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TETRAVALENT ARC-TRANSITIVE GRAPHS OF ORDER $3p^2$

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Abstract

Let s be a positive integer. A graph is *s-transitive* if its automorphism group is transitive on s-arcs but not on $(s+1)$-arcs. Let p be a prime. In this article a complete classification of tetravalent s-transitive graphs of order $3p^2$ is given.

Keywords: s-transitive graphs, symmetric graphs, Cayley graphs.

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1. Introduction

In this paper we consider undirected finite connected graphs without loops or multiple edges. For a graph X we use $V(X)$, $E(X)$ and $\mathrm{Aut}(X)$ to denote its vertex set, edge set and its full automorphism group, respectively. For $u, v \in V(X)$, $\{u, v\}$ is the edge incident to u and v in X, and $N(u)$ is the neighborhood of u in X, that is, the set of vertices adjacent to u in X. A graph X is *locally primitive* if for any vertex $v \in V(X)$, the stabilizer $\mathrm{Aut}(X)_v$ of v in $\mathrm{Aut}(X)$ is primitive on $N(v)$. An *s-arc* in a graph is an ordered $(s+1)$-tuple $(v_0, v_1, \ldots, v_{s-1}, v_s)$ of vertices of the graph such that v_{i-1} is adjacent to v_i for $1 \leq i \leq s$, and $v_{i-1} \neq v_{i+1}$ for $1 \leq i \leq s-1$. For a subgroup $G \leq \mathrm{Aut}(X)$, a graph X is said to be *(G, s)-arc-transitive* or *(G, s)-regular* if G acts transitively or regularly on the set of s-arcs of X, respectively. A (G, s)-arc-transitive graph is said to be *(G, s)-transitive* if it is not $(G, s+1)$-arc-transitive. In particular, an $(\mathrm{Aut}(X), s)$-arc-transitive, $(\mathrm{Aut}(X), s)$-regular or $(\mathrm{Aut}(X), s)$-transitive graph is simply called an *s-arc-transitive*, *s-regular* or *s-transitive* graph, respectively. Note that 0-arc-transitive means *vertex-transitive*, and 1-arc-transitive means *arc-transitive* or *symmetric*. A graph is *edge-transitive* if $\mathrm{Aut}(X)$ is transitive on $E(X)$.

Edge-transitive graphs or s-transitive graphs of small valencies have received considerable attention in the literature. For instance, Tutte [29] initiated the investigation of cubic s-transitive graphs by proving that there exist no cubic s-transitive graphs for $s \geq 6$, and later much subsequent work was done along this line (see [7, 8, 9, 10, 11, 12, 13, 14, 24]). Gardiner and Praeger [15, 16] generally explored the tetravalent symmetric graphs by considering their automorphism groups. Recently, Li *et al.* [22] classified all vertex-primitive symmetric graphs of valency 3 or 4. Moreover, Weiss [31] proved that if X is s-transitive, then $s \in \{1, 2, 3, 4, 5, 7\}$. Let p be a prime. Conder [6] showed that for a fixed integer n and any integer $s > 1$, there are only finitely many cubic s-transitive graphs of order np. Li [20] generalized this result to connected symmetric graphs of any valency, and he also posed the following problem: for small values n and k, classify vertex-transitive locally primitive graphs of order np and valency k.

In this paper we classify all symmetric graphs of order np and valency k for certain values of n and k. The classification of s-transitive graphs of order np and of valency 3 or 4 can be obtained from [4, 5, 30], where $1 \leq n \leq 3$. Feng *et al.* [10, 12, 13] classified cubic s-transitive graphs of order np with $n = 4, 6, 8$ or 10. Recently, Zhou and Feng [35, 36] classified tetravalent s-transitive graphs of order $4p$ or $2p^2$. Also Ghasemi and Zhou [18] classified tetravalent s-transitive graphs of order $4p^2$. In this paper, we prove that there are no tetravalent s-transitive graphs of order $3p^2$, for $s > 1$.

2. Preliminaries

In this section, we introduce some notation and definitions as well as some preliminary results which will be used later in the paper.

For a regular graph X, use $d(X)$ to represent the valency of X, and for any subset B of $V(X)$, the subgraph of X induced by B will be denoted by $[B]$.

For a positive integer n, denote by $\mathbb{Z}_n$ the cyclic group of order n as well as the ring of integers modulo n, by $\mathbb{Z}_n^*$ the multiplicative group of $\mathbb{Z}_n$ consisting of numbers coprime to n, by D_{2n} the dihedral group of order $2n$, and by C_n and K_n the cycle and the complete graph of order n, respectively. We call C_n an *n-cycle*.

Let G be a permutation group on a set Ω and $\alpha \in \Omega$. Denote by G_α the stabilizer of α in G, that is, the subgroup of G fixing the point α. We say that G is *semiregular* on Ω if $G_\alpha = 1$ for every $\alpha \in \Omega$ and *regular* if G is transitive and semiregular. For any $g \in G$, g is said to be *semiregular* if $\langle g \rangle$ is semiregular. The following proposition gives a characterization for Cayley graphs in terms of their automorphism groups.

Proposition 2.1 (Lemma 16.3 [2]). *A graph X is isomorphic to a Cayley graph on a group G if and only if its automorphism group has a subgroup isomorphic* *to G, acting regularly on the vertex set of X.*

Let X be a connected symmetric graph and let $G \leq Aut(X)$ be arc-transitive on X. For a normal subgroup N of G, the *quotient graph* X_N of X relative to the orbits of N is defined as the graph with vertices being the orbits of N on $\mathrm{V}(X)$ and with two orbits adjacent if there is an edge in X between those two orbits. If X_N and X have the same valency, then X is called a *normal cover* of X_N. Let X be a connected tetravalent symmetric graph and N an elementary abelian p-group. A classification of connected tetravalent symmetric graphs was obtained when N has at most two orbits in [15] and a characterization of such graphs was given when X_N is a cycle in [16].

The following proposition is due to Praeger *et al.* (refer to Theorem 1.1 [15] and [27]).

Proposition 2.2. *Let X be a connected tetravalent $(G, 1)$-arc-transitive graph. For each normal subgroup N of G, one of the following holds.*
(1) *N is transitive on $V(X)$,*
(2) *X is bipartite and N acts transitively on each part of the bipartition,*
(3) *N has $r \geq 3$ orbits on $V(X)$, the quotient graph X_N is a cycle of length r, and G induces the full automorphism group D_{2r} on X_N,*
(4) *N has $r \geq 5$ orbits on $V(X)$, N acts semiregularly on $V(X)$, the quotient graph X_N is a connected tetravalent G/N-symmetric graph, and X is a G-normal cover of X_N.*
Moreover, if X is also $(G, 2)$-arc-transitive, then case (3) cannot happen.

The following proposition characterizes the vertex stabilizer of the connected tetravalent s-transitive graphs, which can be deduced from Lemma 2.5 [23], or Proposition 2.8 [22], or Theorem 2.2 [21].

Proposition 2.3. *Let X be a connected tetravalent (G, s)-transitive graph. Let G_v be the stabilizer of a vertex $v \in V(X)$ in G. Then $s = 1, 2, 3, 4$ or 7. Furthermore, either G_v is a 2-group for $s = 1$, or G_v is isomorphic to A_4 or S_4 for $s = 2$; $A_4 \times \mathbb{Z}_3$, $\mathbb{Z}_3 \times S_4$, $S_3 \times S_4$ for $s = 3$; $\mathbb{Z}_3^2 \rtimes \mathrm{GL}(2, 3)$ for $s = 4$; or $[3^5] \rtimes \mathrm{GL}(2, 3)$ for $s = 7$, where $[3^5]$ represents an arbitrary group of order 3^5.*

Let X be a tetravalent one-regular graph of order $3p^2$. If $p \leq 13$, then $|V(X)| = 12, 27, 75, 147, 363$, or 507. Now, a complete census of the tetravalent arc-transitive graphs of order at most 640 has been recently obtained by Potočnik, Spiga and Verret [25, 26]. Therefore, a quick inspection through this list (with the invaluable help of `magma` (see [3])) gives the number of tetravalent one-regular graphs in the case $p \leq 13$. The following Proposition can be extracted from Theorem 3.4 [17].

Proposition 2.4. *Let p be a prime and $p > 13$. A tetravalent graph X of order $3p^2$ is 1-regular if and only if one of the following holds:*

(i) *X is a Cayley graph over $\langle x, y | x^p = y^{6p} = [x, y] = 1\rangle$, with connection set $\{y, y^{-1}, xy, x^{-1}y^{-1}\}$,*

(ii) *X is a connected arc-transitive circulant graph with respect to every connection set S,*

(iii) *X is one of the graphs described in Lemma 8.4 [16].*

Proposition 2.5 (Theorem 1.2 [16])**.** *Let X be a connected tetravalent symmetric graph of order $3p^2$ where $p > 5$ is a prime. Let $A = \mathrm{Aut}(X)$ and let $N = \mathbb{Z}_p^2$ be a minimal normal subgroup of A. Let K denote the kernel of G acting on N-orbits. If the quotient graph X_N is a 3-cycle, then $K_v \cong \mathbb{Z}_2$, and X is one-regular.*

Finally in the following example we introduce $G(3p, r)$, which was first defined in [5].

Example 2.6. *For each positive divisor r of $p - 1$ we use H_r to denote the unique subgroup of $\mathrm{Aut}(\mathbb{Z}_p)$ of order r, which is isomorphic to $\mathbb{Z}_r$. Define a graph $G(3p, r)$ by $V(G(3p, r)) = \{x_i \mid i \in \mathbb{Z}_3, x \in \mathbb{Z}_p\}$, and $E(G(3p, r)) = \{x_i y_{i+1} \mid i \in \mathbb{Z}_3, x, y \in \mathbb{Z}_p, y - x \in H_r\}$. Then $G(3p, r)$ is a connected symmetric graph of order $3p$ and valency $2r$. Also $\mathrm{Aut}(G(3p, p-1)) \cong S_p \times S_3$. For $r \neq p - 1$, $\mathrm{Aut}(G(3p, r))$ is isomorphic to $(\mathbb{Z}_p.H_r).S_3$ and acts regularly on the arc set, where $X.Y$ denotes an extension of X by Y.*

3. Main Results

In this section, we classify tetravalent s-transitive graphs of order $3p^2$ for each prime p. To do so, we need the following lemmas.

Lemma 3.1. *Let p be a prime and let $n > 1$ be an integer. Let X be a connected tetravalent graph of order $3p^n$. If $G \leq \mathrm{Aut}(X)$ is transitive on the arc set of X, then every minimal normal subgroup of G is solvable.*

Proof. Let $v \in V(X)$. Since G is arc-transitive on X, by Proposition 2.3, G_v either is a 2-group or has order dividing $2^4 \cdot 3^6$. It follows that $|G| \mid 2^4 \cdot 3^7 \cdot p^n$ or $|G| = 2^m \cdot 3 \cdot p^n$ for some integer m. Let N be a minimal normal subgroup of G.

Suppose that N is non-solvable. Then $p > 3$ because a $\{2, 3\}$-group is solvable by a theorem of Burnside Theorem 8.5.3 [28]. Since N is minimal, it is a product of isomorphic non-abelian simple groups. Since $|N| \mid 2^4 \cdot 3^7 \cdot p^n$, or $|N| = 2^m \cdot 3 \cdot p^n$ by [19], pp.12–14, each direct factor of N is one of the following: $A_5, A_6, \mathrm{PSL}(2, 7), \mathrm{PSL}(2, 8), \mathrm{PSL}(2, 17), \mathrm{PSL}(3, 3), \mathrm{PSU}(3, 3)$ or $\mathrm{PSU}(4, 2)$.

An inspection of the orders of such groups gives $n = 2$ and $|N| \mid 2^4 \cdot 3^7 \cdot p^n$. It follows that X is $(G, 2)$-arc transitive and we have $N \cong A_5 \times A_5$. Then $p = 5$ and

$|X| = 75$. However, from [32] we know that all tetravalent arc-transitive graphs of order 75 are 1-transitive, a contradiction. ■

Lemma 3.2. *Let X be a connected tetravalent G-arc-transitive graph of order $3p^2$, where $p > 13$. Assume that G has a normal subgroup N of prime order. If N has at least three orbits on $V(X)$, then either X_N is of valency 4 or G is regular on the arcs of X.*

Proof. By our assumption N has at least three orbits on $V(X)$. If N has $r \geq 5$ orbits on $V(X)$, then by Proposition 2.2, X_N has valency 4 and X is a normal cover of X_N. Thus we may suppose that N has $r \geq 3$ orbits. Thus $d(X_N) = 2$ and $|X_N| = 3p$ or $|X_N| = p^2$.

First suppose that $|X_N| = 3p$. Thus $X_N \cong C_{3p}$ and hence $G/K \cong \mathrm{Aut}(C_{3p}) \cong D_{6p}$. Let Δ and Δ' be two adjacent orbits of N in $V(X)$. Then the subgraph $[\Delta\cup\Delta']$ of X induced by $\Delta\cup\Delta'$ has valency 2. Since $p > 13$, one has $[\Delta\cup\Delta'] \cong C_{2p}$. The subgroup K^* of K fixing Δ pointwise also fixes Δ' pointwise. The connectivity of X and the transitivity of G/K on $V(X_N)$ imply that $K^* = 1$, and consequently, $K \leq \mathrm{Aut}([\Delta \cup \Delta']) \cong D_{4p}$. Since K fixes Δ, one has $|K| \leq 2p$. It follows that $|G| = |G/K||K| \leq 12p^2$, and hence G is regular on the arcs of X.

Now suppose that $|X_N| = p^2$. Thus $X_N \cong C_{p^2}$. It follows that $G/K \cong D_{2p^2}$. Let Δ and Δ' be two adjacent orbits of N in $V(X)$. Then the subgraph $[\Delta \cup \Delta']$ of X induced by $\Delta \cup \Delta'$ has valency 2. Clearly, we have $[\Delta \cup \Delta'] \cong C_6$. The subgroup K^* of K fixing Δ pointwise also fixes Δ' pointwise. The connectivity of X and the transitivity of G/K on $V(X_N)$ imply that $K^* = 1$, and consequently, $K \leq \mathrm{Aut}([\Delta \cup \Delta']) \cong D_{12}$. Since K fixes Δ, one has $|K| \leq 6$. It follows that $|G| = |G/K||K| \leq 12p^2$, and hence G is regular on the arcs of X. Now the proof is complete. ■

Theorem 3.3. *Let p be a prime and let X be a connected tetravalent graph of order $3p^2$. Then X is s-transitive for some positive integer s if and only if it is isomorphic to one of the graphs in Proposition* 2.4.

Proof. Let X be a tetravalent s-transitive graph of order $3p^2$ for a positive integer s. By [25, 26], we may assume that $p > 13$. If X is one-regular, then X is one of the graphs in Proposition 2.4 and so $s = 1$. In what follows, we assume that $p > 13$ and that X is not one-regular. Set $A = \mathrm{Aut}(X)$ and let P be a Sylow p-subgroup. Then $|P| = p^2$ and by Lemma 3.1, A is solvable. First we prove a claim.

Claim 1. *P is not normal in A.*

Proof. Suppose to, the contrary that $P \trianglelefteq A$. If P is a minimal normal subgroup of A then by Proposition 2.5, X is one-regular, a contradiction. Suppose that P contains a non-trivial subgroup, say N, which is normal in A. Consider the quotient graph X_N of X relative to the orbit set of N, and let K be the kernel of A on $V(X_N)$. Since $p > 13$, one has $|X_N| = 3p$. By Lemma 3.2 either X is a normal cover of X_N or $d(X_N) = 2$ and X is one-regular. Since X is not one-regular, we may suppose that $d(X_N) = 4$. By [30], $G(3p, 2)$ is the only tetravalent symmetric graph of order $3p$, (see Example 2.6). Also $|\mathrm{Aut}(G(3p, 2))| = 12p$ and $G(3p, 2)$ is one-regular. Thus $|A/M| = 12p$ and so $|A| = 12p^2$. Thus X is one-regular, a contradiction.

Let M be the maximal normal 2-subgroup of A and assume $|M| > 1$. Consider the quotient graph X_M of X relative to the orbit set of M, and let K be the kernel of A acting on $V(X_M)$. Since $p > 13$, every orbit of M has length 2 or 4, a contradiction. So A has no non-trivial normal 2-subgroup. □

Now we are ready to complete the proof. Let M be a minimal normal subgroup of A. Clearly, M is a 3-group or a p-group. First suppose that M is a p-group. Thus $|M| = p$ or p^2. If $|M| = p^2$, then $M = P$ is a Sylow p-subgroup of A. By Claim 1, P is not normal in A, a contradiction. Suppose that $|M| = p$. By Lemma 3.2 either X is a normal cover of X_M or $d(X_M) = 2$ and X is one-regular. Since X is not one-regular, we may suppose that $d(X_M) = 4$. By [30], $G(3p, 2)$ is the only tetravalent symmetric graph of order $3p$ (see Example 2.6). Also $|\mathrm{Aut}(G(3p, 2))| = 12p$ and $G(3p, 2)$ is one-regular. Thus $|A/M| = 12p$ and so $|A| = 12p^2$. Thus X is one-regular, a contradiction.

Now suppose that M is a 3-group. Thus $|X_M| = p^2$. If $d(X_M) = 4$, then by Proposition 2.5, $K = M$ is semiregular on $V(X_M)$. Therefore $K = M \cong \mathbb{Z}_3$. Since $P > 13$, $PM = P \times M$ is abelian. Clearly, PM is transitive on $V(X)$. Thus PM is regular on $V(X)$, because $|PM| = 3p^2$. Thus X is a Cayley graph on abelian group of order $3p^2$. By Theorem 1.2 [1], X is normal. If PM is cyclic, then by [33] X is one-regular, a contradiction. Thus PM is not cyclic. Now by Proposition 3.3 [34], X is one-regular, a contradiction. If $d(X_M) = 2$, then $X_M \cong C_{p^2}$. By Lemma 3.2, X is one-regular, a contradiction. ■

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