

ON SUPER EDGE-ANTIMAGICNESS OF SUBDIVIDED STARS

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Abstract

Enomoto, Llado, Nakamigawa and Ringel (1998) defined the concept of a super $(a, 0)$ -edge-antimagic total labeling and proposed the conjecture that every tree is a super $(a, 0)$ -edge-antimagic total graph. In the support of this conjecture, the present paper deals with different results on super (a, d) -edge-antimagic total labeling of subdivided stars for $d \in \{0, 1, 2, 3\}$.

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1. INTRODUCTION

All graphs in this paper are finite, undirected and simple. For a graph G , $V(G)$ and $E(G)$ denote the vertex-set and the edge-set, respectively. A (v, e) -graph G is a graph such that $|V(G)| = v$ and $|E(G)| = e$. A general reference for graph-theoretic ideas can be seen in [30]. A *labeling* (or *valuation*) of a graph is a map that carries graph elements to numbers (usually to positive or non-negative integers). In this paper, the domain will be the set of all vertices and edges and such a labeling is called a *total labeling*. Some labelings use the vertex-set only or the edge-set only and we shall call them *vertex-labelings* or *edge-labelings*, respectively.

Definition 1.1. An (s, d) -edge-antimagic vertex $((s, d)$ -EAV) labeling of a (v, e) -graph G is a bijective function $\lambda : V(G) \rightarrow \{1, 2, \dots, v\}$ such that the set of edge-sums of all edges in G , $w(xy) = \{\lambda(x) + \lambda(y) : xy \in E(G)\}$, forms an arithmetic progression $\{s, s + d, s + 2d, \dots, s + (e - 1)d\}$, where $s > 0$ and $d \geq 0$ are two fixed integers.

Definition 1.2. A bijection $\lambda : V(G) \cup E(G) \rightarrow \{1, 2, \dots, v + e\}$ is called an (a, d) -edge-antimagic total $((a, d)$ -EAT) labeling of a (v, e) -graph G if the set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in E(G)\}$ forms an arithmetic progression starting from a and having common difference d , where $a > 0$ and $d \geq 0$ are two fixed integers. A graph that admits an (a, d) -EAT labeling is called an (a, d) -EAT graph.

Definition 1.3. If λ is an (a, d) -EAT labeling such that $\lambda(V(G)) = \{1, 2, \dots, v\}$, then λ is called a *super* (a, d) -EAT labeling and G is known as a *super* (a, d) -EAT graph.

In Definitions 1.2 and 1.3, if $d = 0$ then an $(a, 0)$ -EAT labeling is called an *edge-magic total* (EMT) labeling and a *super* $(a, 0)$ -EAT labeling is called a *super edge magic total* (SEMT) labeling. Moreover, in general a is called a *minimum edge-weight* but particularly a *magic constant* when $d = 0$. The definition of an (a, d) -EAT labeling was introduced by Simanjuntak, Bertault and Miller in [27] as a natural extension of *magic valuation* defined by Kotzig and Rosa [19, 20]. A *super* (a, d) -EAT labeling is a natural extension of the notion of *super edge-magic labeling* defined by Enomoto, Llado, Nakamigawa and Ringel. Moreover, Enomoto *et al.* [7] proposed the following conjecture.

Conjecture 1.1. *Every tree admits a super $(a, 0)$ -EAT labeling.*

In the support of this conjecture, many authors have considered a *super* $(a, 0)$ -EAT labeling for different particular classes of trees. Lee and Shah [21] verified this conjecture by a computer search for trees with at most 17 vertices. For different values of d , the results related to a *super* (a, d) -EAT labeling can

be found for w-trees [12], extended w-trees [13, 14], stars [22], subdivided stars [15, 16, 17, 18, 25, 26, 23, 24], path-like trees [3], caterpillars [19, 20, 29], disjoint union of stars and books [9], and wheels, fans and friendship graphs [28], paths and cycles [27] and complete bipartite graphs [1]. For detail studies of a super (a, d) -EAT labeling reader can see [2, 4, 5, 6, 8, 9, 10, 11].

Definition 1.4. Let $n_i \geq 1$, $1 \leq i \leq r$, and $r \geq 3$. A *subdivided star* $T(n_1, n_2, \dots, n_r)$ is a tree obtained by inserting $n_i - 1$ vertices to each of the i th edge of the star $K_{1,r}$, where for all $n_i = 1$, $T(\underbrace{1, 1, \dots, 1}_{r\text{-times}}) \cong K_{1,r}$. Moreover

suppose that $V(G) = \{c\} \cup \{x_i^{l_i} : 1 \leq i \leq r ; 1 \leq l_i \leq n_i\}$ is the vertex-set and $E(G) = \{c x_i^1 : 1 \leq i \leq r\} \cup \{x_i^{l_i} x_i^{l_i+1} : 1 \leq i \leq r ; 1 \leq l_i \leq n_i - 1\}$ is the edge-set of the subdivided star $G \cong T(n_1, n_2, \dots, n_r)$, thus $v = |V(G)| = \sum_{i=1}^r n_i + 1$ and $e = |E(G)| = \sum_{i=1}^r n_i$.

Lu [23, 24] called the subdivided star $T(n_1, n_2, n_3)$ as a three-path tree and proved that it is a super $(a, 0)$ -EAT graph if n_1 and n_2 are odd with $n_3 = n_2 + 1$ or $n_3 = n_2 + 2$. Ngurah *et al.* [25] proved that the subdivided star $T(n_1, n_2, n_3)$ is also a super $(a, 0)$ -EAT graph if $n_3 = n_2 + 3$ or $n_3 = n_2 + 4$. Salman *et al.* [26] found a super $(a, 0)$ -EAT labeling of subdivided stars $T(\underbrace{n, n, n, \dots, n}_{r\text{-times}})$, where

$n \in \{2, 3\}$. Moreover, Javaid *et al.* [15, 16, 17] found the super (a, d) -EAT labelings on different subclasses of subdivided stars for $d \in \{0, 1, 2\}$. However, the investigation of the different results related to a super (a, d) -EAT labeling of the subdivided star $T(n_1, n_2, n_3, \dots, n_r)$ with unequal n_i for $1 \leq i \leq r$ is still open. In this paper, we investigate a super (a, d) -EAT labeling on the subdivided stars for all possible values of d .

2. BASIC RESULTS

In this section, we present some basic results which will be used frequently to prove the main results.

Ngurah *et al.* [25] found lower and upper bounds of the minimum edge-weight a for a subclass of the subdivided stars, which is stated as follows.

Lemma 2.1. *If $T(n_1, n_2, n_3)$ is a super $(a, 0)$ -EAT graph, then $\frac{1}{2l}(5l^2 + 3l + 6) \leq a \leq \frac{1}{2l}(5l^2 + 11l - 6)$, where $l = \sum_{i=1}^3 n_i$.*

The lower and upper bounds of the minimum edge-weight a for another subclass of subdivided stars established by Salman *et al.* [26] are given below.

Lemma 2.2. If $T(\underbrace{n, n, \dots, n}_{n\text{-times}})$ is a super $(a, 0)$ -EAT graph, then $\frac{1}{2l}(5l^2 + (9 - 2n)l + n^2 - n) \leq a \leq \frac{1}{2l}(5l^2 + (2n + 5)l + n - n^2)$, where $l = n^2$.

Moreover, the following lemma presents the lower and upper bound of the minimum edge-weight a for the most generalized subclass of subdivided stars proved by Javaid and Bhatti [17, 18].

Lemma 2.3. If $T(n_1, n_2, n_3, \dots, n_r)$ has a super (a, d) -EAT labeling, then $\frac{1}{2l}(5l^2 + r^2 - 2lr + 9l - r - (l - 1)ld) \leq a \leq \frac{1}{2l}(5l^2 - r^2 + 2lr + 5l + r - (l - 1)ld)$, where $l = \sum_{i=1}^r n_i$ and $d \in \{0, 1, 2, 3\}$.

Bača and Miller [4] state a necessary condition for a graph to be super (a, d) -EAT, which provides an upper bound on the parameter d . Let a (v, e) -graph G be a super (a, d) -EAT. The minimum possible edge-weight is at least $v + 4$. The maximum possible edge-weight is no more than $3v + e - 1$. Thus $a + (e - 1)d \leq 3v + e - 1$ or $d \leq \frac{2v + e - 5}{e - 1}$. For any subdivided star, where $v = e + 1$, it follows that $d \leq 3$.

Let us recall the following proposition which we will use frequently in the proofs of the main results.

Proposition 2.4 [3]. If a (v, e) -graph G has an (s, d) -EAV labeling, then

- (i) G has a super $(s + v + 1, d + 1)$ -EAT labeling,
- (ii) G has a super $(s + v + e, d - 1)$ -EAT labeling.

3. SUPER (a, d) -EAT LABELING OF SUBDIVIDED STARS

This section deals with the main results related to super (a, d) -EAT labelings on more generalized families of subdivided stars for all possible values of d .

3.1. When $n \equiv 0 \pmod{2}$

The results of super (a, d) -EAT labelings for different values of d on the class of subdivided stars $T(n, n + 1, n_3, \dots, n_r)$ when $n \equiv 0 \pmod{2}$ are as follows.

Theorem 3.1. For $n \equiv 0 \pmod{2}$ and $r \geq 3$, $G \cong T(n, n + 1, n_3, \dots, n_r)$ admits a super $(a, 0)$ -EAT labeling with $a = 2v + s - 1$ and a super $(a, 2)$ -EAT labeling with $a = v + s + 1$, where $v = |V(G)|$ and $s = (n + 3) + \sum_{m=3}^r [2^{m-3}(n + 1)]$ and $n_m = 2^{m-2}(n + 1)$ for $3 \leq m \leq r$.

Proof. According to the definition of graph G , we have that $v = (2n + 2) + \sum_{m=3}^r [2^{m-2}(n + 1)]$ and $e = v - 1$. Define $\lambda : V(G) \rightarrow \{1, 2, \dots, v\}$ as follows.

$$\lambda(c) = 1.$$

For even $1 \leq l_i \leq n_i$, where $i = 1, 2$ and $3 \leq i \leq r$, let

$$\lambda(u) = \begin{cases} 1 + \frac{l_1}{2}, & \text{for } u = x_1^{l_1}, \\ (n+2) - \frac{l_2}{2}, & \text{for } u = x_2^{l_2}. \end{cases}$$

$$\lambda(x_i^{l_i}) = (n+2) + \sum_{m=3}^i [2^{m-3}(n+1)] - \frac{l_i}{2}.$$

For odd $1 \leq l_i \leq n_i$ and $\alpha = (n+1) + \sum_{m=3}^r [2^{m-3}(n+1)]$, where $i = 1, 2$ and $3 \leq i \leq r$, let

$$\lambda(u) = \begin{cases} \alpha + \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ (\alpha + n + 2) - \frac{l_2+1}{2}, & \text{for } u = x_2^{l_2}, \end{cases}$$

and $\lambda(x_i^{l_i}) = (\alpha + n + 2) + \sum_{m=3}^i [2^{m-3}(n+1)] - \frac{l_i+1}{2}$.

The set of all edge-sums $\{\lambda(x) + \lambda(y) : xy \in E(G)\}$ generated by the above formulas forms a consecutive integer sequence $(\alpha+1)+1, (\alpha+1)+2, \dots, (\alpha+1)+e$, where $s = \alpha + 2$. Therefore, by Proposition 2.1, λ can be extended to a super $(a, 0)$ -EAT labeling with $a = 2v - 1 + s = 2v + (n+2) + \sum_{m=3}^r [2^{m-3}(n+1)]$ and to a super $(a, 2)$ -EAT labeling with $a = v + 1 + s = v + (n+4) + \sum_{m=3}^r [2^{m-3}(n+1)]$. ■

Theorem 3.2. For $n \equiv 0 \pmod{2}$ and $r \geq 3$, $G \cong T(n, n+1, n_3, \dots, n_r)$ admits a super $(a, 1)$ -EAT labeling with $a = 2v+2$, where $v = |V(G)|$ and $n_m = 2^{m-2}(n+1)$ for $3 \leq m \leq r$.

Proof. We define the vertex labeling $\lambda : V(G) \rightarrow \{1, 2, \dots, v\}$ as follows.

$$\lambda(c) = 1.$$

For $1 \leq l_i \leq n_i$, where $i = 1, 2$ and $3 \leq i \leq r$,

$$\lambda(u) = \begin{cases} l_1 + 1, & \text{for } u = x_1^{l_1}, \\ (2n+3) - l_2, & \text{for } u = x_2^{l_2}, \end{cases}$$

and $\lambda(x_i^{l_i}) = (2n+3) + \sum_{m=3}^i [2^{m-2}(n+1)] - l_i$.

Suppose $\alpha = (2n+2) + \sum_{m=3}^r [2^{m-2}(n+1)]$ and define $\lambda : E(G) \rightarrow \{v+1, v+2, \dots, v+e\}$ as follows. For $l_i = 1$, where $i = 1, 2$ and $3 \leq i \leq r$, let

$$\lambda(cu) = \begin{cases} (2\alpha - 1), & \text{for } u = x_1^1, \\ 2\alpha - (n + 1), & \text{for } u = x_2^1, \end{cases}$$

and $\lambda(cx_i^1) = 2\alpha - (n + 1) - \sum_{m=3}^i [2^{m-3}(n + 1)]$.

For $1 \leq l_i \leq n_i - 1$, where $i = 1, 2$ and $3 \leq i \leq r$,

$$\lambda(x_i^{1_i} x_i^{1_i+1}) = \begin{cases} (2\alpha - 1) - l_1, & \text{for } i = 1, \\ 2\alpha - 2(n + 1) + l_2, & \text{for } i = 2, \end{cases}$$

and $\lambda(x_i^{1_i} x_i^{1_i+1}) = 2\alpha - 2(n + 1) - \sum_{m=3}^i [2^{m-2}(n + 1)] + l_i$, for $3 \leq i \leq r$.

The set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in V(G)\}$ generated by the above formulas forms an integer sequence $2v + 2, 2v + 3, \dots, 2v + 1 + e$ with difference 1. Consequently, λ is a super $(a, 1)$ -EAT labeling with $a = 2v + 2$. ■

Theorem 3.3. For $n \equiv 0 \pmod{2}$ and $r \geq 3$, the graph $G \cong T(n, n+1, n_3, \dots, n_r)$ admits a super $(a, 3)$ -EAT labeling with $a = v + 4$, where $v = |V(G)|$ and $n_m = 2^{m-2}(n + 1)$ for $3 \leq m \leq r$.

Proof. Consider the vertex labeling as in the proof of Theorem 3.2. Now, we define the edge labeling $\lambda : E(G) \rightarrow \{v + 1, v + 2, \dots, v + e\}$ as follows.

For $l_i = 1$, where $i = 1, 2$ and, $3 \leq i \leq r$, let

$$\lambda(cu) = \begin{cases} (\alpha + 1), & \text{for } u = x_1^1, \\ (\alpha + n + 1), & \text{for } u = x_2^1, \end{cases}$$

and $\lambda(cx_i^1) = (\alpha + n + 1) + \sum_{m=3}^i [2^{m-3}(n + 1)]$.

For $1 \leq l_i \leq n_i - 1$,

$$\lambda(x_i^{1_i} x_i^{1_i+1}) = \begin{cases} (\alpha + 1) + l_1, & \text{for } i = 1, \\ (\alpha + 2n + 2) - l_2, & \text{for } i = 2, \end{cases}$$

and $\lambda(x_i^{1_i} x_i^{1_i+1}) = (\alpha + 2n + 2) + \sum_{m=3}^i [2^{m-2}(n + 1)] - l_i$, for $3 \leq i \leq r$.

The set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in V(G)\}$ generated by the above formulas forms an integer sequence $(v + 1) + 3(1), (v + 1) + 3(2), (v + 1) + 3(3), \dots, (v + 1) + 3(e)$ with difference 3. Consequently, λ is a super $(a, 3)$ -EAT labeling with $a = v + 4$. ■

3.2. When $n \equiv 1 \pmod{2}$

The results of super (a, d) -EAT labelings for different values of d on the class of subdivided stars $T(n, n, n+1, n_4, \dots, n_r)$ when $n \equiv 1 \pmod{2}$ are as follows.

Theorem 3.4. For $n \equiv 1 \pmod{2}$ and $r \geq 4$, $G \cong T(n, n, n+1, n_4, \dots, n_r)$ admits a super $(a, 0)$ -EAT labeling with $a = 2v + s - 1$ and a super $(a, 2)$ -EAT labeling with $a = v + s + 1$, where $v = |V(G)|$ and $s = \frac{3(n+1)}{2} + \sum_{m=4}^r [2^{m-4}(n+1)]$, and $n_m = 2^{m-3}(n+1)$ for $4 \leq m \leq r$.

Proof. Let $G \cong T(n, n, n+1, n_4, \dots, n_r)$. Then $v = (3n+2) + \sum_{m=4}^r [2^{m-3}(n+1)]$ and $e = v - 1$. Define $\lambda : V(G) \rightarrow \{1, 2, \dots, v\}$ as follows.

$$\lambda(c) = \frac{n+1}{2}.$$

For even $1 \leq l_i \leq n_i$, where $i = 1, 2, 3$ and $4 \leq i \leq r$,

$$\lambda(u) = \begin{cases} \frac{n+1}{2} - \frac{l_1}{2}, & \text{for } u = x_2^{l_1}, \\ \frac{n+1}{2} + \frac{l_2}{2}, & \text{for } u = x_2^{l_2}, \\ \frac{3(n+1)}{2} - \frac{l_3}{2}, & \text{for } u = x_3^{l_3}. \end{cases}$$

$$\lambda(x_i^{l_i}) = \frac{3(n+1)}{2} + \sum_{m=4}^i [2^{m-4}(n+1)] - \frac{l_i}{2}.$$

For odd $1 \leq l_i \leq n_i$ and $\alpha = \frac{3(n+1)}{2} + \sum_{m=4}^r [2^{m-4}(n+1)]$, where $i = 1, 2, 3$ and $4 \leq i \leq r$,

$$\lambda(u) = \begin{cases} \alpha + \frac{n+3}{2} - \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ \alpha + \frac{n+1}{2} + \frac{l_2+1}{2}, & \text{for } u = x_2^{l_2}, \\ \alpha + \frac{3n+5}{2} - \frac{l_3+1}{2}, & \text{for } u = x_3^{l_3}, \end{cases}$$

and

$$\lambda(x_i^{l_i}) = \alpha + \frac{3n+5}{2} + \sum_{m=4}^i [2^{m-4}(n+1)] - \frac{l_i+1}{2}.$$

The set of all edge-sums $\{\lambda(x) + \lambda(y) : xy \in E(G)\}$ generated by the above formulas forms a consecutive integer sequence $(\alpha+1)+1, (\alpha+1)+2, \dots, (\alpha+1)+e$, where $s = \alpha + 2$. Therefore, by Proposition 2.1, λ can be extended to a super $(a, 0)$ -EAT labeling with $a = 2v + s - 1 = 2v + \frac{3(n+1)}{2} + \sum_{m=4}^r [2^{m-4}(n+1)]$ and to a super $(a, 2)$ -EAT labeling with $a = v + 1 + s = v + \frac{3n+7}{2} + \sum_{m=4}^r [2^{m-4}(n+1)]$. ■

Theorem 3.5. For $n \equiv 1 \pmod{2}$ and $r \geq 4$, $G \cong T(n, n, n+1, n_4, \dots, n_r)$ admits a super $(a, 1)$ -EAT labeling with $a = 2v + 2$, where $v = |V(G)|$ and $n_m = 2^{m-3}(n+1)$ for $4 \leq m \leq r$.

Proof. We define the vertex labeling $\lambda : V(G) \rightarrow \{1, 2, \dots, v\}$ as follows.

$$\lambda(c) = n + 1.$$

For $1 \leq l_i \leq n_i$, where $i = 1, 2, 3$ and $4 \leq i \leq r$, let

$$\lambda(u) = \begin{cases} (n+1) - l_1, & \text{for } u = x_1^{l_1}, \\ (n+1) + l_2, & \text{for } u = x_2^{l_2}, \\ 3(n+1) - l_3, & \text{for } u = x_3^{l_3}, \end{cases}$$

and $\lambda(x_i^{l_i}) = 3(n+1) + \sum_{m=4}^i [2^{m-3}(n+1)] - l_i$.

Suppose that $\alpha = (2n+1) + \sum_{m=3}^r [2^{m-3}(n+1)]$ and define $\lambda : E(G) \rightarrow \{v+1, v+2, \dots, v+e\}$ as follows.

For $l_i = 1$, where $i = 1, 2, 3$ and $4 \leq i \leq r$,

$$\lambda(cu) = \begin{cases} 2\alpha - n, & \text{for } u = x_1^1, \\ 2\alpha - (n+1), & \text{for } u = x_2^1, \\ 2\alpha - (2n+1), & \text{for } u = x_3^1, \end{cases}$$

and $\lambda(cx_i^1) = 2\alpha - (2n+1) - \sum_{m=4}^i [2^{m-4}(n+1)]$ respectively.

For $1 \leq l_i \leq n_i - 1$, $i = 1, 2, 3$ and $4 \leq i \leq r$,

$$\lambda(x_i^{1_i} x_i^{1_i+1}) = \begin{cases} 2\alpha - n + l_1, & \text{for } i = 1, \\ 2\alpha - (n+1) - l_2, & \text{for } i = 2, \\ 2\alpha - (3n+2) + l_3, & \text{for } i = 3, \end{cases}$$

and $\lambda(x_i^{1_i} x_i^{1_i+1}) = 2\alpha - (3n+2) - \sum_{m=4}^i [2^{m-3}(n+1)] + l_i$.

The set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in V(G)\}$ generated by the above formulas forms an integer sequence $2v+2, 2v+3, \dots, 2v+1+e$ with difference 1. Consequently, λ is a super $(a, 1)$ -EAT labeling with $a = 2v+2$. ■

Theorem 3.6. For $n \equiv 1 \pmod{2}$ and $r \geq 4$, $G \cong T(n, n, n+1, n_4, \dots, n_r)$ admits a super $(a, 3)$ -EAT labeling with $a = v+4$, where $v = |V(G)|$ and $n_m = 2^{m-3}(n+1)$ for $4 \leq m \leq r$.

Proof. Consider the vertex labeling as in the proof of Theorem 3.2.2. Now, we define the edge labeling $\lambda : E(G) \rightarrow \{v+1, v+2, \dots, v+e\}$ as follows.

For $l_i = 1$, where $i = 1, 2, 3$ and $4 \leq i \leq r$,

$$\lambda(cu) = \begin{cases} \alpha + n, & \text{for } u = x_1^1, \\ \alpha + (n+1), & \text{for } u = x_2^1, \\ \alpha + (2n+1), & \text{for } u = x_3^1, \end{cases}$$

and $\lambda(cx_i^1) = \alpha + (2n+1) + \sum_{m=4}^i [2^{m-4}(n+1)]$.

For $1 \leq l_i \leq n_i - 1$, where $i = 1, 2, 3$ and $4 \leq i \leq r$,

$$\lambda(x_i^{1_i} x_i^{1_i+1}) = \begin{cases} \alpha + n - l_1, & \text{for } i = 1, \\ \alpha + (n+1) + l_2, & \text{for } i = 2, \\ \alpha + (3n+1) - l_3, & \text{for } i = 3, \end{cases}$$

and $\lambda(x_i^{1_i} x_i^{1_i+1}) = \alpha + (3n+2) + \sum_{m=4}^i [2^{m-3}(n+1)] - l_i$.

The set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in V(G)\}$ generated by the above formulas forms an integer sequence $(v+1) + 3(1), (v+1) + 3(2), (v+1) + 3(3), \dots, (v+1) + 3(e)$ with difference 3. Consequently, λ admits a super $(a, 3)$ -EAT labeling with $a = v+4$. ■

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