] proved that the conjecture above is true. Akiyama, Avis and Era [1] showed a necessary and sufficient condition for a graph to have a $P_{\geq 2}$-factor. Bazgan, Benhamdine, Li and Woźniak [3] posed a toughness condition for the existence of a $P_{\geq 3}$-factor in a graph. Kaneko [8] obtained a criterion for a graph to have a $P_{\geq 3}$-factor. A simpler proof was posed by Kano, Katona and Király [9]. Zhang and Zhou [19] gave a characterization for $P_{\geq 3}$-factor covered graphs.

A graph R is said to be factor-critical if $R - x$ includes a 1-factor (P_2-factor) for any $x \in V(R)$. A graph H is said to be a sun if $H = K_1$, $H = K_2$ or H is the corona of a factor-critical graph R with at least three vertices, i.e., H is obtained from R by adding a new vertex $w = w(v)$ together with a new edge vw for any $v \in V(R)$. A sun with at least six vertices is said to be a big sun. We use $sun(G)$ to denote the number of sun components of G.

Kaneko [8] presented a criterion for a graph to have a $P_{\geq 3}$-factor.

Theorem 1 (Kaneko [8]). *A graph G contains a $P_{\geq 3}$-factor if and only if $sun(G-S) \leq 2|S|$ for any subset S of $V(G)$.*

Zhang and Zhou [19] extended Theorem 1 to $P_{\geq 3}$-factor covered graphs and obtained a characterization for $P_{\geq 3}$-factor covered graphs.

Theorem 2 (Zhang and Zhou [19]). *Let G be a connected graph. Then G is a $P_{\geq 3}$-factor covered graph if and only if $sun(G-S) \leq 2|S| - \varepsilon(S)$ for any subset S of $V(G)$, where $\varepsilon(S)$ is defined by*

$$\varepsilon(S) = \begin{cases} 2, & if\ S \neq \emptyset \text{ and } S \text{ is not an independent set}, \\ 1, & if\ S \neq \emptyset,\ S \text{ is an independent set and there exists a} \\ & \text{non-sun component } of\ G - S, \\ 0, & otherwise. \end{cases}$$

In this paper, we proceed to investigate $P_{\geq 3}$-factor covered graphs and obtain some sufficient conditions for the existence of $P_{\geq 3}$-factor covered graphs. Our main results will be shown in Sections 2, 3 and 4, respectively.

2. TOUGHNESS AND $P_{\geq 3}$-FACTOR COVERED GRAPHS

The toughness $t(G)$ of a graph G was first defined by Chvátal in [5] as follows.

$$t(G) = \min\left\{\frac{|S|}{\omega(G-S)} : S \subseteq V(G), \omega(G-S) \geq 2\right\},$$

if G is not complete; otherwise, $t(G) = +\infty$. Bazgan, Benhamdine, Li and Woźniak [3] showed a toughness condition for the existence of a $P_{\geq 3}$-factor in a graph.

Theorem 3 (Bazgan, Benhamdine, Li and Woźniak [3]). *Let G be a graph with at least three vertices. If $t(G) \geq 1$, then G includes a $P_{\geq 3}$-factor.*

The following theorem is a generalization and improvement of Theorem 3.

Theorem 4. *Let G be a connected graph with at least three vertices. If $t(G) > \frac{2}{3}$, then G is a $P_{\geq 3}$-factor covered graph.*

Remark 5. The result in Theorem 4 is sharp. To see this, we construct a graph $G = K_2 \vee (H_1 \cup H_2 \cup H_3)$, where H_i is a sun for $1 \leq i \leq 3$. Set $S = V(K_2)$. It is easy to see that $sun(G-S) = \omega(G-S) = 3$ and $t(G) = \min\left\{\frac{|X|}{\omega(G-X)} : X \subseteq V(G), \omega(G-X) \geq 2\right\} = \frac{|S|}{\omega(G-S)} = \frac{2}{3}$. Note that $\varepsilon(S) = 2$. Hence, we obtain

$$sun(G-S) = 3 > 2 = 2|S| - \varepsilon(S).$$

In terms of Theorem 2, G is not a $P_{\geq 3}$-factor covered graph.

***Proof of Theorem* 4.** If G is a complete graph, obviously G is a $P_{\geq 3}$-factor covered graph as $|V(G)| \geq 3$. In the following, we assume that G is not a complete graph. Suppose that G satisfies the conditions of in Theorem 4, but it is not a $P_{\geq 3}$-factor covered graph. Then by Theorem 2, there exists a subset S of $V(G)$ such that

$$sun(G - S) > 2|S| - \varepsilon(S). \tag{1}$$

We shall consider three cases by the value of $|S|$ and derive a contradiction in each case.

Case 1. $|S| = 0$. In this case, we have $\varepsilon(S) = 0$. In terms of (1), we obtain

$$sun(G) > 0.$$

According to the integrity of $sun(G)$, we have

$$sun(G) \geq 1. \tag{2}$$

On the other hand, since G is connected, we obtain

$$sun(G) \leq \omega(G) = 1.$$

Combining this with (2), we have

$$sun(G) = \omega(G) = 1. \tag{3}$$

According to (3), $|V(G)| \geq 3$ and the definition of sun, it is easy to see that G is a big sun. We denote by R the factor-critical subgraph of G. For any $u \in V(R)$, we write $X = \{u\}$. Clearly, $\omega(G - X) \geq 2$. In terms of the definition of $t(G)$, we obtain

$$t(G) \leq \frac{|X|}{\omega(G - X)} \leq \frac{1}{2},$$

which contradicts $t(G) > \frac{2}{3}$.

Case 2. $|S| = 1$. In this case, we obtain $\varepsilon(S) \leq 1$. According to (1), we have

$$sun(G - S) > 2|S| - \varepsilon(S) \geq 2 - 1 = 1.$$

In terms of the integrity of $sun(G - S)$, we obtain

$$sun(G - S) \geq 2.$$

Note that $\omega(G - S) \geq sun(G - S)$. Combining this with $t(G) > \frac{2}{3}$, we have

$$\frac{2}{3} < t(G) \leq \frac{|S|}{\omega(G - S)} \leq \frac{|S|}{sun(G - S)} \leq \frac{1}{2},$$

which is a contradiction.

Case 3. $|S| \geq 2$. Note that $\varepsilon(S) \leq 2$. It follows from (1) that

$$sun(G-S) \geq 2|S| - \varepsilon(S) + 1 \geq 2|S| - 1,$$

which implies

$$|S| \leq \frac{sun(G-S)+1}{2} \tag{4}$$

and

$$sun(G-S) \geq 3. \tag{5}$$

In terms of (4), (5), $\omega(G-S) \geq sun(G-S)$ and the definition of $t(G)$, we obtain

$$\begin{aligned} t(G) &\leq \frac{|S|}{\omega(G-S)} \leq \frac{|S|}{sun(G-S)} \leq \frac{sun(G-S)+1}{2sun(G-S)} = \frac{1}{2} + \frac{1}{2sun(G-S)} \\ &\leq \frac{1}{2} + \frac{1}{6} = \frac{2}{3}, \end{aligned}$$

which contradicts $t(G) > \frac{2}{3}$. Theorem 4 is proved. ■

3. Isolated Toughness and $P_{\geq 3}$-Factor Covered Graphs

Yang, Ma and Liu [14] introduced a new parameter, isolated toughness of a graph G, denoted by $I(G)$, which is defined as

$$I(G) = \min\left\{\frac{|S|}{i(G-S)} : S \subseteq V(G), i(G-S) \geq 2\right\},$$

if G is not complete; otherwise, $I(G) = +\infty$. In the following, we investigate the relationship between isolated toughness and $P_{\geq 3}$-factor covered graphs, and obtain an isolated toughness condition for the existence of $P_{\geq 3}$-factor covered graphs. Our main result is the following theorem.

Theorem 6. *Let G be a connected graph with at least three vertices. If $I(G) > \frac{5}{3}$, then G is a $P_{\geq 3}$-factor covered graph.*

Remark 7. Let us show that $I(G) > \frac{5}{3}$ in Theorem 6 cannot be replaced by $I(G) \geq \frac{5}{3}$. We show this by constructing a graph $G = K_2 \vee (3K_2)$. It is easy to see that $I(G) = \frac{5}{3}$. Set $S = V(K_2)$, and so $|S| = 2$. Then by the definition of $\varepsilon(S)$, we obtain $\varepsilon(S) = 2$. Hence, we obtain

$$sun(G-S) = 3 > 2 = 2|S| - \varepsilon(S).$$

In terms of Theorem 2, G is not a $P_{\geq 3}$-factor covered graph.

***Proof of Theorem* 6.** If G is complete, obviously G is a $P_{\geq 3}$-factor covered graph as $|V(G)| \geq 3$. In the following, we assume that G is not complete. Suppose that G satisfies the hypothesis of Theorem 6, but it is not a $P_{\geq 3}$-factor covered graph. Then by Theorem 2, there exists a subset S of $V(G)$ satisfying

$$sun(G-S) \geq 2|S| - \varepsilon(S) + 1. \tag{6}$$

We shall consider three cases by the value of $|S|$ and derive a contradiction in each case.

Case 1. $|S| = 0$. According to the definition of $\varepsilon(S)$, we have $\varepsilon(S) = 0$. Combining this with (6), we obtain

$$sun(G) \geq 1. \tag{7}$$

Note that since $sun(G) \leq \omega(G)$ and G is connected, we have

$$sun(G) \leq \omega(G) = 1. \tag{8}$$

It follows from (7) and (8) that

$$sun(G) = \omega(G) = 1. \tag{9}$$

By (9), $|V(G)| \geq 3$ and the definition of sun, it is easy to see that G is a big sun. We use R to denote the factor-critical subgraph of G and set $U = V(R)$. Apparently, $i(G-U) = |U| \geq 3$. Then by $I(G) > \frac{5}{3}$ and the definition of $I(G)$, we have

$$\frac{5}{3} < I(G) \leq \frac{|U|}{i(G-U)} = 1,$$

which is a contradiction.

Case 2. $|S| = 1$. Clearly, $\varepsilon(S) \leq 1$. In terms of (6), we obtain

$$sun(G-S) \geq 2|S| - \varepsilon(S) + 1 \geq 2. \tag{10}$$

Assume that there exist a isolated vertices, b K_2's and c big sun components $H_1, H_2, \ldots, H_c$, where $|V(H_i)| \geq 6$, in $G-S$. Thus, it follows from (10) that

$$sun(G-S) = a + b + c \geq 2. \tag{11}$$

We choose one vertex from every K_2 component of $G-S$, and use X to denote the set of such vertices. For every H_i, we denote the factor-critical subgraph of H_i by R_i. We choose one vertex $y_i \in V(R_i)$ for $1 \leq i \leq c$, and write $Y = \{y_1, y_2, \ldots, y_c\}$. Apparently, we obtain

$$i(G - (S \cup X \cup Y)) = a + b + c \geq 2.$$

In terms of (6), (11), the definition of $I(G)$, $\varepsilon(S) \leq 1$ and $I(G) > \frac{5}{3}$, we have

$$\frac{5}{3} < I(G) \leq \frac{|S \cup X \cup Y|}{i(G-(S \cup X \cup Y))} = \frac{|S|+b+c}{a+b+c} = \frac{|S|+sun(G-S)-a}{sun(G-S)}$$
$$\leq \frac{|S|+sun(G-S)}{sun(G-S)} \leq \frac{\frac{sun(G-S)+\varepsilon(S)-1}{2}+sun(G-S)}{sun(G-S)} \leq \frac{3}{2},$$

which is a contradiction.

Case 3. $|S| \geq 2$. Note that $\varepsilon(S) \leq 2$. Combining this with (6), we obtain

$$sun(G-S) \geq 2|S| - \varepsilon(S) + 1 \geq 2|S| - 1 \geq 3. \tag{12}$$

Assume that there exist a isolated vertices, b K_2's and c big sun components $H_1, H_2, \ldots, H_c$, where $|V(H_i)| \geq 6$, in $G-S$. Thus, we have $sun(G-S) = a+b+c$. We choose one vertex from each K_2 component of $G-S$, and denote the set of such vertices by X. We use R_i to denote the factor-critical subgraph of H_i for each H_i, and set $Y_i = V(R_i)$. Obviously, $|X| = b$ and $i(H_i - Y_i) = |Y_i| = \frac{|V(H_i)|}{2}$. Put $Y = \bigcup_{i=1}^{c} Y_i$. Then by (12) we obtain

$$i(G-(S \cup X \cup Y)) = a+b+\sum_{i=1}^{c}|Y_i| = a+b+\sum_{i=1}^{c}\frac{|V(H_i)|}{2}$$
$$\geq a+b+c = sun(G-S) \geq 3.$$

Combining this with $I(G) > \frac{5}{3}$ and the definition of $I(G)$, we have

$$\frac{5}{3} < I(G) \leq \frac{|S \cup X \cup Y|}{i(G-(S \cup X \cup Y))} = \frac{|S|+b+\sum_{i=1}^{c}\frac{|V(H_i)|}{2}}{a+b+\sum_{i=1}^{c}\frac{|V(H_i)|}{2}}$$

that is,

$$3|S| > 5a + 2b + 2\sum_{i=1}^{c}\frac{|V(H_i)|}{2}. \tag{13}$$

Note that $|V(H_i)| \geq 6$ and $sun(G-S) = a+b+c$. According to (12) and (13), we have

$$3|S| > 5a+2b+2\sum_{i=1}^{c}\frac{|V(H_i)|}{2} \geq 5a+2b+6c$$
$$\geq 2(a+b+c) = 2sun(G-S) \geq 2(2|S|-1),$$

which implies

$$|S| < 2,$$

which contradicts $|S| \geq 2$. This completes the proof of Theorem 6. ■

4. Regular Graphs and $P_{\geq 3}$-Factor Covered Graphs

Kaneko [8] showed a condition for a regular graph to have a $P_{\geq 3}$-factor.

Theorem 8 (Kaneko [8])**.** *Every regular graph G with degree $r \geq 2$ admits a $P_{\geq 3}$-factor.*

In this section, we mainly study the relationship between regular graphs and $P_{\geq 3}$-factor covered graphs, and obtain a sufficient condition for a regular graph to be a $P_{\geq 3}$-factor covered graph. Our main result is shown in the following, and it is an improvement of Theorem 8.

Theorem 9. *Every regular graph G with degree $r \geq 2$ is a $P_{\geq 3}$-factor covered graph.*

Proof. Without loss of generality, we may assume that G is connected. Otherwise, we consider each connected component of G.

Suppose that G is not a $P_{\geq 3}$-factor covered graph. Then by Theorem 2, there exists a subset S of $V(G)$ satisfying

$$sun(G - S) \geq 2|S| - \varepsilon(S) + 1. \tag{14}$$

Claim 1. $S \neq \emptyset$.

Proof. If $S = \emptyset$, then $\varepsilon(S) = 0$. By (14), we have

$$sun(G) \geq 1.$$

On the other hand, G is connected, and so $sun(G) \leq \omega(G) \leq 1$. Thus, we obtain

$$sun(G) = 1.$$

Obviously, G itself is a sun. Note that $r \geq 2$. Hence, $G \neq K_1$ and $G \neq K_2$. Thus, G is a big sun, which contradicts that G is a regular graph with degree $r \geq 2$. This completes the proof of Claim 1. □

Claim 2. $sun(G - S) \geq 2$.

Proof. According to Claim 1, we have $|S| \geq 1$.

If $|S| = 1$, then $\varepsilon(S) \leq 1$. It follows from (14) that

$$sun(G - S) \geq 2|S| - \varepsilon(S) + 1 \geq 2|S| = 2.$$

In the following, we consider $|S| \geq 2$. In this case, $\varepsilon(S) \leq 2$. Then by (14), we obtain

$$sun(G - S) \geq 2|S| - \varepsilon(S) + 1 \geq 2|S| - 1 \geq 3 > 2.$$

This completes the proof of Claim 2. □

In the following, we assume that there exist a isolated vertices, b K_2's and c big sun components $H_1, H_2, \ldots, H_c$, where $|V(H_i)| \geq 6$, in $G - S$. In terms of Claim 2, we have

$$sun(G - S) = a + b + c \geq 2. \tag{15}$$

For any $x \in V(bK_2)$, the degree of x in bK_2 is 1. For each H_i, H_i has at least three vertices of degree exactly one. Note that G is a regular graph with degree $r \geq 2$. Thus, we obtain

$$ar + 2b(r - 1) + 3c(r - 1) \leq r|S|.$$

Combining this with (14), (15) and $\varepsilon(S) \leq 2$, we have

$$\begin{aligned} ar + 2b(r - 1) + 3c(r - 1) \; &\leq \; r|S| \leq \frac{r}{2}\left(sun(G - S) + \varepsilon(S) - 1\right) \\ &\leq \; \frac{r}{2}\left(sun(G - S) + 1\right) = \frac{r}{2}(a + b + c + 1), \end{aligned}$$

that is,

$$ar + 3br + 5cr - r \leq 4b + 6c. \tag{16}$$

It follows from (15), (16) and $r \geq 2$ that

$$\begin{aligned} 2a + 6b + 10c - 2 \; &= \; 2(a + 3b + 5c - 1) \leq r(a + 3b + 5c - 1) \\ &= \; ar + 3br + 5cr - r \leq 4b + 6c, \end{aligned}$$

which implies

$$a + b + 2c \leq 1.$$

Note that $c \geq 0$. Hence, we obtain

$$a + b + c \leq 1,$$

which contradicts (15). The proof of Theorem 9 is complete. ■

Acknowledgements

The authors would like to thank the anonymous referees for their comments on this paper. This work is supported by the National Natural Science Foundation of China (Grant No. 11371009, 11501256, 61503160) and the National Social Science Foundation of China (Grant No. 14AGL001), and sponsored by 333 Project of Jiangsu Province.

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Received 27 October 2015
Revised 27 August 2016
Accepted 27 August 2016