Discussiones Mathematicae
Graph Theory 34 (2014) 75–84
doi:10.7151/dmgt.1711

SUPERMAGIC GRAPHS HAVING A SATURATED VERTEX[1]

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Abstract

A graph is called supermagic if it admits a labeling of the edges by pairwise different consecutive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. In this paper we establish some conditions for graphs with a saturated vertex to be supermagic. Inter alia we show that complete multipartite graphs $K_{1,n,n}$ and $K_{1,2,\ldots,2}$ are supermagic.

Keywords: supermagic graph, saturated vertex, vertex-magic total labeling.

2010 Mathematics Subject Classification: 05C78.

1. Introduction

We consider finite undirected graphs without loops, multiple edges and isolated vertices. If G is a graph, then $V(G)$ and $E(G)$ stand for the vertex set and the edge set of G, respectively. Cardinalities of these sets are called the *order* and the *size* of G.

Let a graph G and a mapping f from $E(G)$ into positive integers be given. The *index-mapping* of f is the mapping f^* from $V(G)$ into positive integers defined by

$$f^*(v) = \sum_{e \in E(G)} \eta(v,e) f(e) \quad \text{for every } v \in V(G),$$

[1]The work was supported by the Slovak Research and Development Agency under the contract No. APVV-0023-10 and by the Slovak VEGA Grant 1/0652/12.

where $\eta(v, e)$ is equal to 1 when e is an edge incident with a vertex v, and 0 otherwise. An injective mapping f from $E(G)$ to positive integers is called a *magic labeling* of G for an *index* λ if its index-mapping f^* satisfies

$$f^*(v) = \lambda \quad \text{for all } v \in V(G).$$

A magic labeling f of G is called a *supermagic labeling* of G if the set $\{f(e) : e \in E(G)\}$ consists of consecutive positive integers. We say that a graph G is *supermagic* (*magic*) whenever there exists a supermagic (magic) labeling of G.

The concept of magic graphs was introduced by Sedláček [9]. Supermagic graphs were introduced by Stewart [11]. There is by now a considerable number of papers published on magic and supermagic graphs; we single out [7, 10, 6, 4] as being more particularly relevant to the present paper, and refer the reader to [2] for comprehensive references.

Let $G \cup H$ denote the disjoint union of graphs G and H. The *join* $G \oplus H$ of the disjoint graphs G and H is the graph $G \cup H$ together with all edges joining vertices of $V(G)$ and vertices of $V(H)$. The vertex v of a graph H is called *saturated vertex*, if it is adjacent to every other vertex. The graph H with a saturated vertex v is isomorphic to $(H - v) \oplus K_1$. So the graph with a saturated vertex is also denoted by $G \oplus K_1$.

Magic graphs with a saturated vertex were characterized in [10]. In the paper there are also given some conditions for the existence of supermagic graphs $G \oplus K_1$. In [7] there are given other sufficient conditions for existence of such graphs. Similar problems are solved in [6].

In this paper we will deal with supermagic graphs $G \oplus K_1$ for regular graphs G.

2. Vertex-magic Total Labelings

The notion of a vertex-magic total labeling was introduced in [8]. A bijective mapping $g : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ is called a *vertex-magic total labeling* of a graph G if there is a constant h such that

$$g(v) + \sum\nolimits_{e \in E(G)} \eta(v, e) g(e) = h \quad \text{for every } v \in V(G),$$

that is, $g(v) + g^*(v) = h$ for every vertex v. The constant h is called the *magic constant* for g.

For regular graphs G, the supermagic graphs $G \oplus K_1$ can be characterized in the following way.

Theorem 1. *Let G be a d-regular graph of order n. The graph $G \oplus K_1$ is supermagic if and only if G admits a vertex-magic total labeling with magic con-*

stant h such that $(n-d-1)$ is a divisor of the non-negative integer $(n+1)h - n\frac{d+2}{2}\left(n\frac{d+2}{2}+1\right)$.

Proof. Let v be a saturated vertex of $H = G \oplus K_1$.

Suppose that $f : E(H) \to \{a, a+1, \ldots, a+|E(H)|-1\}$ is a supermagic labeling of H for an index λ. As $|E(H)| = n(d+2)/2$, the index λ satisfies

$$\lambda = \frac{2}{|V(H)|}\sum\nolimits_{e\in E(H)} f(e) = \frac{1}{n+1}\left(2a + n\frac{d+2}{2} - 1\right)n\frac{d+2}{2}.$$

Consider the mapping g defined by

$$g(x) = \begin{cases} 1 + f(x) - a & \text{for } x \in E(G), \\ 1 + f(vx) - a & \text{for } x \in V(G). \end{cases}$$

Evidently, g is a bijection from $V(G)\cup E(G)$ onto $\{1, 2, \ldots, |V(G)|+|E(G)|\}$ and $g(w) + g^*(w) = \lambda - (a-1)(d+1) = h$ for any vertex $w \in V(G)$. Therefore g is a vertex-magic total labeling of G. Moreover, we have

$$\begin{aligned} (n+1)h - n\tfrac{d+2}{2}\left(n\tfrac{d+2}{2}+1\right) &= (n+1)\left(\tfrac{1}{n+1}\left(2a + n\tfrac{d+2}{2} - 1\right)n\tfrac{d+2}{2} - (a-1)(d+1)\right) \\ &\quad - n\tfrac{d+2}{2}\left(n\tfrac{d+2}{2}+1\right) \\ &= an(d+2) - n\tfrac{d+2}{2} - (a-1)(d+1)(n+1) - n\tfrac{d+2}{2} \\ &= (a-1)n(d+2) - (a-1)(d+1)(n+1) \\ &= (a-1)(n-d-1) \geq 0. \end{aligned}$$

Thus, $(n-d-1)$ is a divisor of $(n+1)h - n\frac{d+2}{2}\left(n\frac{d+2}{2}+1\right)$.

Now, let us assume that g is a vertex-magic total labeling of G with magic constant h such that $(n-d-1)$ is a divisor of the non-negative integer $(n+1)h - n\frac{d+2}{2}\left(n\frac{d+2}{2}+1\right)$. Then there is a non-negative integer κ such that

$$(n+1)h - n\frac{d+2}{2}\left(n\frac{d+2}{2}+1\right) = \kappa(n-d-1).$$

Consider the mapping f given by

$$f(e) = \begin{cases} \kappa + g(e) & \text{for } e \in E(G), \\ \kappa + g(w) & \text{for } e = vw. \end{cases}$$

Clearly, f is a bijection from $E(H)$ onto $\{\kappa+1, \kappa+2, \ldots, \kappa+|V(G)|+|E(G)|\}$. Moreover, $f^*(v) = \kappa|V(G)| + \sum_{w\in V(G)} g(w)$ and $f^*(w) = h + \kappa(d+1)$ for any vertex $w \in V(G)$. In this case we obtain

$$\begin{aligned} \sum\nolimits_{w\in V(G)} g(w) + |V(G)|h &= 2\sum\nolimits_{x\in V(G)\cup E(G)} g(x) \\ &= (|V(G)|+|E(G)|)(|V(G)|+|E(G)|+1) \\ &= n\tfrac{d+2}{2}\left(n\tfrac{d+2}{2}+1\right). \end{aligned}$$

This implies

$$\begin{aligned}\sum_{w\in V(G)} g(w) &= n\tfrac{d+2}{2}\left(n\tfrac{d+2}{2}+1\right) - nh\\ &= n\tfrac{d+2}{2}\left(n\tfrac{d+2}{2}+1\right) - (n+1)h + h\\ &= h - \kappa(n-d-1).\end{aligned}$$

Hence

$$f^*(v) = \kappa n + h - \kappa(n-d-1) = h + \kappa(d+1),$$

which means that f is a supermagic labeling of H. ∎

Using known results on vertex-magic total labelings of regular graphs of odd order the previous theorem implies the same assertions that was proved in [7] by other methods. Therefore, we apply Theorem 1 to regular graphs of even order and we have immediately

Corollary 2. *Let G be a $2(k-1)$-regular graph of order $2k$. The graph $G \oplus K_1$ is supermagic if and only if G admits a vertex-magic total labeling with magic constant h such that $(2k+1)h \geq 2k^2(2k^2+1)$.*

Corollary 3. *Let G be a k-regular graph of order $2k$. The graph $G\oplus K_1$ is supermagic if and only if G admits a vertex-magic total labeling with magic constant h such that the non-negative integer $(2k+1)h - k(k+2)(k+1)^2$ is an integral multiple of $(k-1)$.*

Note that for 2-regular graphs a vertex-magic total labeling corresponds to an edge-magic total labeling introduced by Kotzig and Rosa. Using this correspondence we can rewrite the following known result for cycles (see [12]).

Proposition 1. *The cycle C_{2k} has a vertex-magic total labeling with magic constant $h = 7k + 1$.*

Now we are able to prove the following assertion.

Theorem 4. *Let G be a d-regular graph of order $2k$. If G contains a Hamilton cycle C such that $G - E(C)$ is supermagic then G admits a vertex-magic total labeling with magic constant $h = 7k + 1 + \frac{1}{2}(d-2)(kd+6k+1)$.*

Proof. Put $H = G - E(C)$. By the assumption there is a supermagic labeling $f : E(H) \to \{1, 2, \dots, |E(H)|\}$. Since $|E(H)| = k(d-2)$, the index of f satisfies

$$\lambda = \frac{(|E(H)|+1)|E(H)|}{|V(G)|} = \frac{1}{2}\big(k(d-2)+1\big)(d-2).$$

Similarly, according to Proposition 1 there exists a vertex-magic total labeling $g : V(C) \cup E(C) \to \{1, 2, \dots, 4k\}$ with magic constant $h = 7k + 1$. Consider the mapping φ defined by

$$\varphi(x)=\begin{cases} g(x) & \text{for } x\in V(C)\cup E(C),\\ 4k+f(x) & \text{for } x\in E(H).\end{cases}$$

Clearly, φ is a bijection from $V(G)\cup E(G)$ onto $\{1,2,\dots,|V(G)|+|E(G)|\}$. Accordingly

$$\begin{aligned}\varphi(w)+\varphi^*(w) &= h+\lambda+4k(d-2)\\ &= 7k+1+\tfrac{1}{2}\big(k(d-2)+1\big)(d-2)+4k(d-2)\\ &= 7k+1+\tfrac{1}{2}(d-2)(kd+6k+1)\end{aligned}$$

for any vertex $w\in V(G)$. Therefore φ is a desired vertex-magic total labeling of G. ■

3. Complete Multipartite Graphs

A *complete k-partite graph* is a graph whose vertices can be partitioned into $k\geq 2$ disjoint classes $V_1,\dots,V_k$ such that two vertices are adjacent if and only if they belong to distinct classes. If $|V_i|=n_i$ for all $i=1,\dots,k$, then the complete k-partite graph is denoted by $K_{n_1,\dots,n_k}$. If $n_i=n$ for all $i=1,\dots,k$, then the complete k-partite graph is regular of degree $(k-1)n$ and is denoted by $K_{k[n]}$. Similarly, if $n_i=n$ for all $i=1,\dots,k$ and $n_{k+1}=p$ then the complete $(k+1)$-partite graph is denoted by $K_{p,k[n]}$.

In this section we characterize supermagic graphs $K_{1,2[n]}=K_{n,n}\oplus K_1$ and $K_{1,k[2]}=K_{k[2]}\oplus K_1$. Let us recall some notions and assertions, which we shall use in the next.

A *k-factor* (or only a *factor*) of a graph is defined to be its k-regular spanning subgraph.

Proposition 2 [3]**.** *If G is a graph decomposable into pairwise edge-disjoint supermagic factors, then G is supermagic.*

For any graph G we define a graph $G^{\bowtie}$ by $V(G^{\bowtie})=\bigcup_{v\in V(G)}\{v^0,v^1\}$ and $E(G^{\bowtie})=\bigcup_{vu\in E(G)}\{v^0u^1,v^1u^0\}\cup\bigcup_{v\in V(G)}\{v^0v^1\}$. In [7] the following result is proved.

Proposition 3 [7]**.** *Let G be a $2r$-regular graph of odd order. If G is Hamiltonian, then $G^{\bowtie}$ is a supermagic graph.*

In [4] the following assertions are proved.

Proposition 4 [4]**.** *Let G be a $4k$-regular bipartite graph which can be decomposed into two edge-disjoint connected $2k$-factors. Then G is a supermagic graph.*

Proposition 5 [4]**.** *Let G be a 6-regular bipartite graph of order $2n$ which can be decomposed into three edge-disjoint 2-factors where the first is isomorphic to $2C_n$ and the others are Hamilton cycles. Then G is a supermagic graph.*

For $X, Y \subseteq V(G)$ the subgraph of a graph G induced by $\{uv \in E(G) : u \in X, v \in Y\}$ is denoted by $G(X,Y)$. The complement of a graph G is denoted by $\overline{G}$. In [5] the following result is proved.

Proposition 6 [5]**.** *Let G be a d-regular bipartite graph of order $2n$ with parts U_1 and U_2. If $n \geq 5$ and d are odd and $\overline{G}(U_1,U_2)$ is a Hamiltonian graph, then the complement of G is a supermagic graph.*

Let n, m and $a_1 < \cdots < a_m \leq \lfloor n/2 \rfloor$ be positive integers. A graph with the vertex set $\{v_0, \ldots, v_{n-1}\}$ and the edge set $\{v_i v_{i+a_j} : 0 \leq i < n, 1 \leq j \leq m\}$, the indices being taken modulo n, is called a *circulant graph* and it is denoted by $C_n(a_1, \ldots, a_m)$. It is easy to see that the circulant graph $C_n(a_1, \ldots, a_m)$ is a regular graph of degree r, where $r = 2m - 1$ when $a_m = n/2$, and $r = 2m$ otherwise. In [1] the following result is proved.

Proposition 7 [1]**.** *Any circulant graph of degree $8k$ is supermagic.*

Now we are able to prove the following assertions.

Theorem 5. *Let C be a Hamilton cycle of the complete bipartite graph $K_{n,n}$, where $n \geq 5$. Then $K_{n,n} - E(C)$ is a supermagic graph.*

Proof. Consider the following cases.

Case A. Let $n \equiv 1 \pmod 2$. It is not difficult to check that the graph $(\overline{C}_n)^{\bowtie}$ is isomorphic to $K_{n,n} - E(C)$. The graph $\overline{C}_n$ is Hamiltonian, $(n-3)$-regular and so, by Proposition 3, $K_{n,n} - E(C)$ is a supermagic graph.

Case B. Let $n \equiv 0 \pmod 2$. Put $k = \frac{n}{2}$. Suppose that $\{v_0, v_1, \ldots, v_{n-1}\}$ and $\{u_0, u_1, \ldots, u_{n-1}\}$ are parts of $K_{n,n}$. The subgraph C^j, $0 \leq j \leq k-1$, induced by $\bigcup_{i=0}^{n-1} \{v_i u_{i+2j}, u_{i+2j} v_{i+1}\}$ (indices are taken modulo n) is a Hamilton cycle of $K_{n,n}$. Moreover, C^0, C^1, $\ldots$, C^{k-1} form a decomposition of $K_{n,n}$ into pairwise edge-disjoint cycles.

If k is odd, then there is an integer $r \geq 1$ such that $k = 2r+1$. In this case the graph $K_{n,n} - E(C^0)$ is regular of degree $4r$ and the sets $\bigcup_{j=1}^{r} E(C^j)$, $\bigcup_{j=r+1}^{2r} E(C^j)$ form its decomposition into two edge-disjoint connected $2r$-factors. Thus, according to Proposition 4, $K_{n,n} - E(C^0)$ is a supermagic graph.

A supermagic labeling of $K_{8,8} - E(C^3)$ is described below by giving the labels of edges in the following matrix.

	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
v_0	8	39	16	11	25	–	–	48
v_1	41	7	33	13	24	29	–	–
v_2	–	42	6	36	17	20	26	–
v_3	–	–	43	5	32	9	23	35
v_4	27	–	–	44	4	40	18	14
v_5	22	34	–	–	45	3	31	12
v_6	19	15	28	–	–	46	2	37
v_7	30	10	21	38	–	–	47	1

Let T_1 and T_2 be subgraphs of $K_{12,12}$ induced by $E(C^0) \cup E(C^1) \cup E(C^2)$ and $E(C^3) \cup E(C^4)$, respectively. A supermagic labeling of T_1 is described below by giving the labels of edges in the following matrix.

	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}
v_0	12	58	24	16	37	–	–	–	–	–	–	72
v_1	61	11	49	19	36	43	–	–	–	–	–	–
v_2	–	62	10	54	25	30	38	–	–	–	–	–
v_3	–	–	63	9	48	13	35	51	–	–	–	–
v_4	–	–	–	64	8	60	26	22	39	–	–	–
v_5	–	–	–	–	65	7	47	14	34	52	–	–
v_6	–	–	–	–	–	66	6	59	27	21	40	–
v_7	–	–	–	–	–	–	67	5	46	18	33	50
v_8	41	–	–	–	–	–	–	68	4	53	28	23
v_9	32	53	–	–	–	–	–	–	69	3	45	17
v_{10}	29	20	42	–	–	–	–	–	–	70	2	56
v_{11}	44	15	31	57	–	–	–	–	–	–	71	1

By Proposition 4, T_2 is a supermagic graph because it is decomposable into two Hamilton cycles. Therefore, according to Proposition 2, the graph $K_{12,12} - E(C^5)$ is supermagic.

For even $k \geq 8$ there is an integer $r \geq 4$ such that $k = 2r$. Let H_1 be a subgraph of $K_{n,n}$ induced by $\{v_0u_{k-1}, v_ku_{n-1}\} \cup \big(E(C^0) - \{u_{k-1}v_k, u_{n-1}v_0\}\big)$. It is easy to see that H_1 is isomorphic to $2C_n$. Similarly, the subgraph H_2 induced by $\big(E(C^0) \cup E(C^r)\big) - E(H_1)$ is a 2-factor of $K_{n,n}$. According to Proposition 5, the subgraph G_1 of $K_{n,n}$ induced by $E(H_1) \cup E(C^1) \cup E(C^{r+1})$ is supermagic. Similarly, the subgraph G_2 induced by $\bigcup_{j=2}^{r-1} E(C^j) \cup \bigcup_{j=r+2}^{2r-2} E(C^j) \cup E(H_2)$ is a $4(r-2)$-regular bipartite graph decomposable into two edge-disjoint connected $2(r-2)$-factors. By Proposition 4, G_2 is a supermagic graph. As G_1 and G_2 are edge-disjoint factors of $K_{n,n} - E(C^{2r-1})$, according to Proposition 2, the graph $K_{n,n} - E(C^{2r-1})$ is supermagic. ■

Corollary 6. *The complete tripartite graph $K_{1,2[n]}$ is supermagic if and only if $n \geq 2$.*

Proof. According to Proposition 1, the graph $C_4 = K_{2,2}$ has a vertex-total labeling with magic constant 15. Vertex-magic total labelings of $K_{3,3}$ and $K_{4,4}$ with magic constants 36 and 70 are described below by giving the labels of vertices and edges in the following matrices.

		u_0	u_1	u_2
		4	3	5
v_0	1	7	15	13
v_1	2	14	12	8
v_2	9	11	6	10

		u_0	u_1	u_2	u_3
		1	4	6	9
v_0	2	24	22	14	8
v_1	3	23	13	12	19
v_2	5	15	11	21	18
v_3	10	7	20	17	16

According to Theorems 4 and 5 the graph $K_{n,n}$, for $n \geq 5$, admits a vertex -magic total labeling with magic constant

$$h = 7n + 1 + \frac{1}{2}(n-2)(n^2+6n+1) = \frac{1}{2}n(n+1)(n+3).$$

Therefore, for every $n \geq 2$, the graph $K_{n,n}$ has a vertex-magic total labeling with magic constant $h = \frac{1}{2}n(n+1)(n+3)$. Moreover,

$$(2n+1)h - n(n+2)(n+1)^2 = \frac{n(n+1)}{2}(n-1) > 0.$$

Thus, by Corollary 3, $K_{n,n} \oplus K_1 = K_{1,2[n]}$ is a supermagic graph.

The opposite implication is obvious. ∎

Theorem 7. *The complement of the circulant graph $C_{2n}(1,n)$ is supermagic for any integer $n \geq 4$.*

Proof. Clearly, the circulant graph $C_{2n}(2,3,\ldots,n-1)$ is a complement of $C_{2n}(1,n)$. Consider the following cases.

Case A. Let $n \equiv 1 \pmod 2$. In this case $C_{2n}(1,n)$ is a 3-regular bipartite graph with parts $U_1 = \{v_0, v_2, \ldots, v_{2n-2}\}$ and $U_2 = \{v_1, v_3, \ldots, v_{2n-1}\}$. Moreover, $C_{2n}(n-2)$ is a Hamilton cycle of $\overline{C_{2n}(1,n)}(U_1,U_2)$. Therefore, by Proposition 6, the complement of $C_{2n}(1,n)$ is a supermagic graph.

Case B. Let $n \equiv 2 \pmod 4$. Then there is an integer k such that $n = 4k+2$. In this case $C_{2n}(2,3,\ldots,n-1)$ is a circulant graph of degree $2(n-2) = 8k$. According to Proposition 7, the complement of $C_{2n}(1,n)$ is supermagic.

Case C. Let $n \equiv 0 \pmod 4$. Then there is an integer k such that $n = 4k$. If $k = 1$, then a supermagic labeling of $C_8(2,3)$ is described below by giving the labels of edges in the following matrix.

	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_0	–	–	2	10	–	9	13	–
v_1	–	–	–	16	4	–	6	8
v_2	2	–	–	–	5	15	–	12
v_3	10	16	–	–	–	7	1	–
v_4	–	4	5	–	–	–	14	11
v_5	9	–	15	7	–	–	–	3
v_6	13	6	–	1	14	–	–	–
v_7	–	8	12	–	11	3	–	–

If $k \geq 2$, then the graph $C_{8k}(2, 3, \ldots, 4k-1)$ is decomposable into factors $F_1 = C_{8k}(2k-1, 4k-1)$ and $F_2 = C_{8k}(2, \ldots, 2k-2, 2k, \ldots, 4k-2)$. The factor F_1 is a 4-regular bipartite graph which can be decomposed into Hamilton cycles $C_{8k}(2k-1)$ and $C_{8k}(4k-1)$. According to Proposition 4, F_1 is a supermagic graph. Similarly, F_2 is a circulant graph of degree $8(k-1)$ and by Proposition 7, it is also supermagic. Finally, by Proposition 2, the complement of $C_{8k}(1, 4k)$ is a supermagic graph. ■

Corollary 8. *The complete multipartite graph $K_{1,n[2]}$ is supermagic if and only if $n \geq 2$.*

Proof. According to Corollary 6, the graph $K_{1,2[2]}$ is a supermagic graph.

A vertex-magic total labeling of $K_{3[2]}$ with magic constant 53 is described below by giving the labels of vertices and edges in the following matrix.

		v_0	v_1	v_2	v_3	v_4	v_5
		1	2	3	7	6	5
v_0	1	–	11	10	–	18	13
v_1	2	11	–	9	16	–	15
v_2	3	10	9	–	14	17	–
v_3	7	–	16	14	–	4	12
v_4	6	18	–	17	4	–	8
v_5	5	13	15	–	12	8	–

Since $K_{n[2]} = C_{2n}(1, 2, \ldots, n-1)$, the complement of $C_{2n}(1, n)$ is isomorphic to $K_{n[2]} - E(C_{2n}(1))$. Thus, according to Theorems 4 and 7, the graph $K_{n[2]}$, for $n \geq 4$, admits a vertex-magic total labeling with magic constant

$$h = 7n + 1 + \frac{1}{2}(2n-4)\big(n(2n-2) + 6n + 1\big) = 2n^3 - 1.$$

Therefore, for every $n \geq 3$, the graph $K_{n[2]}$ has a vertex-magic total labeling with magic constant $h = 2n^3 - 1$. Moreover, for $n \geq 3$, we have

$$(2n+1)h = 4n^4 + 2n(n^2-1) - 1 > 4n^4 + 2n^2 = 2n^2(2n^2+1).$$

Thus, by Corollary 2, $K_{n[2]} \oplus K_1 = K_{1,n[2]}$ is a supermagic graph.

The opposite implication is obvious. ■

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Received 21 May 2012
Revised 13 November 2012
Accepted 14 November 2012