Discussiones Mathematicae
Graph Theory 33 (2013) 649–656
doi:10.7151/dmgt.1683

SOME SHARP BOUNDS ON THE NEGATIVE DECISION NUMBER OF GRAPHS[1]

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Abstract

Let $G = (V, E)$ be a graph. A function $f : V \to \{-1, 1\}$ is called a *bad function* of G if $\sum_{u \in N_G(v)} f(u) \leq 1$ for all $v \in V$, where $N_G(v)$ denotes the set of neighbors of v in G. The *negative decision number* of G, introduced in [12], is the maximum value of $\sum_{v \in V} f(v)$ taken over all bad functions of G. In this paper, we present sharp upper bounds on the negative decision number of a graph in terms of its order, minimum degree, and maximum degree. We also establish a sharp Nordhaus-Gaddum-type inequality for the negative decision number.

Keywords: negative decision number, bad function, sharp upper bounds, Nordhaus-Gaddum results.

2010 Mathematics Subject Classification: 05C69, 05C75.

1. Introduction

All graphs considered in this paper are simple and undirected. We follow [2] in general for notation and terminologies in graph theory. For a graph G, let $V(G)$ and $E(G)$ denote its vertex set and edge set, respectively. The *order* of G is $|V(G)|$. For each vertex $v \in V(G)$, let $N_G(v) = \{u \in V(G) \mid uv \in E(G)\}$ and $N_G[v] = N_G(v) \cup \{v\}$. The *degree* of v (in G) is $d_G(v) := |N_G(v)|$. The *minimum degree* and *maximum degree* of G are $\delta(G) := \min_{v \in V(G)} d_G(v)$ and $\Delta(G) := \max_{v \in V(G)} d_G(v)$, respectively. For an integer r, G is called *r-regular* if $\Delta(G) = \delta(G) = r$, and is called *nearly r-regular* if $\Delta(G) = r$ and $\delta(G) = r-1$. For

[1]This work was supported in part by the National Basic Research Program of China Grant 2011CBA00300, 2011CBA00301, and the National Natural Science Foundation of China Grant 61033001, 61061130540, 61073174.

$S \subseteq V(G)$, $G[S]$ is the subgraph of G induced by S. Let K_n denote the complete graph of order n, and $K_{a,b}$ the complete bipartite graph with two partition parts having order a and b respectively. Let $\overline{G}$ be the *complement* of G, that is, $\overline{G}$ is a graph with vertex set $V(G)$ and edge set $\{uv \mid u, v \in V(G); u \neq v; uv \notin E(G)\}$. For any function $f : V(G) \to \mathbb{R}$, we define $f(S) := \sum_{v \in S} f(v)$ for all $S \subseteq V(G)$, and the *weight* of f is $w(f) := f(V(G))$.

A function $f : V(G) \to \{-1, 1\}$ is called a *bad function* of G if $f(N_G(v)) \leq 1$ for all $v \in V(G)$. The *negative decision number* of G, denoted by $\beta_D(G)$, is the maximum weight of a bad function of G. The negative decision number is introduced in [12], and several variants of this parameter have been studied recently; see e.g. [14, 15]. The negative decision number can be used to model the minimum number of "negative votes" in a social network that can force every individual in the network to have a "negative opinion" under certain rules (see [12]). It can also be regarded as the "dual" of the concept *signed domination*, which has attracted considerable attention and has been extensively explored in the literature (see e.g. [1, 3, 4, 5, 9, 10, 11, 16, 17] and the references therein). We mention that a somewhat similar (but essentially different) graph parameter called the *signed matching number* (with the function defined on the edges instead of vertices, and requiring that the sum of function values of all incident edges to any vertex is at most 1) has been studied in [13]. For a comprehensive treatment and detailed surveys on (earlier) results in domination theory, the reader is referred to [7, 8].

In this paper, we continue the investigation of the negative decision number in graphs. We present sharp upper bounds on the negative decision number of a graph in terms of its order, minimum degree, and maximum degree, from which several interesting results follow directly. We then establish a sharp Nordhaus-Gaddum-type inequality for the negative decision number.

2. Sharp Upper Bounds on $\beta_D(G)$

In this section we present upper bounds on $\beta_D(G)$ in terms of the order, minimum degree, and maximum degree of G. Since an edgeless graph of order n trivially has negative decision number n, throughout this section we will only consider graphs G with $\Delta(G) \geq 1$. For notational convenience, let $I_n = (n \bmod 2)$ for each integer n; that is, I_n is the binary indicator variable of whether n is odd. Obviously $I_n = 1 - I_{n+1} = 1 - I_{n-1}$ for any integer n. It is also easy to see that for every bad function f of G and $v \in V(G)$, $f(N_G(v)) \leq I_{d_G(v)}$.

Theorem 1. *For every graph G of order n, minimum degree δ and maximum degree $\Delta \geq 1$,*

$$\beta_D(G) \leq \min \left\{ n - \Delta + I_\Delta,\ n \cdot \tfrac{\Delta - \delta + I_\Delta + I_\delta}{\Delta + \delta + I_\Delta - I_\delta} \right\}.$$

Proof. Let G be any graph of order n, minimum degree δ and maximum degree Δ, and f a bad function of G of weight $\beta_D(G)$. Let $v \in V(G)$ be a vertex of degree Δ. Then we have $w(f) = f(N_G(v)) + f(V(G) \setminus N_G(v)) \leq I_{d_G(v)} + n - d_G(v) = n - \Delta + I_\Delta$, which proves that $\beta_D(G) \leq n - \Delta + I_\Delta$. Now it suffices to show that $\beta_D(G) \leq n \cdot \frac{\Delta-\delta+I_\Delta+I_\delta}{\Delta+\delta+I_\Delta-I_\delta}$.

When $\delta = \Delta$, the inequality becomes exactly Theorem 4 in [12]. Thus, in the remainder of the proof we suppose $\Delta \geq \delta + 1$. We need to introduce some notations. Let $P = \{v \in V(G) \mid f(v) = 1\}$ and $Q = V(G) \setminus P = \{v \in V(G) \mid f(v) = -1\}$. Let $P_\delta = \{v \in P \mid d_G(v) = \delta\}$, $P_\Delta = \{v \in P \mid d_G(v) = \Delta\}$, and $P_m = P \setminus (P_\delta \cup P_\Delta)$. Define Q_δ, Q_Δ and Q_m analogously. For each $c \in \{\delta, \Delta, m\}$, let $V_c = P_c \cup Q_c$. Finally let $R = \{v \in V(G) \mid d_G(v) \equiv 0 \pmod 2\}$. Clearly $f(N_G(v)) \leq 0$ for all $v \in R$ and $f(N_G(v)) \leq 1$ for all $v \in V(G) \setminus R$. Also, $\delta + 1 \leq d_G(v) \leq \Delta - 1$ for all $v \in V_m$. Thus, we have

$$\begin{aligned}
n - |R| &\geq \sum_{x\in V(G)} f(N_G(x)) = \sum_{x \in V(G)} d_G(x) f(x) \\
&= \delta|P_\delta| + \Delta|P_\Delta| + \sum_{x\in P_m} d_G(x) - \delta|Q_\delta| - \Delta|Q_\Delta| - \sum_{x\in Q_m} d_G(x) \\
&\geq \delta|P_\delta| + \Delta|P_\Delta| + (\delta+1)|P_m| - \delta|Q_\delta| - \Delta|Q_\Delta| - (\Delta-1)|Q_m| \\
&= \delta|V_\delta| + \Delta|V_\Delta| + (\delta+1)|V_m| - 2\delta|Q_\delta| - 2\Delta|Q_\Delta| - (\Delta+\delta)|Q_m| \\
&= \delta n + (\Delta-\delta)|V_\Delta| + |V_m| - (\Delta+\delta)|Q| + (\Delta-\delta)|Q_\delta| - (\Delta-\delta)|Q_\Delta| \\
&\quad \text{(since } n = |V_\delta| + |V_\Delta| + |V_m| \text{ and } |Q| = |Q_\delta| + |Q_\Delta| + |Q_m|) \\
&= \delta n + (\Delta-\delta)|P_\Delta| + |V_m| - (\Delta+\delta)|Q| + (\Delta-\delta)|Q_\delta|.
\end{aligned}$$

By our definition, $V_\delta \subseteq R$ if $\delta \equiv 0 \pmod 2$ and $V_\Delta \subseteq R$ if $\Delta \equiv 0 \pmod 2$, and hence $|R| \geq (1 - I_\delta)|V_\delta| + (1 - I_\Delta)|V_\Delta|$. Therefore,

$$\begin{aligned}
(\Delta+\delta)|Q| &\geq (\delta-1)n + |R| + (\Delta-\delta)|P_\Delta| + |V_m| + (\Delta-\delta)|Q_\delta| \\
&\geq (\delta-1)n + (1-I_\delta)|V_\delta| + (1-I_\Delta)|V_\Delta| + (\Delta-\delta)|P_\Delta| + |V_m| \\
&\quad +(\Delta-\delta)|Q_\delta| \\
&= (\delta-1)n + (1-I_\delta)|P_\delta| + (1-I_\delta)|Q_\delta| + (1-I_\Delta)|P_\Delta| \\
&\quad +(1-I_\Delta)|Q_\Delta| + (\Delta-\delta)|P_\Delta| + |P_m| + |Q_m| + (\Delta-\delta)|Q_\delta| \\
&= (\delta-1)n + (\Delta-\delta+1-I_\Delta)|P_\Delta| + (1-I_\delta)|P_\delta| + |P_m| \\
&\quad +(\Delta-\delta+1-I_\delta)|Q_\delta| + (1-I_\Delta)|Q_\Delta| + |Q_m|.
\end{aligned}$$

Since $\Delta - \delta + 1 - I_\Delta \geq 1 - I_\delta$ and $\Delta - \delta + 1 - I_\delta \geq 1 - I_\Delta$, we obtain

$$\begin{aligned}
(\Delta+\delta)|Q| &\geq (\delta-1)n + (1-I_\delta)(|P_\Delta| + |P_\delta| + |P_m|) \\
&\quad +(1-I_\Delta)(|Q_\delta| + |Q_\Delta| + |Q_m|) \\
&= (\delta-1)n + (1-I_\delta)(n-|Q|) + (1-I_\Delta)|Q| \\
&= (\delta-I_\delta)n + (I_\delta-I_\Delta)|Q|.
\end{aligned}$$

As $\Delta + \delta + I_\Delta - I_\delta > 0$, we have

$$|Q| \geq n \cdot \frac{\delta - I_\delta}{\Delta + \delta + I_\Delta - I_\delta} ,$$

and thus

$$w(f) = n - 2|Q| \leq n\left(1 - \frac{2(\delta - I_\delta)}{\Delta + \delta + I_\Delta - I_\delta}\right) = n \cdot \frac{\Delta - \delta + I_\Delta + I_\delta}{\Delta + \delta + I_\Delta - I_\delta} ,$$

completing the proof of Theorem 1. ∎

We next show that Theorem 1 is best possible for all minimum and maximum degrees. Note that our theorem has a "high degree of sharpness" as it applies not only to specific values of degrees but to *all* of them.

Theorem 2. *For any integers δ and Δ for which $\Delta \geq \delta \geq 0$ and $\Delta \geq 1$, there exists a graph G of order n with minimum degree δ and maximum degree Δ such that*

$$\beta_D(G) = \min\left\{n - \Delta + I_\Delta,\ n \cdot \frac{\Delta - \delta + I_\Delta + I_\delta}{\Delta + \delta + I_\Delta - I_\delta}\right\}.$$

Proof. When $\delta = 1$, let G be the graph $K_{1,\Delta}$, which has minimum degree δ, maximum degree Δ, and order $n = \Delta + 1$. It is clear that $\beta_D(G) = 1 + I_\Delta = \min\left\{n - \Delta + I_\Delta,\ n \cdot \frac{\Delta-\delta+I_\Delta+I_\delta}{\Delta+\delta+I_\Delta-I_\delta}\right\}$. When $\delta = 0$, just take G to be the union of $K_{1,\Delta}$ and K_1 (a single vertex), and the statement still holds. Thus, we assume in what follows that $\Delta \geq \delta \geq 2$.

Let $a = (\Delta + I_\Delta)/2, b = (\delta - I_\delta)/2$, and $m = 2\Delta$. It is easy to verify that a and b are integers satisfying $1 \leq a \leq \Delta$ and $1 \leq b \leq \delta$. Let K be the union of m disjoint copies of $K_{a,b}$. Hence, K itself is a bipartite graph with partition (A, B), where each vertex in A has exactly b neighbors in B and each vertex in B has exactly a neighbors in A. We then add some edges between the vertices in A to make $K[A]$ become $(\delta - b)$-regular. (This can be done in the following way: Imagine that there is a complete graph K_a with vertex set A. Since $|A| = ma$ is even and every complete graph of even order is 1-factorable (see e.g. Theorem 9.1 in [6]), the edges of K_a can be partitioned into $|A| - 1 \geq \delta$ perfect matchings of it. Taking $\delta - b$ of these matchings and adding them to K will make $K[A]$ become $(\delta - b)$-regular.) Similarly, add some edges between vertices in B to make $K[B]$ $(\Delta - a)$-regular. Denote the finally obtained graph by G. Then, G is a graph of order $n = |A| + |B| = m(a + b)$, minimum degree δ, and maximum degree Δ. It is also clear that $d_G(v) = \delta$ for all $v \in A$ and $d_G(u) = \Delta$ for all $u \in B$.

Now define a function $f : A \cup B \to \{-1, 1\}$ by letting $f(v) = 1$ for all $v \in A$ and $f(u) = -1$ for all $u \in B$. Then, for each $v \in A$, $f(N_G(v)) = \delta - 2b = I_\delta \leq 1$, and for each $u \in Q$, $f(N_G(u)) = 2a - \Delta = I_\Delta \leq 1$. Therefore, f is a bad function of G of weight $|A| - |B|$. Since $n = |A| + |B|$ and $|A|/|B| = a/b = (\Delta + I_\Delta)/(\delta - I_\delta)$, we obtain

$$\begin{aligned}\beta_D(G) &\geq w(f) = |A| - |B| = n\left(1 - \frac{2}{|A|/|B| + 1}\right) = n \cdot \frac{\Delta - \delta + I_\Delta + I_\delta}{\Delta + \delta + I_\Delta - I_\delta} \\ &\geq \min\left\{n - \Delta + I_\Delta, n \cdot \frac{\Delta - \delta + I_\Delta + I_\delta}{\Delta + \delta + I_\Delta - I_\delta}\right\}.\end{aligned}$$

By Theorem 1, we have $\beta_D(G) \leq \min\left\{n - \Delta + I_\Delta, n \cdot \frac{\Delta - \delta + I_\Delta + I_\delta}{\Delta + \delta + I_\Delta - I_\delta}\right\}$. Thus,

$$\beta_D(G) = \min\left\{n - \Delta + I_\Delta, n \cdot \frac{\Delta - \delta + I_\Delta + I_\delta}{\Delta + \delta + I_\Delta - I_\delta}\right\},$$

and Theorem 2 is proved. ■

The following corollaries are immediate from Theorem 1.

Corollary 3. *For any nearly r-regular graph G of order n, $\beta_D(G) \leq \frac{n}{r - I_{r-1}}$.*

Proof. Let G be a nearly r-regular graph. Then $\Delta(G) = r$ and $\delta(G) = r - 1$. By Theorem 1, we have

$$\beta_D(G) \leq n \cdot \frac{r - (r-1) + I_r + I_{r-1}}{r + (r-1) + I_r - I_{r-1}} = \frac{2n}{2r - 1 + 1 - 2I_{r-1}} = \frac{n}{r - I_{r-1}},$$

which finishes the proof of Corollary 3. ■

Corollary 4. *Let c be a real number, $0 < c < 1$. Then $\beta_D(G) \leq cn$ for every graph G of order n with $\Delta(G) \leq \frac{(1+c)\delta(G) - 2}{1-c}$.*

Proof. Let $c \in (0, 1)$ and G be a graph of order n, minimum degree δ and maximum degree Δ, such that $\Delta \leq \frac{(1+c)\delta - 2}{1-c}$. Then,

$$(1-c)\Delta \leq (1+c)\delta - (1-c) - (1+c) \leq (1+c)\delta - (1-c)I_\Delta - (1+c)I_\delta\ .$$

Hence, we have

$$\Delta - \delta + I_\Delta + I_\delta \leq c(\Delta + \delta + I_\Delta - I_\delta)\ ,$$

which, together with Theorem 1, indicates that

$$\beta_D(G) \leq n \cdot \frac{\Delta - \delta + I_\Delta + I_\delta}{\Delta + \delta + I_\Delta - I_\delta} \leq cn.$$

The corollary is thus proved. ■

3. Nordhaus-Gaddum Type Inequality for $\beta_D(G)$

In this section we provide a sharp Nordhaus-Gaddum inequality for $\beta_D(G)$. Before presenting our results, we cite a remark from [12] that will be helpful to our proof.

Lemma 5 (Remark at the beginning of Section 2.4 in [12], restated)**.** *For every integer $n \geq 3$, $\beta_D(K_n) = -I_n$.*

Theorem 6. *For any graph G of order $n \geq 1$,*

$$\beta_D(G) + \beta_D(\overline{G}) \leq \begin{cases} 4 & \text{if } n = 2; \\ n + I_n & \text{if } n \neq 2. \end{cases}$$

Moreover, the bound is sharp for every integer $n \geq 1$.

Proof. The theorem is trivial for $n \in \{1, 2\}$. Thus, we assume in what follows that $n \geq 3$. Let G be a graph of order n. We first show that the inequality holds. Consider the following cases.

Case 1. n is even, in which case $n + I_n = n$.

Case 1.1. G is r-regular for some $r \in \{0, 1, \ldots, n-1\}$. If $r = 0$ or $n - 1$, then $G = \overline{K_n}$ or K_n, and hence $\beta_D(G) + \beta_D(\overline{G}) = n + 0 = n$ by Lemma 5. If $1 \leq r \leq n-2$, then $\overline{G}$ is $(n-1-r)$-regular and $1 \leq n-1-r \leq n-2$. Since n is even, exactly one of r and $n-1-r$ is even. Thus, by Theorem 4 in [12], $\beta_D(G) + \beta_D(\overline{G}) \leq \max\{\frac{n}{r}, \frac{n}{n-1-r}\} \leq n$. This case is completed.

Case 1.2. G is nearly r-regular for some $r \in \{1, 2, \ldots, n-1\}$. In this case $\Delta(G) = r, \Delta(\overline{G}) = n - 1 - \delta(G) = n - r$, and $\overline{G}$ is nearly $(n-r)$-regular. We know from Theorem 1 that $\beta_D(G) \leq n - \Delta(G) + I_{\Delta(G)} = n - r + I_r$, and $\beta_D(\overline{G}) \leq n - \Delta(\overline{G}) + I_{\Delta(\overline{G})} = r + I_{n-r} = r + I_r$ (recall that n is even). Thus $\beta_D(G) + \beta_D(\overline{G}) \leq n + 2I_r$. If $I_r = 0$, then $\beta_D(G) + \beta_D(\overline{G}) \leq n$. If $I_r = 1$, then we apply Corollary 3, which tells us that

$$\beta_D(G) + \beta_D(\overline{G}) \leq \frac{n}{r - I_{r-1}} + \frac{n}{n - r - I_{n-r-1}} = \frac{n}{r} + \frac{n}{n-r}.$$

When $2 \leq r \leq n-2$ we have $\frac{n}{r} + \frac{n}{n-r} \leq \frac{n}{2} + \frac{n}{2} = n$, and when $r = 1$ or $n-1$, we have $\frac{n}{r} + \frac{n}{n-r} = n + \frac{n}{n-1} < n + 2$ since $n \geq 3$. As $\beta_D(G) + \beta_D(\overline{G})$ is always an even integer and n is even, it holds that $\beta_D(G) + \beta_D(\overline{G}) \leq n$. Thus this case is finished.

Case 1.3. $\Delta(G) \geq \delta(G) + 2$. In this case $\Delta(\overline{G}) = n - 1 - \delta(G) \geq n + 1 - \Delta(G)$. It is easy to verify that $n_1 - I_{n_1} \geq n_2 - I_{n_2}$ for any integers $n_1 \geq n_2$. Thus, by Theorem 1 we get $\beta_D(\overline{G}) \leq n - (\Delta(\overline{G}) - I_{\Delta(\overline{G})}) \leq n - (n + 1 - \Delta(G) - I_{n+1-\Delta(G)}) =$

$\Delta(G)-I_{\Delta(G)}$, and thus $\beta_D(G)+\beta_D(\overline{G}) \leq (n-\Delta(G)+I_{\Delta(G)})+(\Delta(G)-I_{\Delta(G)}) = n$, completing the proof of this case.

Case 2. n is odd, in which case $n + I_n = n + 1$.

Case 2.1. G is r-regular for some $r \in \{0, 1, \ldots, n-1\}$. In this case $\overline{G}$ is $(n-1-r)$-regular. Since $nr = 2|E(G)|$ and n is odd, r must be even, and thus $n-1-r$ is also even. If $r = 0$ or $n-1$, then $G = \overline{K_n}$ or K_n, and by Lemma 5 $\beta_D(G) + \beta_D(\overline{G}) = n + (-1) = n - 1$. If $2 \leq r \leq n-3$, we have $\beta_D(G) + \beta_D(\overline{G}) \leq 0 < n + 1$ by Theorem 4 in [12]. This case is thus completed.

Case 2.2. $\Delta(G) \geq \delta(G) + 1$. In this case $\Delta(\overline{G}) = n - 1 - \delta(G) \geq n - \Delta(G)$. From Theorem 1 and the fact that $n_1 - I_{n_1} \geq n_2 - I_{n_2}$ whenever $n_1 \geq n_2$, we have $\beta_D(\overline{G}) \leq n - (\Delta(\overline{G}) - I_{\Delta(\overline{G})}) \leq n - (n - \Delta(G) - I_{n-\Delta(G)}) = \Delta(G) + 1 - I_{\Delta(G)}$ (note that n is odd and hence $I_{n-\Delta(G)} = 1 - I_{\Delta(G)}$). Therefore, $\beta_D(G) + \beta_D(\overline{G}) \leq (n - \Delta(G) + I_{\Delta(G)}) + (\Delta(G) + 1 - I_{\Delta(G)}) = n + 1$, completing the proof of this case.

We have established that $\beta_D(G) + \beta_D(\overline{G}) \leq n + I_n$ for all $n \geq 3$. Now we show that the inequality is sharp for all $n \geq 3$. If n is even, taking G to be K_n gives that $\beta_D(G) + \beta_D(\overline{G}) = n + 0 = n + I_n$. If n is odd, let $V(G) = \{x_i, y_i \mid 1 \leq i \leq (n-1)/2\} \cup \{z\}$ and $E(G) = \{x_i y_i \mid 1 \leq i \leq (n-1)/2\}$. Thus, G consists of $(n-1)/2$ independent edges and a single vertex, and clearly $\beta_D(G) = n$. Note that $V(\overline{G}) = V(G)$ and $E(\overline{G}) = \{zx_i \mid 1 \leq i \leq (n-1)/2\} \cup \{zy_i \mid 1 \leq i \leq (n-1)/2\} \cup \{x_i y_j \mid i \neq j\}$. Define a function $f : V(\overline{G}) \to \{-1, 1\}$ as follows: $f(z) = 1$, and for all $1 \leq i \leq (n-1)/2$, $f(x_i) = 1$ and $f(y_i) = -1$. It is easy to verify that f has weight 1, and that $f(N_{\overline{G}}(v)) \leq 1$ for all $v \in V(\overline{G})$. Thus $\beta_D(\overline{G}) \geq 1$ and $\beta_D(G) + \beta_D(\overline{G}) \geq n + 1 = n + I_n$. Since we already proved that $\beta_D(G) + \beta_D(\overline{G}) \leq n + I_n$, we have $\beta_D(G) + \beta_D(\overline{G}) = n + I_n$. This shows the sharpness of the inequality, and hence concludes the proof of Theorem 6. ■

References

[1] W. Chen and E. Song, *Lower bounds on several versions of signed domination number*, Discrete Math. **308** (2008) 1837–1846.
doi:10.1016/j.disc.2006.09.050

[2] R. Diestel, Graph Theory (Fourth Edition, Springer-Verlag, 2010).

[3] J.E. Dunbar, S.T. Hedetniemi, M.A. Henning and P.J. Slater, *Signed domination in graphs*, Graph Theory, Combinatorics, and Applications **1** (1995) 311–322.

[4] O. Favaron, *Signed domination in regular graphs*, Discrete Math. **158** (1996) 287–293.
doi:10.1016/0012-365X(96)00026-X

[5] Z. Füredi and D. Mubayi, *Signed domination in regular graphs and set-systems*, J. Combin. Theory (B) **76** (1999) 223–239.
doi:10.1006/jctb.1999.1905

[6] F. Harary, Graph Theory (Addison-Wesley, 1969).

[7] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: Advanced Topics (Marcel Dekker, 1998).

[8] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of Domination in Graphs (Marcel Dekker, 1998).

[9] M.A. Henning, *Signed total domination in graphs*, Discrete Math. **278** (2004) 109–125.
doi:10.1016/j.disc.2003.06.002

[10] J. Matoušek, *On the signed domination in graphs*, Combinatorica **20** (2000) 103–108.
doi:10.1007/s004930070034

[11] L. Volkmann, *Signed domination and signed domatic numbers of digraphs*, Discuss. Math. Graph Theory **31** (2011) 415–427.
doi:10.7151/dmgt.1555

[12] C. Wang, *The negative decision number in graphs*, Australas. J. Combin. **41** (2008) 263–272.

[13] C. Wang, *The signed matchings in graphs*, Discuss. Math. Graph Theory **28** (2008) 477–486.
doi:10.7151/dmgt.1421

[14] C. Wang, *Lower negative decision number in a graph*, J. Appl. Math. Comput. **34** (2010) 373–384.
doi:10.1007/s12190-009-0327-5

[15] C. Wang, *Voting 'against' in regular and nearly regular graphs*, Appl. Anal. Discrete Math. **4** (2010) 207–218.
doi:10.2298/AADM100213014W

[16] B. Zelinka, *Signed total domination number of a graph*, Czechoslovak Math. J. **51** (2001) 225–229.
doi:10.1023/A:1013782511179

[17] Z. Zhang, B. Xu, Y. Li and L. Liu, *A note on the lower bounds of signed domination number of a graph*, Discrete Math. **195** (1999) 295–298.
doi:0.1016/S0012-365X(98)00189-7

Received 27 July 2011
Revised 20 July 2012
Accepted 23 July 2012