

ON SUPER (a, d) - H -ANTIMAGIC TOTAL COVERING OF STAR RELATED GRAPHS

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Abstract

Let $G = (V(G), E(G))$ be a simple graph and H be a subgraph of G . G admits an H -covering, if every edge in $E(G)$ belongs to at least one subgraph of G that is isomorphic to H . An (a, d) - H -antimagic total labeling of G is a bijection $\lambda : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, |V(G)| + |E(G)|\}$ such that for all subgraphs H' isomorphic to H , the H' weights

$$wt(H') = \sum_{v \in V(H')} \lambda(v) + \sum_{e \in E(H')} \lambda(e)$$

constitute an arithmetic progression $a, a + d, a + 2d, \dots, a + (n - 1)d$ where a and d are positive integers and n is the number of subgraphs of G isomorphic to H . Additionally, the labeling λ is called a super (a, d) - H -antimagic total labeling if $\lambda(V(G)) = \{1, 2, 3, \dots, |V(G)|\}$.

In this paper we study super (a, d) - H -antimagic total labelings of star related graphs $G_u[S_n]$ and caterpillars.

Keywords: super (a, d) - H -antimagic total labeling, star.

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1. INTRODUCTION

Let $G = (V(G), E(G))$ and $H = (V(H), E(H))$ be simple and finite graphs. Let $|V(G)| = p, |E(G)| = q$. An edge covering of G is a family of different subgraphs $H_1, H_2, H_3, \dots, H_k$ such that any edge of $E(G)$ belongs to at least one of the subgraphs H_j 's, $1 \leq j \leq k$. If the H_j are isomorphic to a given graph H , then G admits an H -covering.

Suppose G admits an H -covering. Gutiérrez and Lladó [1] defined an H -magic labeling which is a generalization of Kotzig and Rosa's edge magic total labeling [5]. A bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, p + q\}$ is called an H -magic labeling of G if there exists a positive integer k such that each subgraph H' isomorphic to H satisfies

$$f(H') = \sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e) = k.$$

In this case, we say that G is H magic. When $f(V(G)) = \{1, 2, 3, \dots, p\}$, we say that G is H -super magic.

On the other hand, Inayah *et al.* [2] introduced an (a, d) - H -antimagic total labeling of G which is defined as a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, p + q\}$ such that for all subgraphs H' isomorphic to H , the set of H' -weights

$$wt(H') = \sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e)$$

constitutes an arithmetic progression $a, a + d, a + 2d, \dots, a + (n - 1)d$ where a and d are some positive integers and n is the number of subgraphs isomorphic to H . In this case we say that G is (a, d) - H -antimagic. When $f(V(G)) = \{1, 2, 3, \dots, p\}$, we say that f is a super (a, d) - H -antimagic total labeling and G is super (a, d) - H -antimagic.

In [1] Gutiérrez and Lladó discussed H -supermagic labelings of stars, complete bipartite graphs, paths and cycles. In [6], Lladó and Moragas studied C_h -supermagic labelings of some graphs, namely, wheels, windmills, prisms and books. In [7], Maryati *et al.* proved that some classes of trees such as subdivisions of stars, shrubs and banana tree graphs are P_h -supermagic for some h . In [2], Inayah *et al.* studied some properties of (a, d) - H -antimagic total labeling for any graph and also discussed the (a, d) - C_h -antimagic total labelings of fans. Recently, Inayah, Simanjuntak and Salman [4] proved that there exists a super (a, d) - H -antimagic total labeling for shackles of a connected graph H .

In this paper we study super (a, d) - H -antimagic total labelings of star related graphs $G_u[S_n]$ and caterpillars.

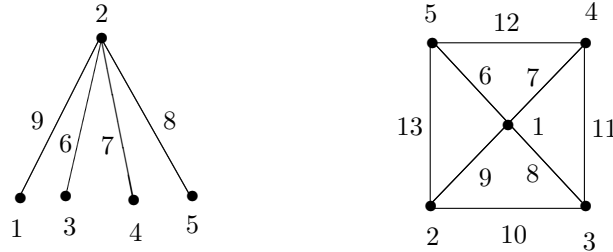


Figure 1. Super $(21, 1)$ - P_3 -antimagic total labeling and super $(33, 1)$ - C_3 -antimagic total labeling.

2. SUM SET PARTITIONS

As in [1, 3, 8], the proofs of our main results are based on the use of sum set partitions. We recall in this section some useful facts on this concept.

Let $x < y$ be positive integers. Throughout the paper we denote by $[x, y]$ to mean $\{i \in \mathbb{N} : x \leq i \leq y\}$. Given a set X of integers and a partition $\mathcal{P} = \{X_1, X_2, \dots, X_k\}$ of X into k parts. We denote by $\sum(\mathcal{P}) = (\sum X_1, \sum X_2, \dots, \sum X_k)$, the sum set partition of \mathcal{P} where $\sum X_i = \sum_{x \in X_i} x$. We will always order the partition in such a way that the sequence of subset sums $\sum X_1 \leq \sum X_2 \leq \dots \leq \sum X_k$ is non decreasing.

When all sets in \mathcal{P} have the same cardinality then we say that \mathcal{P} is an equipartition of X or k -equipartition or a k -balanced multisets of X .

We have the following lemmas.

Lemma 1 [8]. *Let x and y be nonnegative integers. Let $X = [x + 1, x(y + 1)]$ with $|X| = xy$ and $Y = [x(y + 2), 2x(y + 1) - 1]$ with $|Y| = xy$. Then, there exists a partition K of $X \cup Y$ such that $\sum(K)$ is an arithmetic progression starting at $x(y + 3) + 1$ with common difference 2 and hence K is xy -balanced with all its subsets being 2-sets.*

Proof. For each $i \in [1, xy]$, define $K_i = \{a_i, b_i\}$ such that $a_i = x + i$, $b_i = x(y + 2) + i - 1$. Thus $\sum K_i = x(y + 3) + 2i - 1$, for all $i \in [1, xy]$.

Hence, the sum set partition of K , $\sum(K) = (\sum K_1, \sum K_2, \dots, \sum K_{xy})$ forms an arithmetic progression with common difference 2. Therefore, K is xy -balanced with all its subsets being 2-sets. ■

Lemma 2. *Let x, y and z be nonnegative integers. Let $X = [x + 1, x + y]$ with $|X| = y$ and $Y = [x + y + z + 1, x + 2y + z]$ with $|Y| = y$. Then, there exists a partition K of $X \cup Y$ such that*

- (i) $\sum(K)$ is an arithmetic progression starting at $2x + 2y + z + 1$ with common difference 0, and

- (ii) $\sum(K)$ is an arithmetic progression starting at $2x + y + z + 2$ with common difference 2 and hence K is y -balanced with all its subsets being 2-sets.

Proof. (i) For each $i \in [1, y]$ define the sets $K_i = \{a_i, b_i\}$ such that $a_i = x + i$, $b_i = x + 2y + z - i + 1$. Then $\sum K_i = 2x + 2y + z + 1$, for each $i \in [1, y]$.

Hence, the sum set partition of K , $\sum(K) = (\sum K_1, \sum K_2, \dots, \sum K_y)$ forms an arithmetic progression with common difference 0. Therefore, K is y -balanced with all its subsets being 2-sets.

(ii) For each $i \in [1, y]$, we take the sets $K_i = \{a_i, b_i\}$ such that: $a_i = x + i$, $b_i = x + y + z + i$. Then $\sum K_i = 2x + y + z + 2i$, for each $i \in [1, y]$.

Hence, the sum set partition of K , $\sum(K) = (\sum K_1, \sum K_2, \dots, \sum K_y)$ forms an arithmetic progression with common difference 2. Therefore, K is y -balanced with all its subsets being 2-sets. ■

Lemma 3. Let x, y and z be nonnegative integers. Let $X = \{1, 3, 5, \dots, 2y - 1\}$ with $|X| = y$ and $Y = [x + y + z + 1, x + 2y + z]$ with $|Y| = y$. Then, there exists a partition K of $X \cup Y$ such that

- (i) $\sum(K)$ is an arithmetic progression starting at $x + 2y + z + 1$ with common difference 1, and
(ii) $\sum(K)$ is an arithmetic progression starting at $x + y + z + 2$ with common difference 3 and hence K is y -balanced with all its subsets being 2-sets.

Proof. (i) For each $i \in [1, y]$, we define $K_i = \{a_i, b_i\}$ where $a_i = 2i - 1$, $b_i = x + 2y + z - i + 1$. Then $\sum K_i = x + 2y + z + i$, for each $i \in [1, y]$.

Hence, the sum set partition of K , $\sum(K) = (\sum K_1, \sum K_2, \dots, \sum K_y)$ forms an arithmetic progression with common difference 1. Therefore, K is y -balanced and all its subsets are 2-sets.

(ii) For each $i \in [1, y]$, we define $K_i = \{a_i, b_i\}$ where $a_i = 2i - 1$, $b_i = x + y + z + i$. Then $\sum K_i = x + y + z + 3i - 1$, for each $i \in [1, y]$.

Hence, the sum set partition of K , $\sum(K) = (\sum K_1, \sum K_2, \dots, \sum K_y)$ forms an arithmetic progression with common difference 3. Therefore, K is y -balanced and all its subsets are 2-sets. ■

3. MAIN RESULTS

Let G be a (p, q) graph and S_n be a star with n edges. Fix a vertex u of G . Then $G_u[S_n]$ is the graph obtained by identifying the vertex u with the centre of S_n . Let w be any vertex of S_n . Then $G + e$, $e = uw$, is a subgraph of $G_u[S_n]$. In this section, we consider graphs G for which $G_u[S_n]$ contains exactly n subgraphs isomorphic to $G + e$.

Let $G' \cong G_u[S_n]$. Let v_1, v_2, \dots, v_p and w_1, w_2, \dots, w_n be the vertices of G and S_n respectively. Let e_1, e_2, \dots, e_q and $e_{q+1}, e_{q+2}, \dots, e_{q+n}$ be the edges of G and S_n respectively. Then $|V(G')| = p + n$ and $|E(G')| = q + n$.

Lemma 4. *If the graph $G_u[S_n], n \geq 2$, admits a super (a, d) - $(G + e)$ -antimagic total labeling, then $d \leq p + q + 2$.*

Proof. Let $G' \cong G_n[S_n]$. Suppose there exists a bijection $f : V(G') \cup E(G') \rightarrow \{1, 2, 3, \dots, p + q + 2n\}$ which is a super (a, d) - $(G + e)$ -antimagic total labeling of G' . Let $wt(H') = \sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e)$ be the weights of the subgraph H' isomorphic to $G + e$ and let $W = \{w(H') : H' \cong G + e\} = \{a, a + d, a + 2d, \dots, a + (t - 1)d\}$ be the set of H' weights and t be the number of subgraphs. Here $t = n$. Now, it is easy to see that the minimum possible weight of H' is at least $(p + 1)(p + 2)/2 + (q + 1)(p + n) + (q + 1)(q + 2)/2$ i.e., $a \geq (p + 1)(p + 2)/2 + (q + 1)(p + n) + (q + 1)(q + 2)/2$. Also the maximum possible weight of H' is not more than $(p + 1)(p + n) - p(p + 1)/2 + (q + 1)(p + q + 2n) - q(q + 1)/2$, i.e., $a + (t - 1)d \leq (p + 1)(p + n) - p(p + 1)/2 + (q + 1)(p + q + 2n) - q(q + 1)/2$, $(n - 1)d \leq (n - 1)(p + q + 2)$, thus $d \leq p + q + 2$. ■

Theorem 5. *The graph G' admits a super $(\frac{1}{2}(p + q)(p + q + 3) + n(q + 2) + p + 1, 0)$ - $(G + e)$ -antimagic total labeling.*

Proof. Let $Z = [1, p + q + 2n]$ and partition Z into four sets such that $Z = A \cup B \cup C \cup D$ where $A = [1, p], B = [p + 1, p + n], C = [p + n + 1, p + q + n]$ and $D = [p + q + n + 1, p + q + 2n]$. Let $K = B \cup D$ and let $x = p, y = n$ and $z = q$. Then by Lemma 2(i), K is n -balanced multisets with all its subsets being 2-sets and $\sum K_i = 2p + q + 2n + 1$, for each $i \in [1, n]$.

Now we define a total labeling f on G' as follows:

Label the vertices $v_i, 1 \leq i \leq p$ by the elements of A and label the edges $e_i, 1 \leq i \leq q$ by the elements of C in any manner. Next use the elements of K to label all the vertices and edges of the star, use the smaller labels for the vertices and bigger labels for the edges in reverse order. Then for each $i, 1 \leq i \leq n$,

$$\begin{aligned} wt(G + e_{q+i}) &= (1 + 2 + 3 + \dots + p) + (p + n + 1 + p + n + 2, \dots, p + q + n) \\ &+ \sum K_i \\ &= \frac{p(p + 1)}{2} + q(p + n) + \frac{q(q + 1)}{2} + 2p + q + 2n + 1 \\ &= \frac{p(p + 1)}{2} + \frac{q(q + 1)}{2} + (p + n)(q + 2) + q + 1 \\ &= \frac{1}{2}(p + q)(p + q + 3) + n(q + 2) + p + 1. \end{aligned}$$

Hence G' has a super $(\frac{1}{2}(p + q)(p + q + 3) + n(q + 2) + p + 1, 0)$ - $(G + e)$ -antimagic total labeling. ■

Theorem 6. *The graph G' has a super $(\frac{1}{2}[(p+q)^2 + (p+q)(2n+3) + 5n - n^2] + 1, 1)$ - $(G+e)$ -antimagic total labeling.*

Proof. Let $Z = [1, p+q+2n]$ and partition Z into four sets such that $Z = A \cup B \cup C \cup D$ where $A = \{2, 4, \dots, 2n, 2n+1, 2n+2, \dots, p+n\}$, $B = \{1, 3, 5, \dots, 2n-1\}$, $C = [p+n+1, p+q+n]$ and $D = [p+q+n+1, p+q+2n]$. Let $K = B \cup D$ and let $x = p, y = n$ and $z = q$. Then by Lemma 3(i), K is n -balanced multisets with all its subsets being 2-sets and $\sum K_i = p+q+2n+i$, for each $i \in [1, n]$.

Now we define a total labeling f on G' as follows:

Label the vertices $v_i, 1 \leq i \leq p$ with the elements of A and label the edges $e_i, 1 \leq i \leq q$ with the elements of C in any order. Next use the elements of K to label all the vertices and edges of the star, use the smaller labels for the vertices and bigger labels for the edges in reverse order. Then for each $i, 1 \leq i \leq n$,

$$\begin{aligned} wt(G + e_{q+i}) &= 2 + 4 + 6 + \dots + 2n + 2n + 1 + 2n + 2 + \dots + 2n + p - n \\ &\quad + p + n + 1 + p + n + 2 + \dots + p + n + q + \sum K_i \\ &= n(n+1) + (p-n)2n + \frac{(p-n)(p-n+1)}{2} \\ &\quad + q(p+n) + \frac{q(q+1)}{2} + p + q + 2n + i \\ &= \frac{1}{2}((p+q)^2 + (p+q)(2n+3) + 5n - n^2) + i. \end{aligned}$$

Hence G' has a super $(\frac{1}{2}[(p+q)^2 + (p+q)(2n+3) + 5n - n^2] + 1, 1)$ - $(G+e)$ -antimagic total labeling. ■

Theorem 7. *The graph G' has a super $(\frac{1}{2}(p+q)(p+q+3) + (q+1)n + p + 2, 2)$ - $(G+e)$ -antimagic total labeling.*

Proof. Consider the partition of $[1, p+q+2n]$ introduced in the proof of Theorem 5. By Lemma 2(ii), $\sum K_i = 2p+q+n+2i$, for each $i \in [1, n]$.

$$\begin{aligned} wt(G + e_{q+i}) &= \frac{p(p+1)}{2} + q(p+n) + \frac{q(q+1)}{2} + 2p+q+n+2i \\ &= \frac{1}{2}(p+q)(p+q+3) + (q+1)n + p + 2i. \end{aligned}$$

Hence G' has a super $(\frac{1}{2}(p+q)(p+q+3) + (q+1)n + p + 2, 2)$ - $(G+e)$ -antimagic total labeling. ■

Theorem 8. *The graph G' has a super $(\frac{1}{2}[(p+q)^2 + (p+q)(2n+3) - (n-1)(n-2)] + 3, 3)$ - $(G+e)$ -antimagic total labeling.*

Proof. Consider the partition of $[1, p + q + 2n]$ introduced in the proof of Theorem 6. By Lemma 3(ii), $\sum K_i = p + q + n - 1 + 3i$, for each $i \in [1, n]$.

$$\begin{aligned} wt(G + e_{q+i}) &= \frac{n(n+1)}{2} + (p-n)(2n) + \frac{(p-n)(p-n+1)}{2}q(p+n) \\ &\quad + \frac{q(q+1)}{2} + p + q + n - 1 + 3i \\ &= \frac{1}{2}[(p+q)^2 + (p+q)(2n+3) - (n-1)(n-2)] + 3i. \end{aligned}$$

Hence G' has a super $(\frac{1}{2}[(p+q)^2 + (p+q)(2n+3) - (n-1)(n-2)] + 3, 3)$ - $(G + e)$ -antimagic total labeling. ■

Theorem 9. *The graph $G_u[S_2]$ admits a super (a, d) - $(G + e)$ -antimagic total labeling if and only if $d \in \{0, 1, 2, \dots, p + q + 2\}$.*

Proof. By Theorems 5–8, we have $d \in \{0, 1, 2, 3\}$. The weight of G is the same for all the weights of the subgraphs $(G + e_i), i = 1, 2$. So it is enough to find the labels of vertices and edges of the star S_2 . Now, for each $i, 1 \leq i \leq p - 2$ we define the labeling f_i as follows.

$$\begin{aligned} f_i(w_1) &= p - i, & 1 \leq i \leq p - 2, \\ f_i(e_{q+1}) &= p + q + 3, \\ f_i(w_2) &= p + 2, \text{ and} \\ f_i(e_{q+2}) &= p + q + 4. \end{aligned}$$

Thus, the induced sums of the labels of vertices and edges of S_2 are $2p + q + 3 - i$ and $2p + q + 6$. Hence, $d = 3 + i, 1 \leq i \leq p - 2$. Therefore, $d = 4, 5, \dots, p + 1$.

Also for each $i, 1 \leq i \leq q + 1$, we define the labeling f_i as follows

$$\begin{aligned} f_i(w_1) &= 1, \\ f_i(e_{q+1}) &= p + q + 4 - i, 1 \leq i \leq q + 1, \\ f_i(w_2) &= p + 2, \text{ and} \\ f_i(e_{q+2}) &= p + q + 4. \end{aligned}$$

Thus, the induced sums of the labels of vertices and edges of S_2 are $p + q + 5 - i, 2p + q + 6$. Hence $d = p + 1 + i, 1 \leq i \leq q + 1$. Therefore, $d = p + 2, p + 4, \dots, p + q + 2$.

Hence the results follows. ■

Open Problem 10. *For each $d, 4 \leq d \leq p + q + 2$, either find the super (a, d) - $(G + e)$ -antimagic total labeling of the graph $G_u[S_n], n \geq 3$, or prove that this labeling does not exist.*

4. CATERPILLAR

Definition 11. *The backbone of a caterpillar is the graph obtained from it by removing its pendant edges.*

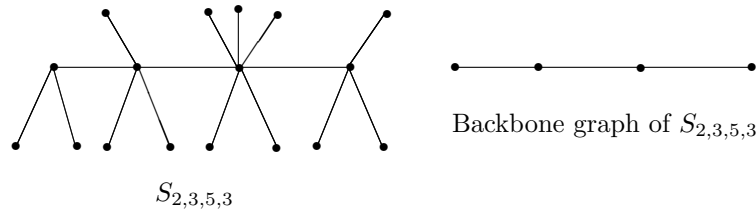


Figure 2

Theorem 12. *A caterpillar S_{n_1, n_2, \dots, n_k} has a super $(2(k + 2)n^2 + 7kn + 2k + 1 + \lceil \frac{k}{2} \rceil, 4n^2)$ - $S_{n,n}$ -antimagic total labeling for $n_1 = n_2 = \dots = n_k = n$.*

Proof. As in [8], let $G \cong S_{n_1, n_2, \dots, n_k}$ with $n_1 = n_2 = \dots = n_k = n$. Then $|V(G)| = k(n + 1)$ and $|E(G)| = k(n + 1) - 1$.

Let $V(G) = \{c_i : 1 \leq i \leq k\} \cup \{v_{ij} : 1 \leq i \leq k, 1 \leq j \leq n\}$ and

$E(G) = \{c_i c_{i+1} : 1 \leq i \leq k - 1\} \cup \{c_i v_{ij} : 1 \leq i \leq k, 1 \leq j \leq n\}$.

Let $Z = [1, 2k(n + 1) - 1]$ and partition Z into four sets such that $Z = A \cup B \cup C \cup D$, where $A = [1, k]$, $B = [k + 1, k(n + 1)]$, $C = [k(n + 1) + 1, k(n + 1) + k - 1]$ and $D = [k(n + 2), 2k(n + 1) - 1]$. Let us take $A = \{x_i : 1 \leq i \leq k\}$ such that

$$x_i = \begin{cases} \lfloor \frac{i}{2} + 1 \rfloor & \text{for odd } i, \\ \lceil \frac{k}{2} \rceil + \frac{i}{2} & \text{for even } i. \end{cases}$$

Let $K = B \cup D$ and let $x = k, y = n$. Then by Lemma 1, K is kn -balanced with all its subsets being 2-sets and $\sum K_i = k(n + 3) + 2i - 1$, for each $i \in [1, k_n]$.

Now we define a total labeling f on G as follows:

Label the vertices of the backbone by the elements of A with the ordering from left to right and label the backbone edges by the elements of C from right to left. Next we use the elements of K to label all the remaining edges and vertices, use the smaller labels for the vertices.

Now for each $1 \leq h \leq k - 1$, we have

$$\begin{aligned} wt(S_{n,n}^h) &= \sum_{j=nh-n+1}^{(h+1)n} [k(n + 3) + 2j - 1] + \frac{h + 1}{2} + \left\lceil \frac{k}{2} \right\rceil + \frac{h + 1}{2} \\ &+ k(n + 1) + k - h = 2kn^2 + 7kn + 2k + 1 + \left\lceil \frac{k}{2} \right\rceil + 4hn^2. \end{aligned}$$

In particular, we obtain that $a = wt(S_{n,n}^1) = 2(k + 2)n^2 + 7kn + 2k + 1 + \lceil \frac{k}{2} \rceil$ and $d = wt(S_{n,n}^{h+1}) - wt(S_{n,n}^h) = 4n^2$, then G has a super $(2(k + 2)n^2 + 7kn + 2k + 1 + \lceil \frac{k}{2} \rceil, 4n^2)$ - $S_{n,n}$ -antimagic total labeling. ■

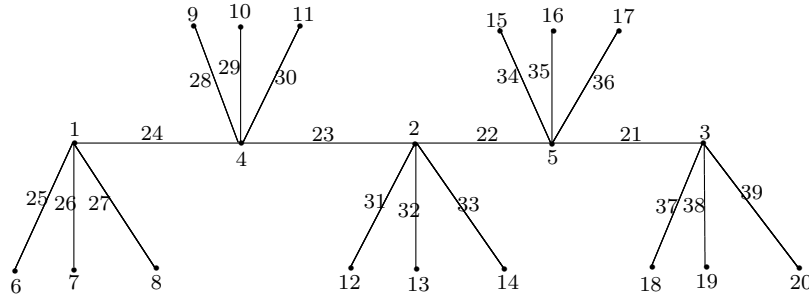


Figure 3. Super $(245, 36)$ - $S_{3,3}$ -antimagic total graph.

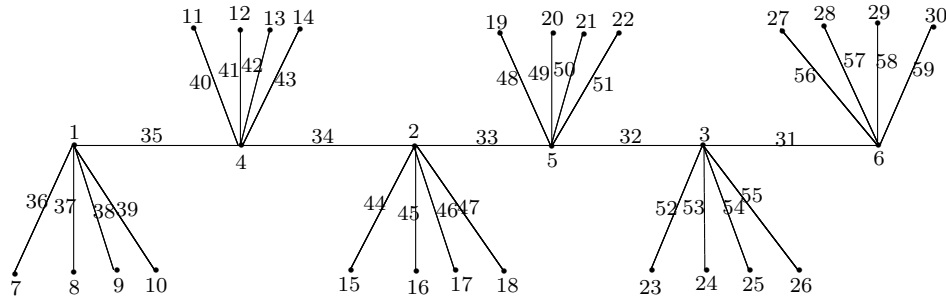


Figure 4. Super $(440, 64)$ - $S_{4,4}$ -antimagic total graph.

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