# GRAPHIC SPLITTING OF COGRAPHIC MATROIDS 

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#### Abstract

In this paper, we obtain a forbidden minor characterization of a cographic matroid $M$ for which the splitting matroid $M_{x, y}$ is graphic for every pair $x, y$ of elements of $M$.


Keywords: binary matroid, graphic matroid, cographic matroid, minor, splitting operation.
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## 1. Introduction

Fleischner [3] introduced the idea of splitting a vertex of degree at least three in a connected graph and used the operation to characterize Eulerian graphs. Figure 1 shows the graph $G_{x, y}$ that is obtained from $G$ by splitting away the edges $x$ and $y$ from the vertex $v$.


Figure 1
Welsh [11] proved that a binary matroid is Eulerian if and only if its dual is bipartite.

It is easy to see that a binary matroid $M$ is Eulerian if and only if the sum of columns of $A$ is zero, where $A$ is a matrix over $G F(2)$ that represents $M$. Raghunathan et al. [7] proved that a binary matroid $M$ is Eulerian if and only if $M_{x, y}$ is Eulerian for every pair of elements $x$ and $y$.

The matroid notations and terminology used here will follow Oxley [6]. We adopt the convention that every graph mentioned in this paper is loopless and coloopless.

Raghunathan et al. [7] extended the splitting operation from graphs to binary matroids as follows:

Definition 1.1. Let $M=M[A]$ be a binary matroid and suppose $x, y \in E(M)$. Let $A_{x, y}$ be the matrix obtained from $A$ by adjoining the row that is zero everywhere except for the entries of 1 in the columns labelled by $x$ and $y$. The splitting matroid $M_{x, y}$ is defined to be the vector matroid of the matrix $A_{x, y}$.
Example 1.2. Consider the Fano matroid $F_{7}=M$ on the set $E=\{1,2,3,4,5,6$, $7\}$. Let $A$ denote the standard matrix representation with respect to the basis $B=\{1,2,3\}$ of $M$ over $G F(2)$, so that

$$
A=\left[\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 1
\end{array}\right]
$$

Then splitting of $M$ by the pair 2 and 4, i.e. the matroid $M_{2,4}$, is represented by the matrix

$$
A_{2,4}=\left[\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0
\end{array}\right]
$$

Let $M(G)$ and $M^{*}(G)$ denote the cycle matroid and the cocycle matroid, respectively of a graph $G$. Various properties of a splitting matroid are obtained in $[1,2,5,7,8,9]$ and $[10]$.

The splitting operation on a graphic matroid may not yield a graphic matroid. Shikare and Waphare [10] characterized graphic matroids whose splitting matroids for every pair of elements are also graphic. Also, cographicness of a matroid may not be preserved under the splitting operation. Borse, Shikare, and Dalvi [2] obtained a forbidden-minor characterization for this class.


Figure 2

Further, the splitting operation on a cographic matroid may not yield a graphic matroid. In this paper, we characterize those cographic matroids $M$ for which $M_{x, y}$ is graphic for every pair $x, y \in E(M)$. The following is the main theorem.

Theorem 1.3. The splitting operation, by any pair of elements, on a cographic matroid yields a graphic matroid if and only if it has no minor isomorphic to any of the cycle matroids $M\left(G_{1}\right)$ and $M\left(G_{2}\right)$, where $G_{1}$ and $G_{2}$ are the graphs depicted in Figure 2.

## 2. Graphic Splitting of Cographic Matroids

Firstly, we give some results which are used in the proof of the main result.
Lemma 2.1 [7]. Let $M=(S, \mathcal{C})$ be a binary matroid on a set $S$ together with the set $\mathcal{C}$ of circuits. Then $M_{x, y}=\left(S, \mathcal{C}^{\prime}\right)$ with $\mathcal{C}^{\prime}=\mathcal{C}_{0} \cup \mathcal{C}_{1}$, where $\mathcal{C}_{0}=\{C \in \mathcal{C}: x, y \in$ $C$ or $x \notin C, y \notin C\} ;$ and $\mathcal{C}_{1}=\left\{C_{1} \cup C_{2}: C_{1}, C_{2} \in \mathcal{C}, x \in C_{1}, y \in C_{2}, C_{1} \cap C_{2}=\emptyset\right.$ and $C_{1} \cup C_{2}$ contains no member of $\left.\mathcal{C}_{0}\right\}$.

Lemma $2.2[5,10]$. Let $x$ and $y$ be elements of a binary matroid $M$ and let $r(M)$ denote the rank of $M$. Then the following statements hold.
(i) $M_{x, y}=M$ if and only if $x$ and $y$ are in series in $M$ or both $x$ and $y$ are coloops in $M$,
(ii) $r\left(M_{x, y}\right)=r(M)+1$ if and only if $M \neq M_{x, y}$,
(iii) if $x_{1}, x_{2}$ are in series in $M$, then they are in series in $M_{x, y}$.
(iv) If $C^{*}$ is a cocircuit of $M$ containing $x, y$ with $\left|C^{*}\right| \geq 3$, then $C^{*}-\{x, y\}$ is a cocircuit of $M_{x, y}$; and
(v) $M_{x, y} /\{x\}$ is Eulerian if and only if $M$ is Eulerian.

Theorem 2.3 [6]. A binary matroid is graphic if and only if it has no minor isomorphic to $F_{7}, F_{7}^{*}, M^{*}\left(K_{5}\right)$ or $M^{*}\left(K_{3,3}\right)$.

Theorem 2.4 [6]. A binary matroid is cographic if and only if it has no minor isomorphic to $F_{7}, F_{7}^{*}, M\left(K_{5}\right)$ or $M\left(K_{3,3}\right)$.

Notation. For the sake of convenience, let $\mathcal{F}=\left\{F_{7}, F_{7}^{*}, M^{*}\left(K_{5}\right), M^{*}\left(K_{3,3}\right)\right\}$.
Lemma 2.5. Let $M$ be a cographic matroid and let $x, y \in E(M)$ such that $M_{x, y}$ is not graphic. Then there is a minor $N$ of $M$ with $\{x, y\} \subset E(N)$ such that $N_{x, y} /\{x\} \cong F$ or $N_{x, y} /\{x, y\} \cong F$ for some $F \in \mathcal{F}$ and further, $N$ has no non-trivial series class except possibly a series class which contains $x$ and $y$.

Proof. As in the proof of Theorem 2.3 in [10], there exists a minor $N$ of $M$ such that $N_{x, y} /\{x\} \cong F$ or $N_{x, y} /\{x, y\} \cong F$ for some $F \in \mathcal{F}$. If $x$ and $y$ are not in
series in $N$, then $N$ has no non-trivial series class. Suppose $x$ and $y$ are in series in $N$. Then, $N=N_{x, y}$. Since $F$ does not have any 2-cocircuit, every 2-cocircuit of $N$ must contain $x$ or $y$. Hence $N$ has at most one non-trivial series class.

Definition 2.6. Let $M$ be a cographic matroid and let $F \in \mathcal{F}$. We say that $M$ is minimal with respect to $F$ if there exist two elements $x$ and $y$ of $M$ such that $M_{x, y} /\{x\} \cong F$ or $M_{x, y} /\{x, y\} \cong F$ and further, $M$ has no non-trivial series class except possibly a series class which contains $x$ and $y$.

Corollary 2.7. Let $M$ be a cographic matroid. For any $x, y \in E(M)$, the matroid $M_{x, y}$ is graphic if and only if $M$ has no minor isomorphic to a minimal matroid with respect to any $F \in \mathcal{F}$.

Proof. The proof follows from Lemma 2.2 and Lemma 2.5.
Lemma 2.8. Let $M$ be a minimal matroid with respect to $F$ for some $F \in \mathcal{F}$ and let $x, y$ be two elements of $M$ such that either $M_{x, y} /\{x\} \cong F$ or $M_{x, y} /\{x, y\} \cong F$. Then
(i) $M$ has neither loops nor coloops,
(ii) if $M_{x, y} /\{x, y\} \cong F$ or $M_{x, y} /\{x\} \cong M^{*}\left(K_{5}\right)$, then $M$ has at most one 2circuit.

Proof. The proof follows from Lemmas 2.1, 2.2 and the fact that $F$ does not contain loops, coloops and 2-circuits.

Lemma 2.9 [10]. A graph is minimal with respect to the matroid $F_{7}$ or $F_{7}^{*}$ if and only if it is isomorphic to one of the three graphs $G_{1}, G_{2}$ and $G_{3}$ in Figure 3.


Figure 3
Lemma 2.10 [10]. A graph is minimal with respect to the matroid $M^{*}\left(K_{3,3}\right)$ if and only if it is isomorphic to one of the four graphs $G_{4}, G_{5}, G_{6}$ and $G_{7}$ presented in Figure 4.


Figure 4
Lemma 2.11 [10]. A graph is minimal with respect to the matroid $M^{*}\left(K_{5}\right)$ if and only if it is isomorphic to $G_{8}$ and $G_{9}$ presented in Figure 5.


Figure 5
Lemma 2.12. Let $M$ be a cographic matroid. Then $M$ is minimal with respect to the matroid $F_{7}$ or $F_{7}^{*}$ if and only if $M$ is isomorphic to one of the cycle matroids $M\left(G_{1}\right), M\left(G_{2}\right)$ and $M\left(G_{3}\right)$, where $G_{1}, G_{2}$ and $G_{3}$ are the graphs in Figure 6.


Figure 6
Proof. From the matrix representation it follows that $M\left(G_{1}\right)_{x, y} /\{x\} \cong F_{7}$, $M\left(G_{2}\right)_{x, y} /\{x, y\} \cong F_{7}$ and $M\left(G_{3}\right)_{x, y} /\{x\} \cong F_{7}^{*}$. Therefore, $M\left(G_{1}\right), M\left(G_{2}\right)$ and $M\left(G_{3}\right)$ are minimal with respect to $F_{7}$ or $F_{7}^{*}$.

Conversely, suppose $M$ is minimal with respect to $F_{7}$ or $F_{7}^{*}$. Then there exist elements $x, y$ such that $M_{x, y} /\{x\} \cong F_{7}$, or $M_{x, y} /\{x, y\} \cong F_{7}$, or $M_{x, y} /\{x\} \cong F_{7}^{*}$ or $M_{x, y} /\{x, y\} \cong F_{7}^{*}$. Suppose $x$ and $y$ are in series. Then, by Lemma 2.2(i), $M=M_{x, y}$. Therefore, $M$ has $F_{7}$ or $F_{7}^{*}$ as a minor, which is a contradiction to Theorem 2.4. Hence $x$ and $y$ are not in series in $M$. Thus, no two elements of $M$ are in series in $M$. Now, the proof follows from Lemma 2.9.

Lemma 2.13. Let $M$ be a cographic matroid. Then $M$ is minimal with respect to the matroid $M^{*}\left(K_{3,3}\right)$ or $M^{*}\left(K_{5}\right)$ if and only if $M$ is isomorphic to one of $M\left(G_{i}\right)$ for $i=5,6,7,12$ and to one of $M^{*}\left(G_{j}\right)$ for $j=4,8,9,10,11,13,14,15$, where the graphs $G_{i}$ 's and $G_{j}$ 's are shown in Figure 7.


Figure 7

Proof. From the matrix representation, it follows that

$$
\begin{aligned}
& M^{*}\left(G_{4}\right)_{x, y} /\{x\} \cong M^{*}\left(K_{3,3}\right), M\left(G_{5}\right)_{x, y} /\{x\} \cong M^{*}\left(K_{3,3}\right), \\
& M\left(G_{6}\right)_{x, y} /\{x, y\} \cong M^{*}\left(K_{3,3}\right), M\left(G_{7}\right)_{x, y} /\{x, y\} \cong M^{*}\left(K_{3,3}\right), \\
& M^{*}\left(G_{8}\right)_{x, y} /\{x, y\} \cong M^{*}\left(K_{3,3}\right), M^{*}\left(G_{9}\right)_{x, y} /\{x, y\} \cong M^{*}\left(K_{3,3}\right), \\
& M^{*}\left(G_{10}\right)_{x, y} /\{x, y\} \cong M^{*}\left(K_{3,3}\right), M^{*}\left(G_{11}\right)_{x, y} /\{x, y\} \cong M^{*}\left(K_{3,3}\right), \\
& M\left(G_{12}\right)_{x, y} /\{x\} \cong M^{*}\left(K_{5}\right), M^{*}\left(G_{13}\right)_{x, y} /\{x\} \cong M^{*}\left(K_{5}\right), \\
& M^{*}\left(G_{14}\right), \\
& x, y /\{x\} \cong M^{*}\left(K_{5}\right) \text { and } M^{*}\left(G_{15}\right)_{x, y} /\{x, y\} \cong M^{*}\left(K_{5}\right) .
\end{aligned}
$$

Therefore, $M\left(G_{i}\right)$ for $i=5,6,7,12$ and $M^{*}\left(G_{j}\right)$ for $j=4,8,9,10,11,13,14,15$ are minimal with respect to the matroid $M^{*}\left(K_{3,3}\right)$ or $M^{*}\left(K_{5}\right)$.

Conversely, suppose that $M$ is a minimal matroid with respect to the matroid $M^{*}\left(K_{3,3}\right)$ or $M^{*}\left(K_{5}\right)$. Then there exist elements $x$ and $y$ of $M$ such that $M_{x, y} /\{x\} \cong M^{*}\left(K_{3,3}\right)$ or $M_{x, y} /\{x, y\} \cong M^{*}\left(K_{3,3}\right)$ or $M_{x, y} /\{x\} \cong M^{*}\left(K_{5}\right)$ or $M_{x, y} /\{x, y\} \cong M^{*}\left(K_{5}\right)$.

Suppose $x$ and $y$ are in series in $M$. Then, by Lemma $2.2(\mathrm{i}), M=M_{x, y}$. Hence $M /\{x\} \cong M^{*}\left(K_{3,3}\right)$ or $M /\{x, y\} \cong M^{*}\left(K_{3,3}\right)$ or $M /\{x\} \cong M^{*}\left(K_{5}\right)$ or $M /\{x, y\} \cong M^{*}\left(K_{5}\right)$; i.e. $M^{*} \backslash\{x\} \cong M\left(K_{3,3}\right)$ or $M^{*} \backslash\{x, y\} \cong M\left(K_{3,3}\right)$ or $M^{*} \backslash\{x\} \cong M\left(K_{5}\right)$ or $M^{*} \backslash\{x, y\} \cong M\left(K_{5}\right)$. Since $x$ and $y$ are in parallel in $M^{*}$, it follows that $M \cong M^{*}\left(G_{i}\right)$ for $i=4,8,9,13,15$.

Now, suppose $x$ and $y$ are not in series in $M$. Then $M \neq M_{x, y}$. By Lemma 2.2(ii), $r\left(M_{x, y}\right)=r(M)+1$.

Case (i). $M_{x, y} /\{x\} \cong M^{*}\left(K_{3,3}\right)$. We claim that $M$ is graphic. By Theorems 2.3 and 2.4 , it suffices to prove that $M$ does not have any of the matroids $F_{7}, F_{7}^{*}, M^{*}\left(K_{3,3}\right)$ and $M^{*}\left(K_{5}\right)$ as a minor. As $M$ is cographic, $F_{7}$ and $F_{7}^{*}$ are excluded minors for $M$. Further, $|E(M)|=10$ and, by Lemma 2.2 (ii), $r(M)=r\left(M_{x, y}\right)-1=r\left(M_{x, y} /\{x\}\right)=r\left(M^{*}\left(K_{3,3}\right)\right)=4$. Hence $M$ cannot have a minor isomorphic to $M^{*}\left(K_{5}\right)$. Assume that $M$ has a minor isomorphic to $M^{*}\left(K_{3,3}\right)$. There exists an element $q$ in $M$ such that $M \backslash q \cong M^{*}\left(K_{3,3}\right)$. Therefore $M^{*} / q \cong M\left(K_{3,3}\right)$. Since $M^{*}\left(K_{3,3}\right)$ is Eulerian, by Lemma 2.2(v), $M$ is Eulerian and hence $M^{*}$ is bipartite. By Lemm 2.8(i), $q$ is neither a loop nor a coloop. Hence there exists a circuit $C$ in $M^{*}$ containing $q$. Since $C$ is an even circuit, $C / q$ is an odd circuit in $M^{*} / q \cong M\left(K_{3,3}\right)$, a contradiction. Thus $M$ is graphic. Hence $M \cong M(G)$, where $G$ is a planar graph. It follows from the proof of Lemma 2.10 that $M \cong M\left(G_{5}\right)$ of Figure 7 .

Case (ii). $M_{x, y} /\{x, y\} \cong M^{*}\left(K_{3,3}\right)$. If $M$ is graphic, then by Lemma 2.10, $M \cong M\left(G_{6}\right)$ or $M\left(G_{7}\right)$ of Figure 7. Suppose that $M$ is not graphic. As $M$ is cographic, $M \cong M^{*}(G)$ for some graph $G$. Further, $G$ has 7 vertices and 11 edges because $r\left(M^{*}\right)=6$. As $\left|E\left(M^{*}\left(K_{3,3}\right)\right)\right|=9, r\left(M^{*}\left(K_{3,3}\right)\right)=4, M \backslash\{p\} /\{q\} \cong$ $M^{*}\left(K_{3,3}\right)$ for some elements $p, q$ of $M$. Therefore $M^{*} /\{p\} \backslash\{q\} \cong M\left(K_{3,3}\right)$. Since $M$ has no 2-cocircuit, $G$ is simple. Further, $G$ is non-planar. By Lemma 2.8(ii), $M$ has at most one 2-circuit and hence $G$ has at most one vertex of degree 2 . Therefore, the degree sequence of $G$ is $(4,3,3,3,3,3,3),(4,4,3,3,3,3,2)$ or (5,3,3,3,3,3,2).

Consider the degree sequence ( $5,3,3,3,3,3,2$ ). A non-planar simple graph with degree sequence $(5,3,3,3,3,3,2)$ can be obtained from a non-planar simple graph with degree sequence ( $4,3,3,3,3,2$ ) or ( $5,3,3,3,2,2$ ) by adding a vertex of degree 2 . But there is no non-planar simple graph with any of these two degree sequences see [4]. So, we discard the degree sequence ( $5,3,3,3,3,3,2$ ).

Since all cocircuits of $M^{*}\left(K_{3,3}\right)$ are even and $M$ has no odd cocircuit, the graph $G$ cannot have an $i$-circuit containing both $x$ and $y$ for $i=3,4,5,7$.

(i)

(ii)

(iii)

(iv)

Figure 8
Now, consider the degree sequence $(4,3,3,3,3,3,3)$. By [10], there is only one nonplanar simple graph of degree sequence $(4,3,3,3,3,3,3)$, as shown in Figure 8(iv).

In this graph every pair of edges is contained in an $i$-circuit, for some $i=3,4,5,7$. Hence we discard this graph.

A non-planar simple graph with degree sequence ( $4,4,3,3,3,3,2$ ) can be obtained from a non-planar simple graph with degree sequence $(3,3,3,3,3,3)$ or $(4,4,3,3,2,2)$ by adding a vertex of degree 2 . It follows from [4] that every nonplanar simple graph with degree sequence $(4,4,3,3,3,3,2)$ is isomorphic to one of the first three graphs of Figure 8. Graph (i) is discarded because every pair of edges is contained in an $i$-circuit for some $i=3,4,5,7$. The remaining two graphs are nothing but the graphs $G_{10}$ and $G_{11}$ in the statement of the lemma.

Case (iii). $M_{x, y} /\{x\} \cong M^{*}\left(K_{5}\right)$. If $M$ is graphic, then, by Lemma 2.11, we get two graphs which one of them is a graph (iv) of Figure 8, which is already discarded. So, $M \cong M\left(G_{12}\right)$ of Figure 7. Suppose that $M$ is not graphic. As $M$ is cographic, $M=M^{*}(G)$ for some non-planar graph $G$. Further, $G$ has 6 vertices and 11 edges because $r\left(M^{*}\right)=5$. By Lemma 2.8(i), $M$ has no loops and coloops and also no two elements of $M$ are in series, $G$ is simple and has minimum degree at least 2. Also, by Lemma 2.8(ii), $G$ has at most one vertex of degree 2. Hence the degree sequence of $G$ is $(4,4,4,4,3,3)$ or $(4,4,4,4,4,2)$. By [4], the graph $G_{14}$ of Figure 7 is the only one non-planar simple graph with the degree sequence $(4,4,4,4,3,3)$. Also, there is only one non-planar simple graph with degree sequence $(4,4,4,4,4,2)$ see [4]. In this graph, any pair of edges are either in a 3 -circuit or a 4 -circuit. If $G$ is isomorphic to this graph, then $x, y$ belong to a 3 -cocircuit or a 4 -cocircuit $C^{*}$ of $M$ and hence $C^{*}-\{x, y\}$ is a 1-cocircuit or a 2-cocircuit in $M_{x, y} /\{x\}$, a contradiction.

Case (iv). $M_{x, y} /\{x, y\} \cong M^{*}\left(K_{5}\right)$. First we show that $M$ is graphic. Suppose that $M$ is not graphic. Then $M$ has $M^{*}\left(K_{5}\right)$ or $M^{*}\left(K_{3,3}\right)$ as a minor. On the contrary, suppose $M$ has $M^{*}\left(K_{5}\right)$ or $M^{*}\left(K_{3,3}\right)$ as a minor. As $r(M)=7$ and $|E(M)|=12, M \backslash\{p\} /\{q\} \cong M^{*}\left(K_{5}\right)$ for some elements $p, q \in E(M)$. This implies that $M^{*} /\{p\} \backslash\{q\} \cong M\left(K_{5}\right)$. Also, $M /\{n, m, s\} \cong M^{*}\left(K_{3,3}\right)$ for some elements $n, m, s \in E(M)$. This implies that $M^{*} \backslash\{n, m, s\} \cong M\left(K_{3,3}\right)$. Thus $M \cong M^{*}(G)$, where $G$ is a non-planar simple graph with 6 vertices and 12 edges. By Lemma 2.8(ii), $G$ has at most one vertex of degree 2. Therefore the degree sequence of $G$ is $(4,4,4,4,4,4),(5,4,4,4,4,3),(5,5,4,4,3,3),(5,5,5,3,3,3)$ or $(5,5,4,4,4,2)$. By [4], there is only one non-planar simple graph for each of these sequences, as shown in Figure 9.


It follows from the nature of cocircuits of $M^{*}\left(K_{5}\right)$, that both $x, y$ do not belong to an $i$-circuit for $i=3,4$ nor to a $j$-cocircuit for $j=3,4,5,7$. These conditions are not satisfied by any pair of edges of the first 4 graphs of Figure 9. Hence we discard these graphs. Further, in the graph (v) of Figure 9 each pair of edges belongs to an $i$-circuit for $i=3,4$ and to a $j$-cocircuit for $j=3,4,5,7$, except the pairs $(1,4),(1,11)$ and $(4,7)$. For these pairs, there is a 5 -circuit in $M_{x, y} /\{x, y\}$ and hence it cannot be isomorphic to $M^{*}\left(K_{5}\right)$ since $M^{*}\left(K_{5}\right)$ has 5 circuits of size 4 and 10 circuits of size 6 . Thus $G$ cannot be obtained from this graph. So $M$ does not have $M^{*}\left(K_{5}\right)$ or $M^{*}\left(K_{3,3}\right)$ as a minor. We conclude that $M$ is graphic. Now the proof follows from Lemma 2.11

Now, we use Lemmas 2.12 and 2.13 to prove Theorem 1.3.
Proof of Theorem 1.3. Let $M$ be a cographic matroid. On combining Corollary 2.7 and Lemmas 2.12 and 2.13, it follows that $M_{x, y}$ is graphic for every pair $\{x, y\}$ of elements of $M$ if and only if $M$ has no minor isomorphic to any of the matroids $M\left(G_{i}\right), i=1,2,3,5,6,7,12$ and $M^{*}\left(G_{j}\right), j=4,8,9,10,11,13,14,15$ where the graphs $G_{i}$ and $G_{j}$ are shown in the statements of the Lemmas 2.12 and 2.13. However, we have $M\left(G_{3}\right) \cong M\left(G_{2}\right) \backslash\{e\} \cong M\left(G_{5}\right) \backslash\{2, w\} \cong$ $M\left(G_{6}\right) /\{2\} \backslash\{6, w\} \cong M\left(G_{7}\right) /\{2\} \backslash\{3,5\} \cong M\left(G_{12}\right) \backslash\{1\} /\{v, 2\} ; M^{*}\left(G_{1}\right) \cong$ $M\left(G_{4}\right) \backslash\{x, e\} \cong M\left(G_{8}\right) \backslash\{x, y, f\} \cong M\left(G_{9}\right) \backslash\{x, y, g\} \cong M\left(G_{10}\right) /\{11\} \backslash\{9, y\}$ and $M^{*}\left(G_{3}\right) \cong M\left(G_{11}\right) /\{6, y\} \backslash\{11\} \cong M\left(G_{13}\right) \backslash\{1,2, x\} \cong M\left(G_{14}\right) /\{y\} \backslash\{2,3\} \cong$ $M\left(G_{15}\right) \backslash\{e, f, x, y\}$.

This means that

$$
M\left(G_{1}\right) \cong M^{*}\left(G_{4}\right) /\{x, e\} \cong M^{*}\left(G_{8}\right) /\{x, y, f\} \cong M^{*}\left(G_{9}\right) /\{x, y, g\}
$$

$$
\cong M^{*}\left(G_{10}\right) /\{9, y\} \backslash\{11\} \text { and } M\left(G_{3}\right) \cong M^{*}\left(G_{11}\right) /\{11\} \backslash\{6, y\}
$$

$$
\cong M^{*}\left(G_{13}\right) /\{1,2, x\} \cong M^{*}\left(G_{14}\right) /\{2,3\} \backslash\{y\} \cong M^{*}\left(G_{15}\right) /\{e, f, x, y\}
$$

Thus, $M_{x, y}$ is graphic if and only if $M$ has no minor isomorphic to any of the matroids $M\left(G_{i}\right)$ for $i=1,3$. But the graphs $G_{i}$ are precisely the graphs given in the statement of the theorem. This completes the proof.

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