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ON SUPER EDGE-ANTIMAGICNESS OF SUBDIVIDED STARS

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Abstract

Enomoto, Llado, Nakamigawa and Ringel (1998) defined the concept of a super $(a, 0)$-edge-antimagic total labeling and proposed the conjecture that every tree is a super $(a, 0)$-edge-antimagic total graph. In the support of this conjecture, the present paper deals with different results on super (a, d)-edge-antimagic total labeling of subdivided stars for $d \in \{0, 1, 2, 3\}$.

Keywords: super (a, d)-EAT labeling, stars, subdivision of stars.

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1. Introduction

All graphs in this paper are finite, undirected and simple. For a graph G, $V(G)$ and $E(G)$ denote the vertex-set and the edge-set, respectively. A (v, e)-graph G is a graph such that $|V(G)| = v$ and $|E(G)| = e$. A general reference for graph-theoretic ideas can be seen in [30]. A *labeling* (or *valuation*) of a graph is a map that carries graph elements to numbers (usually to positive or non-negative integers). In this paper, the domain will be the set of all vertices and edges and such a labeling is called a *total labeling*. Some labelings use the vertex-set only or the edge-set only and we shall call them *vertex-labelings* or *edge-labelings*, respectively.

Definition 1.1. An (s, d)-*edge-antimagic vertex* ((s, d)-EAV) labeling of a (v, e)-graph G is a bijective function $\lambda : V(G) \to \{1, 2, \ldots, v\}$ such that the set of edge-sums of all edges in G, $w(xy) = \{\lambda(x) + \lambda(y) : xy \in E(G)\}$, forms an arithmetic progression $\{s, s + d, s + 2d, \ldots, s + (e - 1)d\}$, where $s > 0$ and $d \geq 0$ are two fixed integers.

Definition 1.2. A bijection $\lambda : V(G) \cup E(G) \to \{1, 2, \ldots, v + e\}$ is called an (a, d)-*edge-antimagic total* ((a, d)-EAT) labeling of a (v, e)-graph G if the set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in V(G)\}$ forms an arithmetic progression starting from a and having common difference d, where $a > 0$ and $d \geq 0$ are two fixed integers. A graph that admits an (a, d)-EAT labeling is called an (a, d)-EAT graph.

Definition 1.3. If λ is an (a, d)-EAT labeling such that $\lambda(V(G)) = \{1, 2, \ldots, v\}$, then λ is called a *super* (a, d)-EAT *labeling* and G is known as a *super* (a, d)-EAT *graph*.

In Definitions 1.2 and 1.3, if $d = 0$ then an $(a, 0)$-EAT labeling is called an *edge-magic total* (EMT) labeling and a super $(a, 0)$-EAT labeling is called a *super edge magic total* (SEMT) *labeling*. Moreover, in general a is called a *minimum edge-weight* but particularly a *magic constant* when $d = 0$. The definition of an (a, d)-EAT labeling was introduced by Simanjuntak, Bertault and Miller in [27] as a natural extension of *magic valuation* defined by Kotzig and Rosa [19, 20]. A super (a, d)-EAT labeling is a natural extension of the notion of *super edge-magic labeling* defined by Enomoto, Llado, Nakamigawa and Ringel. Moreover, Enomoto *et al.* [7] proposed the following conjecture.

Conjecture 1.1. *Every tree admits a super* $(a, 0)$*-EAT labeling.*

In the support of this conjecture, many authors have considered a super $(a, 0)$-EAT labeling for different particular classes of trees. Lee and Shah [21] verified this conjecture by a computer search for trees with at most 17 vertices. For different values of d, the results related to a super (a, d)-EAT labeling can be found for w-trees [12], extended w-trees [13, 14], stars [22], subdivided stars [15, 16, 17, 18, 25, 26, 23, 24], path-like trees [3], caterpillars [19, 20, 29], disjoint union of stars and books [9], and wheels, fans and friendship graphs [28], paths and cycles [27] and complete bipartite graphs [1]. For detail studies of a super (a,d)-EAT labeling reader can see [2, 4, 5, 6, 8, 9, 10, 11].

Definition 1.4. Let $n_i \geq 1$, $1 \leq i \leq r$, and $r \geq 3$. A *subdivided star* $T(n_1, n_2, \ldots, n_r)$ is a tree obtained by inserting $n_i - 1$ vertices to each of the ith edge of the star $K_{1,r}$, where for all $n_i = 1$, $T\underbrace{(1, 1, \ldots, 1)}_{r-times} \cong K_{1,r}$. Moreover suppose that $V(G) = \{c\} \cup \{x_i^{l_i} \ : \ 1 \leq i \leq r \ ; \ 1 \leq l_i \leq n_i\}$ is the vertex-set and $E(G) = \{c\ x_i^1 \ : \ 1 \leq i \leq r\} \cup \{x_i^{l_i} x_i^{l_i+1} \ : \ 1 \leq i \leq r \ ; \ 1 \leq l_i \leq n_i - 1\}$ is the edge-set of the subdivided star $G \cong T(n_1, n_2, \ldots, n_r)$, thus $v = |V(G)| = \sum_{i=1}^{r} n_i + 1$ and $e = |E(G)| = \sum_{i=1}^{r} n_i$.

Lu [23, 24] called the subdivided star $T(n_1, n_2, n_3)$ as a three-path tree and proved that it is a super $(a, 0)$-EAT graph if n_1 and n_2 are odd with $n_3 = n_2 + 1$ or $n_3 = n_2 + 2$. Ngurah *et al.* [25] proved that the subdivided star $T(n_1, n_2, n_3)$ is also a super $(a, 0)$-EAT graph if $n_3 = n_2 + 3$ or $n_3 = n_2 + 4$. Salman *et al.* [26] found a super $(a, 0)$-EAT labeling of subdivided stars $T\underbrace{(n, n, n, \ldots, n)}_{r-times}$, where $n \in \{2, 3\}$. Moreover, Javaid *et al.* [15, 16, 17] found the super (a, d)-EAT labelings on different subclasses of subdivided stars for $d \in \{0, 1, 2\}$. However, the investigation of the different results related to a super (a, d)-EAT labeling of the subdivided star $T(n_1, n_2, n_3, \ldots, n_r)$ with unequal n_i for $1 \leq i \leq r$ is still open. In this paper, we investigate a super (a, d)-EAT labeling on the subdivided stars for all possible values of d.

2. BASIC RESULTS

In this section, we present some basic results which will be used frequently to prove the main results.

Ngurah *et al.* [25] found lower and upper bounds of the minimum edge-weight a for a subclass of the subdivided stars, which is stated as follows.

Lemma 2.1. *If $T(n_1, n_2, n_3)$ is a super $(a, 0)$-EAT graph, then* $\frac{1}{2l}(5l^2 + 3l + 6) \leq a \leq \frac{1}{2l}(5l^2 + 11l - 6)$, *where* $l = \sum_{i=1}^{3} n_i$.

The lower and upper bounds of the minimum edge-weight a for another subclass of subdivided stars established by Salman *et al.* [26] are given below.

Lemma 2.2. *If* $T\underbrace{(n, n, \ldots, n)}_{n-times}$ *is a super* $(a, 0)$*-EAT graph, then* $\frac{1}{2l}(5l^2 + (9 - 2n)l + n^2 - n) \leq a \leq \frac{1}{2l}(5l^2 + (2n+5)l + n - n^2)$*, where* $l = n^2$.

Moreover, the following lemma presents the lower and upper bound of the minimum edge-weight a for the most generalized subclass of subdivided stars proved by Javaid and Bhatti [17, 18].

Lemma 2.3. *If* $T(n_1, n_2, n_3, \ldots, n_r)$ *has a super* (a, d)*-EAT labeling, then* $\frac{1}{2l}(5l^2 + r^2 - 2lr + 9l - r - (l-1)ld) \leq a \leq \frac{1}{2l}(5l^2 - r^2 + 2lr + 5l + r - (l-1)ld)$*, where* $l = \sum_{i=1}^{r} n_i$ *and* $d \in \{0, 1, 2, 3\}$.

Bača and Miller [4] state a necessary condition for a graph to be super (a, d)-EAT, which provides an upper bound on the parameter d. Let a (v, e)-graph G be a super (a, d)-EAT. The minimum possible edge-weight is at least $v + 4$. The maximum possible edge-weight is no more than $3v + e - 1$. Thus $a + (e-1)d \leq 3v + e - 1$ or $d \leq \frac{2v+e-5}{e-1}$. For any subdivided star, where $v = e + 1$, it follows that $d \leq 3$.

Let us recall the following proposition which we will use frequently in the proofs of the main results.

Proposition 2.4 [3]**.** *If a* (v, e)*-graph* G *has an* (s, d)*-EAV labeling, then*

(i) G *has a super* $(s + v + 1, d + 1)$*-EAT labeling,*

(ii) G *has a super* $(s + v + e, d - 1)$*-EAT labeling.*

3. Super (a, d)-EAT Labeling of Subdivided Stars

This section deals with the main results related to super (a, d)-EAT labelings on more generalized families of subdivided stars for all possible values of d.

3.1. When $n \equiv 0 \pmod 2$

The results of super (a, d)-EAT labelings for different values of d on the class of subdivided stars $T(n, n+1, n_3, \ldots, n_r)$ when $n \equiv 0 \pmod 2$ are as follows.

Theorem 3.1. *For* $n \equiv 0 \pmod 2$ *and* $r \geq 3$*,* $G \cong T(n, n+1, n_3, \ldots, n_r)$ *admits a super* $(a, 0)$*-EAT labeling with* $a = 2v + s - 1$ *and a super* $(a, 2)$*-EAT labeling with* $a = v + s + 1$*, where* $v = |V(G)|$ *and* $s = (n+3) + \sum_{m=3}^{r}[2^{m-3}(n+1)]$ *and* $n_m = 2^{m-2}(n+1)$ *for* $3 \leq m \leq r$.

Proof. According to the definition of graph G, we have that $v = (2n+2) + \sum_{m=3}^{r}[2^{m-2}(n+1)]$ and $e = v - 1$. Define $\lambda : V(G) \rightarrow \{1, 2, \ldots, v\}$ as follows.

$$\lambda(c) = 1.$$

For even $1 \le l_i \le n_i$, where $i = 1, 2$ and $3 \le i \le r$, let

$$\lambda(u) = \begin{cases} 1 + \frac{l_1}{2}, & \text{for } u = x_1^{l_1}, \\ (n+2) - \frac{l_2}{2}, & \text{for } u = x_2^{l_2}. \end{cases}$$

$$\lambda(x_i^{l_i}) = (n+2) + \sum_{m=3}^{i} [2^{m-3}(n+1)] - \frac{l_i}{2}.$$

For odd $1 \le l_i \le n_i$ and $\alpha = (n+1) + \sum_{m=3}^{r} [2^{m-3}(n+1)]$, where $i = 1, 2$ and $3 \le i \le r$, let

$$\lambda(u) = \begin{cases} \alpha + \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ (\alpha+n+2) - \frac{l_2+1}{2} & \text{for } u = x_2^{l_2}, \end{cases}$$

and $\lambda(x_i^{l_i}) = (\alpha+n+2) + \sum_{m=3}^{i} [2^{m-3}(n+1)] - \frac{l_i+1}{2}$.

The set of all edge-sums $\{\lambda(x) + \lambda(y) : xy \in E(G)\}$ generated by the above formulas forms a consecutive integer sequence $(\alpha+1)+1, (\alpha+1)+2, \ldots, (\alpha+1)+e$, where $s = \alpha + 2$. Therefore, by Proposition 2.1, λ can be extended to a super $(a, 0)$-EAT labeling with $a = 2v-1+s = 2v+(n+2)+\sum_{m=3}^{r} [2^{m-3}(n+1)]$ and to a super $(a, 2)$-EAT labeling with $a = v+1+s = v+(n+4)+\sum_{m=3}^{r} [2^{m-3}(n+1)]$. ∎

Theorem 3.2. *For $n \equiv 0 \pmod 2$ and $r \ge 3$, $G \cong T(n, n+1, n_3, \ldots, n_r)$ admits a super $(a, 1)$-EAT labeling with $a = 2v+2$, where $v = |V(G)|$ and $n_m = 2^{m-2}(n+1)$ for $3 \le m \le r$.*

Proof. We define the vertex labeling $\lambda : V(G) \to \{1, 2, \ldots, v\}$ as follows.

$$\lambda(c) = 1.$$

For $1 \le l_i \le n_i$, where $i = 1, 2$ and $3 \le i \le r$,

$$\lambda(u) = \begin{cases} l_1 + 1, & \text{for } u = x_1^{l_1}, \\ (2n+3) - l_2, & \text{for } u = x_2^{l_2}, \end{cases}$$

and $\lambda(x_i^{l_i}) = (2n+3) + \sum_{m=3}^{i} [2^{m-2}(n+1)] - l_i$.

Suppose $\alpha = (2n+2) + \sum_{m=3}^{r} [2^{m-2}(n+1)]$ and define $\lambda : E(G) \to \{v+1, v+2, \ldots, v+e\}$ as follows. For $l_i = 1$, where $i = 1, 2$ and $3 \le i \le r$, let

$$\lambda(cu) = \begin{cases} (2\alpha - 1), & \text{for } u = x_1^1, \\ \\ 2\alpha - (n+1), & \text{for } u = x_2^1, \end{cases}$$

and $\lambda(cx_i^1) = 2\alpha - (n+1) - \sum_{m=3}^{i}[2^{m-3}(n+1)]$.

For $1 \le l_i \le n_i - 1$, where $i = 1, 2$ and $3 \le i \le r$,

$$\lambda(x_i^{1_i} x_i^{1_i+1}) = \begin{cases} (2\alpha - 1) - l_1, & \text{for } i = 1, \\ \\ 2\alpha - 2(n+1) + l_2, & \text{for } i = 2, \end{cases}$$

and $\lambda(x_i^{1_i} x_i^{1_i+1}) = 2\alpha - 2(n+1) - \sum_{m=3}^{i}[2^{m-2}(n+1)] + l_i$, for $3 \le i \le r$.

The set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in V(G)\}$ generated by the above formulas forms an integer sequence $2v+2, 2v+3, \ldots, 2v+1+e$ with difference 1. Consequently, λ is a super $(a,1)$-EAT labeling with $a = 2v+2$. ■

Theorem 3.3. *For $n \equiv 0 \pmod 2$ and $r \ge 3$, the graph $G \cong T(n, n{+}1, n_3, \ldots, n_r)$ admits a super $(a,3)$-EAT labeling with $a = v+4$, where $v = |V(G)|$ and $n_m = 2^{m-2}(n+1)$ for $3 \le m \le r$.*

Proof. Consider the vertex labeling as in the proof of Theorem 3.2. Now, we define the edge labeling $\lambda : E(G) \to \{v+1, v+2, \ldots, v+e\}$ as follows.

For $l_i = 1$, where $i = 1, 2$ and, $3 \le i \le r$, let

$$\lambda(cu) = \begin{cases} (\alpha + 1), & \text{for } u = x_1^1, \\ \\ (\alpha + n + 1), & \text{for } u = x_2^1, \end{cases}$$

and $\lambda(cx_i^1) = (\alpha + n + 1) + \sum_{m=3}^{i}[2^{m-3}(n+1)]$.

For $1 \le l_i \le n_i - 1$,

$$\lambda(x_i^{1_i} x_i^{1_i+1}) = \begin{cases} (\alpha + 1) + l_1, & \text{for } i = 1, \\ \\ (\alpha + 2n + 2) - l_2, & \text{for } i = 2, \end{cases}$$

and $\lambda(x_i^{1_i} x_i^{1_i+1}) = (\alpha + 2n + 2) + \sum_{m=3}^{i}[2^{m-2}(n+1)] - l_i$, for $3 \le i \le r$.

The set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in V(G)\}$ generated by the above formulas forms an integer sequence $(v+1)+3(1), (v+1)+3(2), (v+1)+3(3), \ldots, (v+1)+3(e)$ with difference 3. Consequently, λ is a super $(a,3)$-EAT labeling with $a = v+4$. ■

3.2. When $n \equiv 1 \pmod 2$

The results of super (a,d)-EAT labelings for different values of d on the class of subdivided stars $T(n,n,n+1,n_4,\ldots,n_r)$ when $n \equiv 1 \pmod 2$ are as follows.

Theorem 3.4. *For $n \equiv 1 \pmod 2$ and $r \geq 4$, $G \cong T(n,n,n+1,n_4,\ldots,n_r)$ admits a super $(a,0)$-EAT labeling with $a = 2v+s-1$ and a super $(a,2)$-EAT labeling with $a = v+s+1$, where $v = |V(G)|$ and $s = \frac{3(n+1)}{2} + \sum_{m=4}^{r}[2^{m-4}(n+1)]$, and $n_m = 2^{m-3}(n+1)$ for $4 \leq m \leq r$.*

Proof. Let $G \cong T(n,n,n+1,n_4,\ldots,n_r)$. Then $v = (3n+2)+\sum_{m=4}^{r}[2^{m-3}(n+1)]$ and $e = v-1$. Define $\lambda : V(G) \to \{1,2,\ldots,v\}$ as follows.

$$\lambda(c) = \frac{n+1}{2}.$$

For even $1 \leq l_i \leq n_i$, where $i = 1,2,3$ and $4 \leq i \leq r$,

$$\lambda(u) = \begin{cases} \frac{n+1}{2} - \frac{l_1}{2}, & \text{for } u = x_2^{l_1}, \\ \frac{n+1}{2} + \frac{l_2}{2}, & \text{for } u = x_2^{l_2}, \\ \\ \frac{3(n+1)}{2} - \frac{l_3}{2}, & \text{for } u = x_3^{l_3}. \end{cases}$$

$$\lambda(x_i^{l_i}) = \frac{3(n+1)}{2} + \sum_{m=4}^{i}[2^{m-4}(n+1)] - \frac{l_i}{2}.$$

For odd $1 \leq l_i \leq n_i$ and $\alpha = \frac{3(n+1)}{2} + \sum_{m=4}^{r}[2^{m-4}(n+1)]$, where $i = 1,2,3$ and $4 \leq i \leq r$,

$$\lambda(u) = \begin{cases} \alpha + \frac{n+3}{2} - \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ \\ \alpha + \frac{n+1}{2} + \frac{l_2+1}{2}, & \text{for } u = x_2^{l_2}, \\ \\ \alpha + \frac{3n+5}{2} - \frac{l_3+1}{2}, & \text{for } u = x_3^{l_3}, \end{cases}$$

and

$$\lambda(x_i^{l_i}) = \alpha + \frac{3n+5}{2} + \sum_{m=4}^{i}[2^{m-4}(n+1)] - \frac{l_i+1}{2}.$$

The set of all edge-sums $\{\lambda(x)+\lambda(y) : xy \in E(G)\}$ generated by the above formulas forms a consecutive integer sequence $(\alpha+1)+1, (\alpha+1)+2, \ldots, (\alpha+1)+e$, where $s = \alpha+2$. Therefore, by Proposition 2.1, λ can be extended to a super $(a,0)$-EAT labeling with $a = 2v+s-1 = 2v+\frac{3(n+1)}{2}+\sum_{m=4}^{r}[2^{m-4}(n+1)]$ and to a super $(a,2)$-EAT labeling with $a = v+1+s = v+\frac{3n+7}{2}+\sum_{m=4}^{r}[2^{m-4}(n+1)]$. ∎

Theorem 3.5. *For $n \equiv 1 \pmod 2$ and $r \geq 4$, $G \cong T(n, n, n+1, n_4, \ldots, n_r)$ admits a super $(a, 1)$-EAT labeling with $a = 2v + 2$, where $v = |V(G)|$ and $n_m = 2^{m-3}(n+1)$ for $4 \leq m \leq r$.*

Proof. We define the vertex labeling $\lambda : V(G) \to \{1, 2, \ldots, v\}$ as follows.

$$\lambda(c) = n + 1.$$

For $1 \leq l_i \leq n_i$, where $i = 1, 2, 3$ and $4 \leq i \leq r$, let

$$\lambda(u) = \begin{cases} (n+1) - l_1, & \text{for } u = x_1^{l_1}, \\ (n+1) + l_2, & \text{for } u = x_2^{l_2}, \\ 3(n+1) - l_3, & \text{for } u = x_3^{l_3}, \end{cases}$$

and $\lambda(x_i^{l_i}) = 3(n+1) + \sum_{m=4}^{i}[2^{m-3}(n+1)] - l_i$.

Suppose that $\alpha = (2n+1) + \sum_{m=3}^{r}[2^{m-3}(n+1)]$ and define $\lambda : E(G) \to \{v+1, v+2, \ldots, v+e\}$ as follows.

For $l_i = 1$, where $i = 1, 2, 3$ and $4 \leq i \leq r$,

$$\lambda(cu) = \begin{cases} 2\alpha - n, & \text{for } u = x_1^1, \\ 2\alpha - (n+1), & \text{for } u = x_2^1, \\ 2\alpha - (2n+1), & \text{for } u = x_3^1, \end{cases}$$

and $\lambda(cx_i^1) = 2\alpha - (2n+1) - \sum_{m=4}^{i}[2^{m-4}(n+1)]$ respectively.

For $1 \leq l_i \leq n_i - 1$, $i = 1, 2, 3$ and $4 \leq i \leq r$,

$$\lambda(x_i^{1_i} x_i^{1_i+1}) = \begin{cases} 2\alpha - n + l_1, & \text{for } i = 1, \\ 2\alpha - (n+1) - l_2, & \text{for } i = 2, \\ 2\alpha - (3n+2) + l_3, & \text{for } i = 3, \end{cases}$$

and $\lambda(x_i^{1_i} x_i^{1_i+1}) = 2\alpha - (3n+2) - \sum_{m=4}^{i}[2^{m-3}(n+1)] + l_i$.

The set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in V(G)\}$ generated by the above formulas forms an integer sequence $2v+2, 2v+3, \ldots, 2v+1+e$ with difference 1. Consequently, λ is a super $(a, 1)$-EAT labeling with $a = 2v + 2$. ■

Theorem 3.6. *For $n \equiv 1 (\mathrm{mod}\ 2)$ and $r \geq 4$, $G \cong T(n, n, n+1, n_4, \ldots, n_r)$ admits a super $(a, 3)$-EAT labeling with $a = v + 4$, where $v = |V(G)|$ and $n_m = 2^{m-3}(n+1)$ for $4 \leq m \leq r$.*

Proof. Consider the vertex labeling as in the proof of Theorem 3.2.2. Now, we define the edge labeling $\lambda : E(G) \to \{v+1, v+2, \ldots, v+e\}$ as follows.

For $l_i = 1$, where $i = 1, 2, 3$ and $4 \le i \le r$,

$$\lambda(cu) = \begin{cases} \alpha + n, & \text{for } u = x_1^1, \\ \alpha + (n+1), & \text{for } u = x_2^1, \\ \alpha + (2n+1), & \text{for } u = x_3^1, \end{cases}$$

and $\lambda(cx_i^1) = \alpha + (2n+1) + \sum_{m=4}^{i}[2^{m-4}(n+1)]$.

For $1 \le l_i \le n_i - 1$, where $i = 1, 2, 3$ and $4 \le i \le r$,

$$\lambda(x_i^{1_i} x_i^{1_i+1}) = \begin{cases} \alpha + n - l_1, & \text{for } i = 1, \\ \alpha + (n+1) + l_2, & \text{for } i = 2, \\ \alpha + (3n+1) - l_3, & \text{for } i = 3, \end{cases}$$

and $\lambda(x_i^{1_i} x_i^{1_i+1}) = \alpha + (3n+2) + \sum_{m=4}^{i}[2^{m-3}(n+1)] - l_i$.

The set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in V(G)\}$ generated by the above formulas forms an integer sequence $(v+1)+3(1), (v+1)+3(2), (v+1)+3(3), \ldots, (v+1)+3(e)$ with difference 3. Consequently, λ admits a super $(a, 3)$-EAT labeling with $a = v + 4$. ∎

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