Discussiones Mathematicae
Graph Theory 37 (2017) 953–961
doi:10.7151/dmgt.1956

RELATING 2-RAINBOW DOMINATION TO ROMAN DOMINATION

José D. Alvarado

Simone Dantas

Instituto de Matemática e Estatística
Universidade Federal Fluminense, Niterói, Brazil

e-mail: josealvarado.mat17@gmail.com
sdantas@im.uff.br

AND

Dieter Rautenbach

Institute of Optimization and Operations Research
Ulm University, Ulm, Germany

e-mail: dieter.rautenbach@uni-ulm.de

Abstract

For a graph G, let $\gamma_R(G)$ and $\gamma_{r2}(G)$ denote the Roman domination number of G and the 2-rainbow domination number of G, respectively. It is known that $\gamma_{r2}(G) \leq \gamma_R(G) \leq \frac{3}{2}\gamma_{r2}(G)$. Fujita and Furuya [*Difference between 2-rainbow domination and Roman domination in graphs*, Discrete Appl. Math. **161** (2013) 806–812] present some kind of characterization of the graphs G for which $\gamma_R(G) - \gamma_{r2}(G) = k$ for some integer k. Unfortunately, their result does not lead to an algorithm that allows to recognize these graphs efficiently.

We show that for every fixed non-negative integer k, the recognition of the connected K_4-free graphs G with $\gamma_R(G) - \gamma_{r2}(G) = k$ is NP-hard, which implies that there is most likely no good characterization of these graphs. We characterize the graphs G such that $\gamma_{r2}(H) = \gamma_R(H)$ for every induced subgraph H of G, and collect several properties of the graphs G with $\gamma_R(G) = \frac{3}{2}\gamma_{r2}(G)$.

Keywords: 2-rainbow domination, Roman domination.

2010 Mathematics Subject Classification: 05C69.

1. Introduction

We consider finite, simple, and undirected graphs, and use standard terminology and notation.

Rainbow domination was introduced in [3]. Here we consider the special case of 2-rainbow domination. A 2-*rainbow dominating function* of a graph G is a function $f : V(G) \to 2^{\{1,2\}}$ such that $\bigcup_{v\in N_G(u)} f(v) = \{1,2\}$ for every vertex u of G with $f(u) = \emptyset$. For a set X of vertices of G, let $|f(X)| = \sum_{u\in X} |f(u)|$, and let the *weight* $w(f)$ of f be $|f(V(G))|$. The 2-*rainbow domination number* $\gamma_{r2}(G)$ of G is the minimum weight of a 2-rainbow dominating function of G, and a 2-rainbow dominating function of G of weight $\gamma_{r2}(G)$ is *minimum*.

Roman domination was introduced in [7]. A *Roman dominating function* of a graph G is a function $g : V(G) \to \{0,1,2\}$ such that every vertex u of G with $g(u) = 0$ has a neighbor v with $g(v) = 2$. For a set X of vertices of G, let $g(X) = \sum_{u\in X} g(u)$, and let the *weight* $w(g)$ of g be $g(V(G))$. The *Roman domination number* $\gamma_R(G)$ of G is the minimum weight of a Roman dominating function of G, and a Roman dominating function of G of weight $\gamma_R(G)$ is *minimum*.

For a positive integer k, let $[k]$ be the set of positive integers at most k.

The definitions of 2-rainbow domination on the one hand and Roman domination on the other hand clearly exhibit certain similarities. It is therefore not surprising that these notions are related. For later reference, we include the simple proof of the following known results.

Theorem 1 (Wu and Xing [9], Chellali and Rad [5], and Fujita and Furuya [6]). *If G is a graph, then $\gamma_{r2}(G) \le \gamma_R(G) \le \frac{3}{2}\gamma_{r2}(G)$.*

Proof. These inequalities follow immediately from two simple observations. If g is a Roman dominating function of G, then

$$f : V(G) \to 2^{\{1,2\}} : x \mapsto \begin{cases} \emptyset, & \text{if } g(x) = 0, \\ \{1\}, & \text{if } g(x) = 1, \text{ and} \\ \{1,2\}, & \text{if } g(x) = 2 \end{cases}$$

is a 2-rainbow dominating function of G of weight $w(f) \le w(g)$. Similarly, if f is a 2-rainbow dominating function of G, and $|f^{-1}(\{1\})| \ge |f^{-1}(\{2\})|$, then

$$g : V(G) \to \{0,1,2\} : x \mapsto \begin{cases} 0, & \text{if } f(x) = \emptyset, \\ 1, & \text{if } f(x) = \{1\}, \text{ and} \\ 2, & \text{otherwise} \end{cases}$$

is a Roman dominating function of G of weight

$$\begin{aligned} w(g) &= |f^{-1}(\{1\})| + 2|f^{-1}(\{2\})| + 2|f^{-1}(\{1,2\})| \\ &\leq \frac{3}{2}|f^{-1}(\{1\})| + \frac{3}{2}|f^{-1}(\{2\})| + 2|f^{-1}(\{1,2\})| \qquad (1) \\ &\leq \frac{3}{2}|f^{-1}(\{1\})| + \frac{3}{2}|f^{-1}(\{2\})| + 3|f^{-1}(\{1,2\})| \qquad (2) \\ &= \frac{3}{2}w(f). \end{aligned}$$

■

Fujita and Furuya [6] present some kind of characterization of the connected graphs G for which $\gamma_R(G) - \gamma_{r2}(G) = k$ for some non-negative integer k at most $\frac{1}{2}\gamma_{r2}(G)$ (cf. Corollary 3.6 in [6]). Unfortunately, their result does not lead to an algorithm that allows to recognize these graphs efficiently.

In the present note we show that for every fixed non-negative integer k, the recognition of the connected K_4-free graphs G with $\gamma_R(G) - \gamma_{r2}(G) = k$ is NP-hard, which implies that there is most likely no good characterization of these graphs. In view of this negative result, we characterize the graphs G such that $\gamma_{r2}(H) = \gamma_R(H)$ for every induced subgraph H of G, and also establish a similar result for the equality $\gamma_R(H) = \frac{3}{2}\gamma_{r2}(H)$. The graphs G that satisfy $\gamma_R(G) = \frac{3}{2}\gamma_{r2}(G)$ seem far more restricted and we collect several of their properties.

For further related results on these parameters refer to [8, 1, 2].

2. Results

We begin with our hardness results.

Theorem 2. *It is NP-hard to decide whether $\gamma_{r2}(G) = \gamma_R(G)$ for a given connected K_4-free graph G.*

Proof. We describe a reduction from 3SAT. Therefore, let $\mathcal{F}$ be an instance of 3SAT with m clauses $C_1, \ldots, C_m$ over n boolean variables $x_1, \ldots, x_n$. Clearly, we may assume that $m \geq 2$. We will construct a connected K_4-free graph G whose order is polynomially bounded in terms of n and m such that $\mathcal{F}$ is satisfiable if and only if $\gamma_{r2}(G) = \gamma_R(G)$.

- For every variable x_i, we create a copy G_i of the diamond $K_4 - e$, and denote the two vertices of degree 3 in G_i by x_i and $\bar{x}_i$.
- For every clause C_j, we create a vertex C_j.
- For every literal $x \in \{x_1, \ldots, x_n\} \cup \{\bar{x}_1, \ldots, \bar{x}_n\}$ and every clause C_j such that x appears in C_j, we add the edge xC_j.
- Finally, we add an induced path uvw of order 3 and all possible edges between $\{u, w\}$ and $\{C_1, \ldots, C_m\}$.

This completes the construction of G. Clearly, G is connected and K_4-free, and has order $4n + m + 3$.

Let f be a 2-rainbow dominating function of G. It is easy to see that $|f(V(G_i))| \geq 2$ for every $i \in [n]$, and $|f(\{C_1, \ldots, C_m\} \cup \{u, v, w\})| \geq 2$, which implies $\gamma_{r2}(G) \geq 2n + 2$. Since

$$x \mapsto \begin{cases} \{1\}, & x \in \{u, x_1, \ldots, x_n\}, \\ \{2\}, & x \in \{w, \bar{x}_1, \ldots, \bar{x}_n\}, \text{ and} \\ \emptyset, & \text{otherwise} \end{cases}$$

defines a 2-rainbow dominating function of weight $2n + 2$, we obtain $\gamma_{r2}(G) = 2n + 2$, which implies $\gamma_R(G) \geq 2n + 2$.

In remains to show that $\mathcal{F}$ is satisfiable if and only if $\gamma_R(G) = 2n + 2$.

Suppose that $\gamma_R(G) = 2n + 2$. Let g be a minimum Roman dominating function of G. It is easy to see that $g(V(G_i)) \geq 2$ for every $i \in [n]$, and $g(\{C_1, \ldots, C_m\} \cup \{u, v, w\}) \geq 2$. Since $\gamma_R(G) = 2n+2$, all these inequalities are satisfied with equality, and considering u, v, and w, it follows easily that $g(v) = 2$, and that every vertex C_j has a neighbor x in $\{x_1, \ldots, x_n\} \cup \{\bar{x}_1, \ldots, \bar{x}_n\}$ with $g(x) = 2$. Therefore, these latter vertices indicate a satisfying truth assignment for $\mathcal{F}$.

Conversely, suppose that $\mathcal{F}$ is satisfiable, and consider a satisfying truth assignment. The function

$$x \mapsto \begin{cases} 2, & x = v, \\ 2, & x \in \{x_1, \ldots, x_n\} \cup \{\bar{x}_1, \ldots, \bar{x}_n\} \text{ such that } x \text{ is true, and} \\ 0, & \text{otherwise} \end{cases}$$

defines a Roman dominating function of G of weight $2n+2$, which implies $\gamma_R(G) = 2n + 2$, and completes the proof. ■

If G is a graph, then $\gamma_{r2}(G \cup C_4) = \gamma_{r2}(G) + 2$ and $\gamma_R(G \cup C_4) = \gamma_R(G) + 3$. Furthermore, if G has k components $G_1, \ldots, G_k$, the star $K_{1,k+2}$ has endvertices $u_1, \ldots, u_{k+2}$, and G' arises from $G \cup K_{1,k+2}$ by adding an edge between u_i and one vertex of G_i for every $i \in [k]$, then G' is connected, and satisfies $\gamma_{r2}(G') = \gamma_{r2}(G) + 2$ and $\gamma_R(G') = \gamma_R(G) + 2$. In combination with Theorem 2, these observations immediately imply the following.

Corollary 3. *Let k be a positive integer. It is NP-hard to decide whether $\gamma_R(G) - \gamma_{r2}(G) = k$ for a given connected K_4-free graph G.*

We proceed to our results concerning induced subgraphs.

Theorem 4. *A graph G satisfies $\gamma_{r2}(H) = \gamma_R(H)$ for every induced subgraph H of G if and only if G is $\{P_5, C_5, C_4\}$-free.*

Proof. Since $\gamma_{r2}(H) < \gamma_R(H)$ for every graph H in $\{P_5, C_5, C_4\}$, the necessity follows immediately. In view of Theorem 1, in order to complete the proof, it suffices to show that every $\{P_5, C_5, C_4\}$-free graph G satisfies $\gamma_R(G) \leq \gamma_{r2}(G)$. Therefore, let G be a $\{P_5, C_5, C_4\}$-free graph, and let f be a minimum 2-rainbow dominating function of G such that $|f^{-1}(\{1,2\})|$ is as large as possible. For $F \in 2^{\{1,2\}}$, let $V_F = f^{-1}(F)$. Let

$$g : V(G) \to \{0,1,2\} : x \mapsto \begin{cases} 0, & \text{if } x \in V_\emptyset, \\ 1, & \text{if } x \in V_{\{1\}} \cup V_{\{2\}}, \text{ and} \\ 2, & \text{if } x \in V_{\{1,2\}}. \end{cases}$$

Note that $w(g) = w(f)$. If g is a Roman dominating function of G, then $\gamma_R(G) \leq \gamma_{r2}(G)$. Hence, we may assume that g is not a Roman dominating function of G, which implies the existence of a vertex u in $V_\emptyset$ that has a neighbor v_1 in $V_{\{1\}}$ as well as a neighbor v_2 in $V_{\{2\}}$ but no neighbor in $V_{\{1,2\}}$. We say that v_1uv_2 is a *special path*.

First, suppose that there is no special path v_1uv_2 such that v_1 and v_2 are adjacent, that is, every special path is induced. Let v_1uv_2 be a special path. Let

$$f_1 : V(G) \to 2^{\{1,2\}} : x \mapsto \begin{cases} \emptyset, & \text{if } x \in \{v_1, v_2\}, \\ \{1,2\}, & \text{if } x = u, \text{ and} \\ f(x), & \text{otherwise.} \end{cases}$$

Since $w(f_1) = w(f)$, the choice of f implies that f_1 is not a 2-rainbow dominating function of G. By symmetry, we may therefore assume that there is a special path $v_1u'v_2'$ with $u' \notin N_G[u]$. By our assumption, v_1 is not adjacent to v_2 or to v_2'. Since G is C_4-free, u' is not adjacent to v_2, which implies that v_2' is distinct from v_2. Since G is C_4-free, u is not adjacent to v_2'. Now, $G[\{v_2', u', v_1, u, v_2\}]$ is C_5 or P_5 depending on whether v_2 and v_2' are adjacent or not, which is a contradiction. Hence, there is a special path that is not induced. If v_1uv_2 is a special path, and v_1 is adjacent to v_2, then we say that v_1uv_2 is a *special triangle*.

Let U be a set of vertices of maximum order such that every vertex in U belongs to some special triangle T with $V(T) \subseteq U$. Since there is at least one special triangle, the set U is not empty. For $F \in \{\emptyset, \{1\}, \{2\}\}$, let $U_F = U \cap V_F$. By symmetry, we may assume that $|U_{\{1\}}| \geq |U_{\{2\}}|$. Let

$$f_2 : V(G) \to 2^{\{1,2\}} : x \mapsto \begin{cases} \emptyset, & \text{if } x \in U_{\{1\}}, \\ \{1,2\}, & \text{if } x \in U_{\{2\}}, \text{ and} \\ f(x), & \text{otherwise.} \end{cases}$$

Since $w(f_2) \leq w(f)$, the choice of f implies that f_2 is not a 2-rainbow dominating function of G. Since every vertex in $U_\emptyset \cup U_{\{1\}}$ has a neighbor in $U_{\{2\}}$, together with the definition of U, this implies the existence of a special triangle v_1uv_2 as well as a special path $v_1u'v_2'$ such that

- $u \in U_\emptyset$, $v_1 \in U_{\{1\}}$, and $v_2 \in U_{\{2\}}$,
- $u', v_2' \notin U$, and
- u' is not adjacent to v_2.

If v_1 and v_2' are adjacent, then $v_1u'v_2'$ is a special triangle, and adding u' and v_2' to U yields a contradiction to the choice of U. Hence, v_1 is not adjacent to v_2'. Since G is C_4-free, v_2 is not adjacent to v_2'. Let

$$f_3 : V(G) \to 2^{\{1,2\}} : x \mapsto \begin{cases} \emptyset, & \text{if } x = v_2, \\ \{1,2\}, & \text{if } x = v_1, \text{ and} \\ f(x), & \text{otherwise.} \end{cases}$$

Since $w(f_3) = w(f)$, the choice of f implies that f_3 is not a 2-rainbow dominating function of G. This implies the existence of a vertex $u'' \in V_\emptyset$ that is adjacent to v_2 but not to v_1. Since G is C_4-free, u' is not adjacent to u''. Now, $G[\{v_2', u', v_1, v_2, u''\}]$ is C_5 or P_5 depending on whether v_2' and u'' are adjacent or not, which is a contradiction and completes the proof. ■

For a positive integer k, let $\mathcal{G}_k\left(\gamma_R, \frac{3}{2}\gamma_{r2}\right)$ be the set of all graphs G such that $\gamma_R(H) = \frac{3}{2}\gamma_{r2}(H)$ for every induced subgraph H of G with $\gamma_{r2}(H) \geq k$, that is,

$$\mathcal{G}_k\left(\gamma_R, \frac{3}{2}\gamma_{r2}\right) = \left\{G : \forall\, H \subseteq_{\text{ind}} G : \gamma_{r2}(H) \geq k \Rightarrow \gamma_R(H) = \frac{3}{2}\gamma_{r2}(H)\right\}.$$

Since $\gamma_{r2}(K_1) = 1 = \gamma_R(K_1)$, the set $\mathcal{G}_1\left(\gamma_R, \frac{3}{2}\gamma_{r2}\right)$ contains no graph of positive order. Since $\gamma_{r2}(K_2) = \gamma_R(K_2) = \gamma_{r2}(\bar{K}_2) = \gamma_R(\bar{K}_2) = 2$, the set $\mathcal{G}_2\left(\gamma_R, \frac{3}{2}\gamma_{r2}\right)$ contains the null graph, and K_1.

Theorem 5. *A graph G belongs to $\mathcal{G}_3\left(\gamma_R, \frac{3}{2}\gamma_{r2}\right)$ if and only if G is $\{\bar{K}_3, K_2 \cup K_1\}$-free.*

Proof. Since $\gamma_{r2}(\bar{K}_3) = \gamma_R(\bar{K}_3) = \gamma_{r2}(K_2 \cup K_1) = \gamma_R(K_2 \cup K_1) = 3$, the graphs in $\mathcal{G}_3\left(\gamma_R, \frac{3}{2}\gamma_{r2}\right)$ are $\{\bar{K}_3, K_2 \cup K_1\}$-free. In view of Theorem 1, in order to complete the proof, it suffices to show that no graph G exists that is both $\{\bar{K}_3, K_2 \cup K_1\}$-free and satisfies $\gamma_{r2}(G) \geq 3$. For a contradiction, suppose that G is a $\{\bar{K}_3, K_2 \cup K_1\}$-free graph with $\gamma_{r2}(G) \geq 3$. Since $\gamma_{r2}(G) \geq 3$, the graph G is not complete. Let u and v be two distinct vertices of G that are not adjacent. Since G is $\{\bar{K}_3, K_2 \cup K_1\}$-free, we obtain $N_G(u) = N_G(v) = V(G) \setminus \{u, v\}$. This im- plies that

$$x \mapsto \begin{cases} \{1\}, & x = u, \\ \{2\}, & x = v, \text{ and} \\ \emptyset, & \text{otherwise} \end{cases}$$

defines a 2-rainbow dominating function of G of weight 2, which is a contradiction. ■

Theorem 2 implies that the graphs G with $\gamma_{r2}(G) = \gamma_R(G)$ do not have a simple structure. In contrast to that, the graphs G with $\gamma_R(G) = \frac{3}{2}\gamma_{r2}(G)$ seem far more restricted. In fact, it is conceivable that these graphs can be recognized in polynomial time. In our last result, we collect several of their properties.

Theorem 6. *If G is a graph with $\gamma_R(G) = \frac{3}{2}\gamma_{r2}(G)$, then every minimum 2-rainbow dominating function f of G has the following properties, where $V_F = f^{-1}(F)$ and $n_F = |V_F|$ for $F \in 2^{\{1,2\}}$.*

(i) $n_{\{1\}} = n_{\{2\}}$ *and* $n_{\{1,2\}} = 0$.
(ii) *There are no edges between $V_{\{1\}}$ and $V_{\{2\}}$.*
(iii) *For $i \in [2]$, the maximum degree of $G[V_{\{i\}}]$ is at most 1.*
(iv) *For $i \in [2]$, every vertex in $V_\emptyset$ has at least 1 and at most 2 neighbors in $V_{\{i\}}$.*
(v) *For $i \in [2]$, every vertex u in $V_{\{i\}}$ has at least 2 neighbors v in $V_\emptyset$ with $N_G(v) \cap V_{\{i\}} = \{u\}$.*

Proof. Let G be a graph with $\gamma_R(G) = \frac{3}{2}\gamma_{r2}(G)$, and let f be a minimum 2-rainbow dominating function of G.

(i) Since the inequality (1) in the proof of Theorem 1 is satisfied with equality, we obtain $n_{\{1\}} = n_{\{2\}}$. Similarly, since (2) is satisfied with equality, we obtain $n_{\{1,2\}} = 0$. Since f is a 2-rainbow dominating function of G and $n_{\{1,2\}} = 0$, every vertex in $V_\emptyset$ has a neighbor in $V_{\{1\}}$ as well as a neighbor in $V_{\{2\}}$.

(ii) If v_1 in $V_{\{1\}}$ is adjacent to v_2 in $V_{\{2\}}$, then

$$x \mapsto \begin{cases} 0, & x \in \{v_2\} \cup V_\emptyset, \\ 1, & x \in V_{\{2\}} \setminus \{v_2\}, \text{ and} \\ 2, & x \in V_{\{1\}} \end{cases}$$

defines a Roman dominating function of G of weight $\frac{3}{2}\gamma_{r2}(G) - 1$, which is a contradiction.

(iii) If u in $V_{\{i\}}$ is adjacent to two distinct vertices v and w in $V_{\{i\}}$, then

$$x \mapsto \begin{cases} 0, & x \in \{v, w\} \cup V_\emptyset, \\ 1, & x \in V_{\{i\}} \setminus \{u, v, w\}, \text{ and} \\ 2, & x \in \{u\} \cup V_{\{3-i\}} \end{cases}$$

defines a Roman dominating function of G of weight $\frac{3}{2}\gamma_{r2}(G) - 1$, which is a contradiction.

(iv) As observed above, every vertex in $V_\emptyset$ has a neighbor in $V_{\{i\}}$. If u in $V_\emptyset$ is adjacent to three distinct vertices v_1, v_2, and v_3 in $V_{\{i\}}$, then

$$x \mapsto \begin{cases} 0, & x \in \{v_1, v_2, v_3\} \cup (V_\emptyset \setminus \{u\}), \\ 1, & x \in V_{\{i\}} \setminus \{v_1, v_2, v_3\}, \text{ and} \\ 2, & x \in \{u\} \cup V_{\{3-i\}} \end{cases}$$

defines a Roman dominating function of G of weight $\frac{3}{2}\gamma_{r2}(G) - 1$, which is a contradiction.

(v) Let $i \in [2]$ and let $u \in V_{\{i\}}$. Let $P(u) = \{v \in V_\emptyset : N_G(v) \cap V_{\{i\}} = \{u\}\}$. If $P(u) = \emptyset$, then

$$x \mapsto \begin{cases} 0, & x \in V_\emptyset, \\ 1, & x \in V_{\{3-i\}} \cup \{u\}, \text{ and} \\ 2, & x \in V_{\{i\}} \setminus \{u\} \end{cases}$$

defines a Roman dominating function of G of weight $\frac{3}{2}\gamma_{r2}(G) - 1$, which is a contradiction. Hence, $P(u)$ is non-empty. If $P(u) = \{v\}$, then let w be a neighbor of v in $V_{\{3-i\}}$. Now,

$$x \mapsto \begin{cases} 0, & x \in \{u, w\} \cup (V_\emptyset \setminus \{v\}), \\ 1, & x \in V_{\{3-i\}} \setminus \{w\}, \text{ and} \\ 2, & x \in \{v\} \cup (V_{\{i\}} \setminus \{u\}) \end{cases}$$

defines a Roman dominating function of G of weight $\frac{3}{2}\gamma_{r2}(G) - 1$, which is a contradiction. ■

Acknowledgment

J.D. Alvarado and S. Dantas were partially supported by FAPERJ, CNPq, and CAPES. D. Rautenbach was partially supported by CAPES.

References

[1] J.D. Alvarado, S. Dantas and D. Rautenbach, *Averaging 2-rainbow domination and Roman domination*, Discrete Appl. Math. **205** (2016) 202–207. doi:10.1016/j.dam.2016.01.021

[2] J.D. Alvarado, S. Dantas and D. Rautenbach, *Relating 2-rainbow domination to weak Roman domination*, arXiv:1507.04899.

[3] B. Brešar, M.A. Henning and D.F. Rall, *Rainbow domination in graphs*, Taiwanese J. Math. **12** (2008) 213–225.

[4] M. Chellali, T.W. Haynes and S.T. Hedetniemi, *Bounds on weak roman and* 2-*rainbow domination numbers*, Discrete Appl. Math. **178** (2014) 27–32. doi:10.1016/j.dam.2014.06.016

[5] M. Chellali and N.J. Rad, *On* 2-*rainbow domination and Roman domination in graphs*, Australas. J. Combin. **56** (2013) 85–93.

[6] S. Fujita and M. Furuya, *Difference between* 2-*rainbow domination and Roman domination in graphs*, Discrete Appl. Math. **161** (2013) 806–812. doi:10.1016/j.dam.2012.10.017

[7] I. Stewart, *Defend the Roman empire!*, Sci. Amer. **281** (1999) 136–139. doi:10.1038/scientificamerican1299-136

[8] Y. Wu and N.J. Rad, *Bounds on the* 2-*rainbow domination number of graphs*, arXiv:1005.0988v1.

[9] Y. Wu and H. Xing, *Note on* 2-*rainbow domination and Roman domination in graphs*, Appl. Math. Lett. **23** (2010) 706–709. doi:10.1016/j.aml.2010.02.012

Received 3 December 2015
Revised 7 July 2016
Accepted 8 August 2016