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A NOTE ON THE LOCATING-TOTAL DOMINATION IN GRAPHS

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Abstract

In this paper we obtain a sharp (improved) lower bound on the locating-total domination number of a graph, and show that the decision problem for the locating-total domination is NP-complete.

Keywords: dominating set, total dominating set, locating-dominating set, locating-total dominating set, regular graphs.

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1. Introduction

A set S of vertices in a graph G is called a *dominating set* of G if every vertex in $V(G) \setminus S$ is adjacent to some vertex in S. The set S is said to be a *total dominating set* of G if every vertex in $V(G)$ is adjacent to some vertex in S. The domination problem is to determine the minimum cardinality of all *dominating sets* in G. Similarly, the total domination problem is the problem of determining the minimum cardinality of such sets in G. A *locating-dominating set* in a connected graph G is a dominating set S of G such that for every pair of vertices u and v in $V(G) \setminus S$, $N(u) \cap S \neq N(v) \cap S$. The minimum cardinality of a locating-dominating set of G is the *locating-domination number* $\gamma^L(G)$ [6]. A *locating-total dominating set* in a connected graph G is a total dominating set S of G such that for every pair of vertices u and v in $V(G) \setminus S$, $N(u) \cap S \neq N(v) \cap S$. The minimum cardinality of a locating total-dominating set of G is the *locating-total domination number* $\gamma_t^L(G)$ [6]. Determining if an arbitrary graph has a dominating set and locating-dominating set of a given size are well-known NP-complete problems [1, 5].

Total domination plays a role in the problem of placing monitoring devices in a system in such a way that every site in the system, including the monitors, is adjacent to a monitor site so that, if a monitor goes down, then an adjacent monitor can still protect the system. Installing the minimum number of expensive sensors in the system which will transmit a signal at the detection of faults and uniquely determine the location of the faults motivates the concept of locating-dominating sets and locating-total dominating sets [6].

The locating-total domination problem has been discussed for trees [2, 3], cubic graphs and grid graphs [8], corona and composition of graphs [10], claw-free cubic graphs [7], and so on.

The paper is organized as follows. In Section 2, we obtain an improved bound for locating-total domination of regular graphs. Further we prove that the bound is tight for certain families of regular graphs. In Section 3, we prove that the locating-total domination problem is NP-complete.

2. Lower Bound for the Locating-Total Domination Number

All graphs considered in this paper are simple and connected.

Let $G = (V, E)$ be a graph and $S \subseteq V(G)$, a dominating set of G. By the *shadow* of a vertex $u \in V(G)$ on S, we mean the set $S_u = S \cap N[u]$ where $N[u] = N(u) \cup \{u\}$. The *profile* of $u \in V(G)$ is defined to be the $(d_G(u)+1)$-tuple $\pi(u)$ with entries $|S_x|$ where $x \in N[u]$, in ascending order. The *share* of a vertex

$u \in S$ in S is defined by

$$\gamma(u, S) = \sum_{x \in N[u]} \frac{1}{|S_x|}.$$

When the set S is clear from the context, we refer to $\gamma(u, S)$ simply as the share of u and denote it by $\gamma(u)$.

The following lemma is a powerful tool in obtaining lower bounds on various flavors of domination numbers. This result was given in [11].

Lemma 2.1 [11]**.** *Let G be a graph of order n and let S be a dominating set of G. Then $\sum_{u \in S} \gamma(u) = n$.*

In what follows, we give an improved lower bound for $\gamma_t^L(G)$ when G is regular.

2.1. Improved lower bound for regular graphs

Henning *et al.* [8] have proved that the locating-total domination number for a graph G satisfies the inequalities $\gamma_t^L(G) \geq \lfloor \log_2 n \rfloor$ and $\gamma_t^L(G) \geq (diameter(G) + 1)/2$.

In this section, we have obtained an improved lower bound for the locating-total domination number for regular graphs. For proving the main result, we need the following.

Lemma 2.2. *Let S be a locating-total dominating set of a k-regular graph G of order n, for some positive integer $k \geq 2$. Then $\gamma(u) \leq \frac{k+2}{2}$, for each $u \in S$.*

Proof. Let $u \in S$. Since S is a total dominating set, at least one vertex v in $N(u)$ belongs to S. Now for any two distinct vertices x and y of $N[u]$ we claim that $|S_x| = |S_y| = 1$ is not possible. For, if $|S_x| = |S_y| = 1$, then $N(x) \cap S = N(y) \cap S$, a contradiction. Therefore $|S_x| = 1$ for at most one vertex x of $N[u]$. For all vertices $y \neq x$ in $N[u]$, $|S_y| \geq 2$. Hence for all vertices $y \neq x$ in $N[u]$, $\frac{1}{|S_y|} \leq \frac{1}{2}$. Thus we have $\gamma(u) = \sum_{w \in N[u]} \frac{1}{|S_w|} \leq 1 + k(\frac{1}{2}) = \frac{k+2}{2}$. ■

Theorem 2.3. *Let G be a k-regular graph of order n. Then $\gamma_t^L(G) \geq \left\lceil \frac{2n}{k+2} \right\rceil$.*

Proof. Let S be a locating-total dominating set of G. By Lemma 2.2, we have $\gamma(u) \leq \frac{k+2}{2}$, for all $u \in S$. By Lemma 2.1, $n = \sum_{u \in S} \gamma(u) \leq \frac{k+2}{2} |S|$. Therefore $|S| \geq \left\lceil \frac{2n}{k+2} \right\rceil$. ■

Remark 1. For a given k, there exists an integer n, n large, such that $\left\lceil \frac{2n}{k+2} \right\rceil > \lfloor \log_2 n \rfloor$. Such a pair of numbers is denoted by $n(k)$. Thus our bound obtained in Theorem 2.3 is better than the bound obtained by Henning *et al.* [8].

In the sequel we prove that the lower bound obtained in Theorem 2.3 is sharp for extended cycle-of-ladders and circulant networks. Without loss of generality we refer to the vertices in these graphs by their labels.

2.2. γ_t^L of extended cycle-of-ladder $ECL(2l, s)$

In [4], Fang introduced a network called cycle-of-ladder and proved that it is a spanning subgraph of the hypercube network, thereby proving that hypercube network is bipancyclic. In this section, we derive a new network from cycle-of-ladder and call it the extended cycle-of-ladder network.

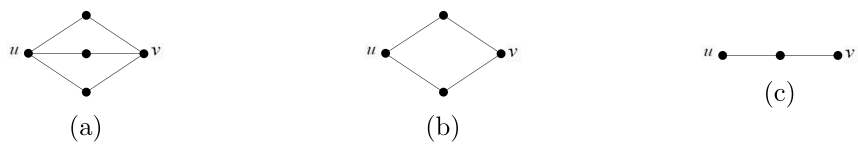


Figure 1. Illustrates the proof of Proposition 2.

Definition [9]**.** The n-ladder graph L of length n is defined as $P_2 \times P_{n+1}$, where P_{n+1} is a path on $n+1$ vertices, $n \geq 1$.

The graph obtained via this definition has the advantage of looking like a ladder having two rails and $n+1$ rungs between them. The length of the ladder is defined as n.

Definition [4]**.** A cycle-of-ladder is a graph containing a cycle C_b of length $2l$ called the bone cycle and l ladders $L_1, L_2, \ldots, L_l$ with $R_b(1), R_b(2), \ldots, R_b(l)$ as the bottom rungs such that $R_b(i)$'s are respectively the alternate edges in C_b, $1 \leq i \leq l$. We denote the cycle-of-ladder as $CL(2l, s)$, where l and s represent the number of ladders and the length of each ladder, respectively.

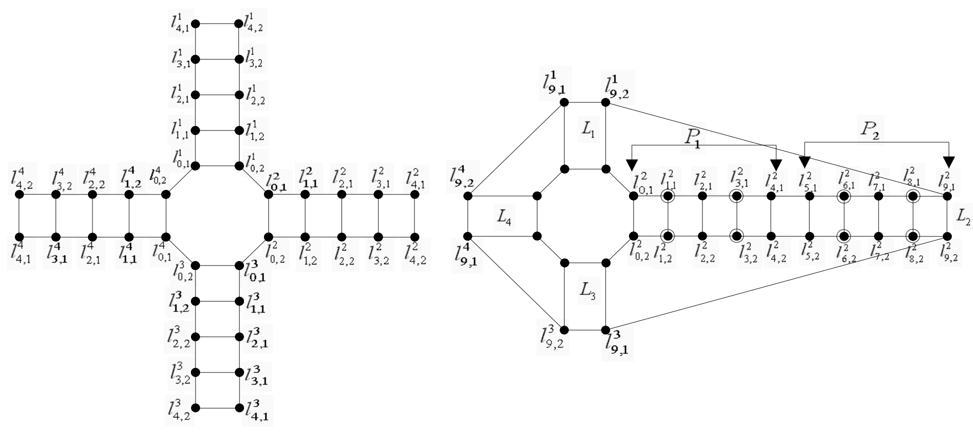
For convenience we label the vertices of L_i as $l_{j,1}^i$ and $l_{j,2}^i$ where $0 \leq j \leq s$ and $1 \leq i \leq l$ in $CL(2l, s)$. Figure 2(a) illustrates $(l_{0,1}^1, l_{0,2}^1, l_{0,1}^2, l_{0,2}^2, l_{0,1}^3, l_{0,2}^3, l_{0,1}^4, l_{0,2}^4, l_{0,1}^1)$ as the bone cycle and the edges $(l_{0,1}^1, l_{0,2}^1), (l_{0,1}^2, l_{0,2}^2), (l_{0,1}^3, l_{0,2}^3), (l_{0,1}^4, l_{0,2}^4)$ as $R_b(1), R_b(2), R_b(3)$ and $R_b(4)$, respectively.

We add l number of edges to $CL(2l, s)$ to obtain a 3-regular graph and call it the extended cycle-of-ladder $ECL(2l, s)$.

Definition. The extended cycle-of-ladder $ECL(2l, s)$ is obtained from $CL(2l, s)$ by adding edges between $l_{s,2}^i$ and $l_{s,1}^{i+1}$, where $1 \leq i \leq l-1$, and between $l_{s,2}^l$ and $l_{s,1}^1$.

Proposition 2. *Let G be an extended cycle-of-ladder $ECL(2l, s)$ with $l \equiv 0 \pmod 2$ and $s \equiv 4 \pmod 5$. Then $\gamma_t^L(ECL(2l, s)) = 4l(s+1)/5$.*

Proof. Label the vertices of $L(i)$ as $l^i_{j,1}$ and $l^i_{j,2}$ where $0 \le j \le s$ and $1 \le i \le l$ in $ECL(2l, s)$. See Figure 2(b). Since $s \equiv 4 \pmod 5$, $s + 1$ is a multiple of 5. We have $s + 1$ rungs in each ladder L_i, $1 \le i \le l$. Partition the $s + 1$ rungs into sets $P_1, P_2, \ldots, P_{(s+1)/5}$ of five consecutive rungs beginning from the bottom rung in each ladder. Let S contain the vertices in the second and fourth rungs of each partition. In other words, $S = \bigcup_{1\le i\le\lceil s/5\rceil} \bigcup_{1\le j\le l/2} \left\{ l^{2j-1}_{5i-4,1}, l^{2j-1}_{5i-2,1}, l^{2j-1}_{5i-4,2}, l^{2j-1}_{5i-2,2}, l^{2j}_{5i-4,1}, l^{2j}_{5i-2,1}, l^{2j}_{5i-4,2}, l^{2j}_{5i-2,2} \right\}$. We claim that S is a minimum locating-total dominating set of $ECL(2l, s)$. Clearly S is a total dominating set. We have only to prove that S is a locating-total dominating set of $ECL(2l, s)$. Let $u, v \in V \setminus S$. If u and v are in different ladders, then $N(u) \cap S \ne N(v) \cap S$. Suppose u and v are in the same ladder, say L. Suppose $N(u) \cap S = N(v) \cap S$. If $|N(u) \cap S| = |N(v) \cap S| = 3$, then u, v and the three vertices adjacent to both u and v induce a subgraph shown in Figure 1(a), which is not possible by the definition of extended cycle-of-ladder. If $|N(u) \cap S| = |N(v) \cap S| = 2$, then u, v and the two vertices adjacent to both u and v induce a subgraph shown in Figure 1(b), which is not possible by the choice of S. Now $|N(u) \cap S| = |N(v) \cap S| = 1$ is not possible (see Figure 1(c)), since at least one of u, v has two vertices of S adjacent to it, contradicting $N(u) \cap S = N(v) \cap S$. Thus S is a locating-total dominating set in $ECL(2l, s)$. Now $|S| = 8(\lceil s/5\rceil)(l/2) = 4l(s + 1)/5$. By Theorem 2.3, $\gamma^L_t(ECL(2l, s)) = 4l(s + 1)/5$. ■



(a) (b)

Figure 2. (a) $CL(8, 4)$.
(b) Vertices in a locating-total dominating set of $ECL(8, 9)$ are circled.

2.3. γ_t^L of circulant graph $G(n, \pm\{1,2\})$

Definition [12]**.** The undirected circulant graph $G(n, \pm S)$, where $S \subseteq \{1, 2, \ldots, j\}$, $1 \leq j \leq \lfloor n/2 \rfloor$, the vertex set $V = \{0, 1, \ldots, n-1\}$ and the edge set $E = \{(i,k) : |k-i| \equiv s (\text{mod } n), s \in S\}$.

For brevity, we use the label $0, 1, 2, \ldots, n-1$ as $1, 2, \ldots, n$ in $G(n, \pm S)$.

Proposition 3. *Let G be a circulant graph $G(n, \pm\{1,2\})$ where $n \geq 7$. Then $\gamma_t^L(G(n, \pm\{1,2\})) = \lceil n/3 \rceil$ if $n \equiv 0, 1, 2, 4$ (mod 6).*

Proof. Label the vertices of $G(n, \pm\{1,2\})$ from 1 to n, sequentially with clockwise sense. We begin with the case when $n \equiv 0$ (mod 6), where all labels are taken modulo n. Let $S = \bigcup_{1 \leq k \leq n/6} \{n-6k+3, n-6k+1\}$, $1 \leq k \leq n/6$. We claim that S is a locating-total dominating set of $G(n, \pm\{1,2\})$. Let $N_{V \setminus S}(S)$ denote the set of all neighborhood in $V \setminus S$ of members of S. For $1 \leq k \leq n/6$, it is easy to see that $N_{V \setminus S}(S) = N(S) \cap V \setminus S = V \setminus S$.

Moreover $(n-6k+3, n-6k+1)$ is an edge in $G(n, \pm\{1,2\})$. Therefore S is a total dominating set in $G(n, \pm\{1,2\})$. We have only to show that S is a locating-total dominating set. For $1 \leq k \leq n/6$, $N(n-6k+2) \cap S = \{n-6k+3, n-6k+1\}$, $N(n-6k+4) \cap S = \{n-6k+3\}$, $\mathrm{N}(n-6k+5) \cap S = \{n-6k+3, n-6k+7\}$ and $N(n-6k+6) \cap S = \{n-6k+7\}$, which are all distinct. Now $|S| = 2(n/6) = \lceil n/3 \rceil$. See Figure 3(a). By Theorem 2.3, $\gamma_t^L(G(n, \pm\{1,2\})) = 2n/(k+2) = 2n/(4+2) = n/3$, when $n \equiv 0$ (mod 6).

When $n \equiv 1, 2$ (mod 6), $S = \bigcup_{1 \leq k \leq n/6} \{n-6k+3, n-6k+1\} \cup \{1\}$, $1 \leq k \leq \lfloor n/6 \rfloor$; and when $n \equiv 4$ (mod 6), $S = \bigcup_{1 \leq k \leq n/6} \{n-6k+3, n-6k+1\} \cup \{1, 3\}$, $1 \leq k \leq \lfloor n/6 \rfloor$ are respectively the minimum locating-total dominating sets in $G(n, \pm\{1,2\})$. Thus by Theorem 2.3, $\gamma_t^L(G(n, \pm\{1,2\})) = \lceil n/2 \rceil$, when $n \equiv 0, 1, 2, 4$ (mod 6). See Figure 3(b). ■

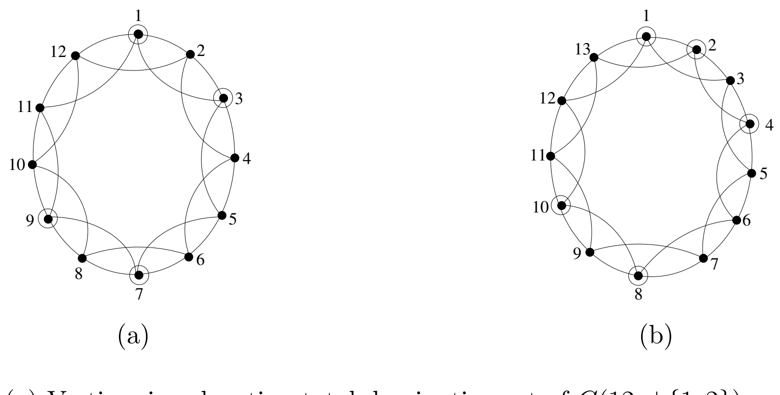


Figure 3. (a) Vertices in a locating-total dominating set of $G(12, \pm\{1,2\})$ and (b) vertices in a locating-total dominating set of $G(13, \pm\{1,2\})$ are circled.

3. Locating-Total Domination Problem is NP-Complete

The locating-domination problem and locating-total domination problem are not equivalent. In other words, it is not possible to derive a minimum locating-dominating set from a minimum locating-total dominating set and vice-versa. For example, consider the graph G shown in Figure 4. In G the minimum locating-dominating set $T = \{2, 5, 7\}$ and hence $\gamma^L(G) = 3$ (see Figure 4(a)). Now, in G the minimum locating-total dominating set $S = \{2, 3, 7, 8\}$ and hence $\gamma_t^L(G) = 4$ (see Figure 4(b)). Locating-domination problem is NP-complete [1]. In this section we prove locating-total domination problem is NP-complete.

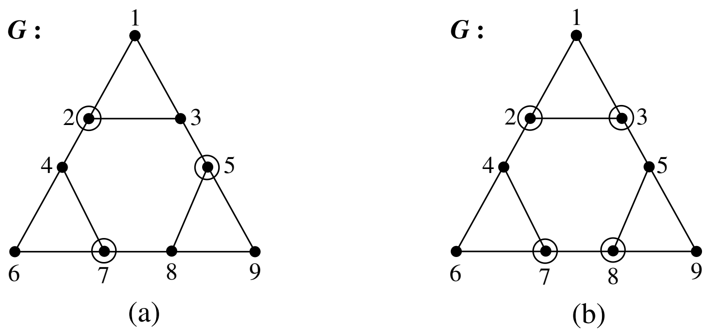


Figure 4. (a) Vertices in a locating-dominating set of G are circled.
(b) Vertices in a locating-total dominating set of G are circled.

Theorem 3.1. *The following decision problem is NP-complete:*
Name: locating-total dominating set ($LTDS$).
Instance: a connected graph $G = (V, E)$ *and an integer* $k \leq |V|$.
Question: is there a locating-total dominating set $S \subseteq V$ *of size at most* k?

Proof. We polynomially reduce 3-SAT to $LTDS$. We consider any instance of 3-SAT, $\mathbb{C} = \{C_1, C_2, \ldots, C_m\}$ over the set of variables $X = \{x_1, x_2, \ldots, x_n\}$. For each variable x_i of X, we construct the graph $G_{x_i} = (V_{x_i}, E_{x_i})$ with $V_{x_i} = \{a_i, b_i, c_i, d_i, e_i, x_i, \overline{x_i}\}$ and $E_{x_i} = \{a_i x_i, a_i\overline{x_i}, a_i c_i, b_i c_i, c_i x_i, c_i\overline{x_i}, d_i x_i, d_i\overline{x_i},\ d_i e_i\}$, $1 \leq i \leq n$.

Next for each clause $C_j = \{u_{j,1}, u_{j,2}, u_{j,3}\}$, we construct the graph $G_{C_j} = (V_{C_j},\ E_{C_j})$, with $V_{C_j} = \{\alpha_j, \beta_j, \gamma_j, \mu_j\}$ and $E_{C_j} = \{\alpha_j\beta_j, \beta_j\gamma_j, \gamma_j\mu_j, \gamma_j\eta_j, \mu_j\eta_j\}$, $1 \leq j \leq m$.

Finally, given formula $F = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ we construct $G = (V, E)$ with

$$V = \left(\bigcup_{i=1}^{n} V_{x_i}\right) \cup \left(\bigcup_{i=1}^{n} V_{C_j}\right),$$

$$E = \left(\bigcup_{i=1}^{n} E_{x_i}\right) \cup \left(\bigcup_{i=1}^{n} E_{C_j}\right) \cup \left(\bigcup_{i=1}^{n} \{\alpha_j u_{j,1}, \alpha_j u_{j,2}, \alpha_j u_{j,3}\}\right).$$

We set $k = 3n+2m$; we see that $|V| = 7n+5m$ and $|E| = 9n+8m$. See Figure 5 with $n = 3$ and $m = 2$. In Figure 5, $F = C_1 \wedge C_2$, where $C_1 = (u_{1,1} \vee u_{1,2} \vee u_{1,3}) = (x_1 \vee x_2 \vee x_3)$ and $C_2 = (u_{2,1} \vee u_{2,2} \vee u_{2,3}) = (x_1 \vee \overline{x_2} \vee \overline{x_3})$.

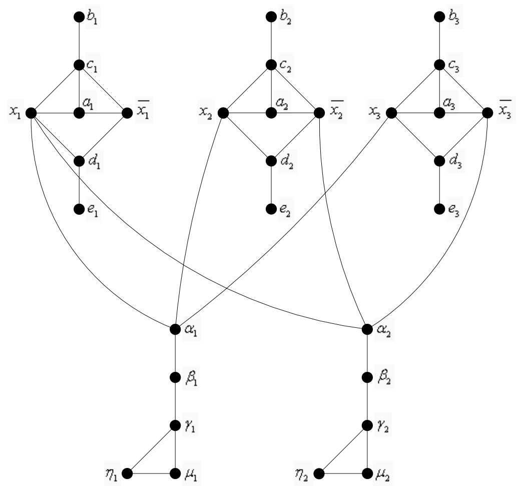


Figure 5. Graph of formula $F = (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3})$.

(i) If F is satisfied, we can construct a locating-total dominating set $S \subseteq V$, of size k, as follows. For all j and i where $1 \le j \le m, 1 \le i \le n$, let S contain $\gamma_j, \mu_j, c_i, d_i$, and whichever of x_i and $\overline{x_i}$ that has been set True. The set S thus constructed has size $3n + 2m = k$. Clearly S is a total dominating set of G. We have only to show that S is a locating-total dominating set. Without loss of generality, assume that $x_i \in S$; then $N(a_i) \cap S = \{c_i, x_i\}, N(b_i) \cap S = \{c_i\}, N(\overline{x_i}) \cap S = \{c_i, d_i\}, N(\overline{e_i}) \cap S = \{d_i\}$; moreover, $N(\beta_j) \cap S = \{\gamma_j\}$, $N(\eta_j) \cap S = \{\gamma_j, \mu_j\}$ and using the assumption that each clause contains at least one true literal, at least one vertex of type x_i or $\overline{x}_i$ will be in $N(\alpha_j) \cap S$.

(ii) Now we assume that there is a subset S of V, of size at most k, which is a locating-total dominating set. It is clear that for all j, either $\{\gamma_j, \mu_j\} \in S$ or $\{\gamma_j, \eta_j\} \in S$. Suppose not. If $\{\beta_j, \gamma_j\} \in S$, then $N(\mu_j) \cap S = \{\gamma_j\} = N(\eta_j) \cap S$ and, if $\{\alpha_j, \beta_j\} \in S$, then μ_j and η_j are not dominated and, if $\{\mu_j, \eta_j\} \in S$, then β_j is not dominated. Thus in all cases, either $\{\gamma_j, \mu_j\} \in S$ or $\{\gamma_j, \eta_j\} \in S$ and α_j must be dominated by another vertex.

Let us now consider the sets $S \cap V_{x_i}$; we claim that at least three elements in V_{x_i} are necessary to make $N(u) \cap S \neq N(v) \cap S$ for all u and v in $V_{x_i} \setminus S$, and that, moreover, if we manage with exactly three, then exactly one of x_i belongs to S. Indeed, suppose first that x_i or $\overline{x_i}$ are in S. Then since two more elements are necessary in V_{x_i} to locate b_i and e_i, either $|S \cap V_{x_i}| \geq 4$ or $|S \cap V_{x_i}| = 3$ and exactly one of x_i and $\overline{x_i}$ belongs to S. Suppose next that neither x_i nor $\overline{x_i}$ are in S. Then, in order to locate and separate a_i, b_i and c_i, and d_i and e_i, at least three elements in $V_{x_i} \setminus \{x_i, \overline{x_i}\}$ are necessary. Now if $\{a_i, c_i, d_i\} \subset S$, then $N(x_i) \cap S = N(\overline{x_i}) \cap S = \{a_i, c_i, d_i\}$; this implies that x_i or $\overline{x_i}$ is located by a vertex of type α. This however contradicts the assumption on the size of $|S|$, since already $3n + 2m$ other vertices necessarily belong to S.

Now, we know that S contains exactly k elements; in particular, exactly two vertices belong to V_{C_j} and exactly three vertices are in V_{x_i}, with exactly one of x_i and $\overline{x_i}$ in S.

Thus, setting $x_i =$ True if $S \cap \{x_i, \overline{x_i}\} = \{x_i\}$ and $x_i =$ False if $S \cap \{x_i, \overline{x_i}\} = \{\overline{x_i}\}$ is a valid truth assignment for the variables of X. Now in order to locate α_j at least one vertex of type x_i or $\overline{x}_i$ must be in S, corresponding to one of the three literals in the clause C_j. This means that C_j contains at least one true literal and it holds for all j. Hence we have a truth assignment which satisfies F. ■

We end the paper with the followings problems.

Problem 1. Can Theorem 3.1 be improved for bipartite graphs and chordal graphs?

Problem 2. Can improved bounds for locating-total domination number be obtained for interval graphs and split graphs?

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