Discussiones Mathematicae
Graph Theory 32 (2012) 19–29

MEDIAN OF A GRAPH WITH RESPECT TO EDGES

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Abstract

For any vertex v and any edge e in a non-trivial connected graph G, the *distance sum* $d(v)$ of v is $d(v) = \sum_{u \in V} d(v, u)$, the *vertex-to-edge distance sum* $d_1(v)$ of v is $d_1(v) = \sum_{e \in E} d(v, e)$, the *edge-to-vertex distance sum* $d_2(e)$ of e is $d_2(e) = \sum_{v \in V} d(e, v)$ and the *edge-to-edge distance sum* $d_3(e)$ of e is $d_3(e) = \sum_{f \in E} d(e, f)$. The set $M(G)$ of all vertices v for which $d(v)$ is minimum is the *median* of G; the set $M_1(G)$ of all vertices v for which $d_1(v)$ is minimum is the *vertex-to-edge median* of G; the set $M_2(G)$ of all edges e for which $d_2(e)$ is minimum is the *edge-to-vertex median* of G; and the set $M_3(G)$ of all edges e for which $d_3(e)$ is minimum is the *edge-to-edge median* of G. We determine these medians for some classes of graphs. We prove that the edge-to-edge median of a graph is the same as the median of its line graph. It is shown that the center and the median; the vertex-to-edge center and the vertex-to-edge median; the edge-to-vertex center and the edge-to-vertex median; and the edge-to-edge center and the edge-to-edge median of a graph are not only different but can be arbitrarily far apart.

Keywords: median, vertex-to-edge median, edge-to-vertex median, edge-to-edge median.

2010 Mathematics Subject Classification: 05C12.

1. Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q, respectively. For basic definitions and terminology, we refer to [1,3]. The *distance* $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G. A $u - v$ path of length $d(u, v)$ is a $u - v$ *geodesic* in G.

It is known that the distance d is a metric on the vertex set V. The *eccentricity* $e(v)$ of a vertex v is $e(v) = \max\{d(v,u) : u \in V\}$ and the collection of vertices with minimum eccentricity is called the *center* of G and is denoted by $C(G)$. A detailed study of center of a graph is found in [2,5]. The *distance sum* of a vertex v is $d(v) = \sum_{u \in V} d(v,u)$, and the collection of vertices with minimum distance sum is called the *median* of G and is denoted by $M(G)$. The *line graph* of a given graph G is the graph $L(G)$, whose vertices are the edges of G with two vertices of $L(G)$ adjacent whenever the corresponding edges of G are adjacent in G. A *block* of a graph is a maximal connected subgraph having no cut-vertices. A graph G with all its blocks complete is called a *block graph*.

Two areas in which the concept of centrality in graphs and networks is widely applied are facility location problems and social networks. Many problems of finding the "best" site for a facility in a graph or network are in one of the two categories: (i) minimax location problems and (ii) minisum location problems. For example, if one is locating an emergency response facility such as fire service station or police station, then the main problem is to minimize the distance from the location of the facility to the vertex farthest from it. On the other hand, if one is locating a service facility such as post office or electricity office, then the main problem is to minimize the sum of the distances from the location of the facility to all the vertices of the graph. The minimax location problem and the minisum location problem refer to the center and the median, respectively, of a graph. These problems are of the vertex-serves-vertex type, where both the "facility" and the "customer" will be located on vertices. The nature of facility (such as super highway or railway line) to be constructed could necessitate selecting a structure (such as path) rather than just a vertex at which to locate a facility. Similarly, the facility may be required to service structures or areas within the network, and not just vertices. In view of this Slater [7] extended this concept of vertex centrality to more structural situations and proposed that four classes of facility location problems should be considered: (i) vertex-serves-vertex, (ii) vertex-serves-structure, (iii) structure-serves-vertex, and (iv) structure-serves-structure. Further, Slater [8] studied in detail the structure-serves-vertex problem by taking the structure to be a path, leading to the concepts of *path center*, *path median* and *path centroid* of a graph.

For subsets $S, T \subseteq V$ and any vertex v, let $d(v,S) = \min\{d(v,u) : u \in S\}$ and $d(S,T) = \min\{d(x,y) : x \in S, y \in T\}$. In particular, if $f = xy$ and $g = wz$ are edges, then $d(v,f) = \min\{d(v,x), d(v,y)\}$ and $d(f,g) = \min\{d(x,w), d(x,z), d(y,w), d(y,z)\}$. To define and develop the general problem, Slater [7] introduced the definition, let $C = \{C_i : i \in I\}$ and $S = \{S_j : j \in J\}$, where each C_i and each S_j is a subset of V. Let $e_s(C_i) = \max\{d(C_i, S_j) : j \in J\}$; C_i is called the (C,S)-*center* if $e_s(C_i) \leq e_s(C_k)$ for all $k \in I$. Actually, depending upon the problem, one may wish to include other conditions. For example, one might also require the minimality condition that there does not exist $C_h \subseteq C_i$ with $C_h \neq C_i$ and $e_s(C_h) = e_s(C_i)$, as Slater [8] did for path centers.

Let $d_s(C_i) = \sum_{j \in J} d(C_i, S_j)$; C_i is called the (C, S)-*median* if $d_s(C_i) \leq d_s(C_k)$ for all $k \in I$. In view of this definition, the problem viz. the vertex-serves-structure, structure-serves-vertex, and structure-serves-structure situations was studied in [6] for center by taking the structure to be an edge.

Definition [6]**.** For any vertex v in a connected graph G, the vertex-to-edge eccentricity $e_1(v)$ of v is $e_1(v) = \max\{d(v, e) : e \in E\}$. A vertex v for which $e_1(v)$ is minimum is called a vertex-to-edge central vertex of G and the set of all vertex-to-edge central vertices of G is the vertex-to-edge center $C_1(G)$ of G.

Example 1.1 [6]**.** For the graph G given in Figure 1.1, $C(G) = \{v_3, v_4, v_5\}$, $C_1(G) = \{v_1, v_2, v_3, v_4, v_5, v_7\}$. The eccentricities and the vertex-to-edge eccentricities of the vertices of G in Figure 1.1 are given in Table 1.1.

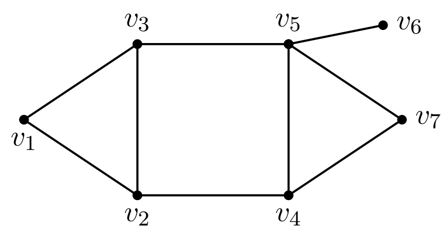

Figure 1.1. The graph G in Example 1.1 with $C(G) \neq C_1(G)$.

Definition [6]**.** For any edge e in a connected graph G, the edge-to-vertex eccentricity $e_2(e)$ of e is $e_2(e) = \max\{d(e, v) : v \in V\}$. Any edge e for which $e_2(e)$ is minimum is called an edge-to-vertex central edge of G and the set of all edge-to-vertex central edges of G is the edge-to-vertex center $C_2(G)$ of G.

v	v_1	v_2	v_3	v_4	v_5	v_6	v_7
$e(v)$	3	3	2	2	2	3	3
$e_1(v)$	2	2	2	2	2	3	2

Table 1.1. The eccentricities and the vertex-to-edge eccentricities of the graph G in Example 1.1.

Definition [6]**.** For any edge e in a connected graph G, the edge-to-edge eccentricity $e_3(e)$ of e is $e_3(e) = \max\{d(e, f) : f \in E\}$. Any edge e for which $e_3(e)$ is minimum is called an edge-to-edge central edge of G and the set of all edge-to-edge central edges of G is the edge-to-edge center $C_3(G)$ of G.

e	v_1v_2	v_1v_3	v_2v_3	v_2v_4	v_3v_5	v_4v_5	v_4v_7	v_5v_6	v_5v_7
$e_2(e)$	3	2	2	2	1	2	2	2	2
$e_3(e)$	2	2	1	1	1	1	2	2	2

Table 1.2. The edge-to-vertex and the edge-to-edge eccentricities of the graph G in Figure 1.1.

Example 1.2 [6]**.** For the graph G given in Figure 1.1, $C_2(G) = \{v_3v_5\}$ and $C_3(G) = \{v_2v_3, v_2v_4, v_3v_5, v_4v_5\}$. Both these types of eccentricities of edges of G in Figure 1.1 are given in Table 1.2.

Centrality concepts have interesting applications in social networks [4,5]. In a social network, an edge represents two individuals having "a common interest" and hence the study of the center or the median of a graph with respect to edges has interesting applications in social networks. In this paper, we study the problem viz. the vertex-serves-structure, structure-serves-vertex, and structure-serves-structure situations for median by taking the structure to be an edge.

We need the following theorem in the sequel.

Theorem 1.3 [9]**.** *The median of a tree consists of either a single vertex or two adjacent vertices.*

2. Median with Respect to an Edge

Definition. For a vertex v in a connected graph G, the *vertex-to-edge distance sum* $d_1(v)$ of v is $d_1(v) = \sum_{e \in E} d(v, e)$. A vertex v for which $d_1(v)$ is minimum is called a *vertex-to-edge median of* G and the set of all vertex-to-edge median vertices of G is the *vertex-to-edge median* $M_1(G)$ of G.

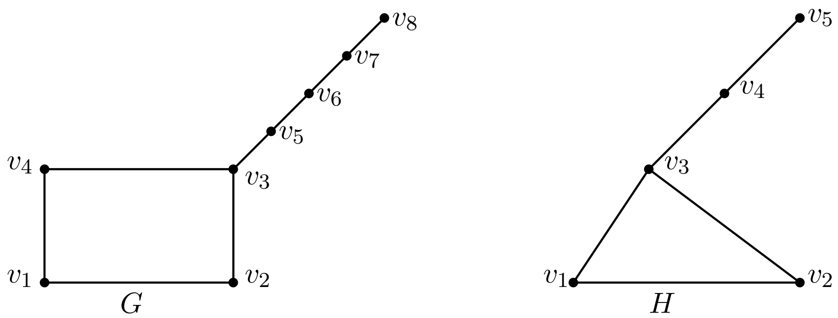

Figure 2.1. Graphs G and H in Example 2.1 with $M(G) \neq M_1(G)$; and $M(H) = M_1(H)$.

Example 2.1. For the graphs G and H given in Figure 2.1, $M(G) = \{v_3, v_5\}$, $M_1(G) = \{v_3\}$, $M(H) = \{v_3\}$ and $M_1(H) = \{v_3\}$. The distance sums and the vertex-to-edge distance sums of the vertices of G and H in Figure 2.1 are given in Table 2.1 and Table 2.2, respectively.

v	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
$d(v)$	22	18	14	18	14	16	20	26
$d_1(v)$	16	12	8	12	9	12	17	24

Table 2.1. $M(G) = \{v_3, v_5\}$ and $M_1(G) = \{v_3\}$ for the graph G in Example 2.1.

v	v_1	v_2	v_3	v_4	v_5
$d(v)$	7	7	5	6	9
$d_1(v)$	4	4	2	4	8

Table 2.2. $M(H) = M_1(H) = \{v_3\}$ for the graph H in Example 2.1.

Remark 2.2. The subgraph induced by the median $M(G)$ or the subgraph induced by the vertex-to-edge median $M_1(G)$ of a connected graph G need not be connected. For the graph G given in Figure 2.2, $M(G) = M_1(G) = \{v_2, v_4\}$.

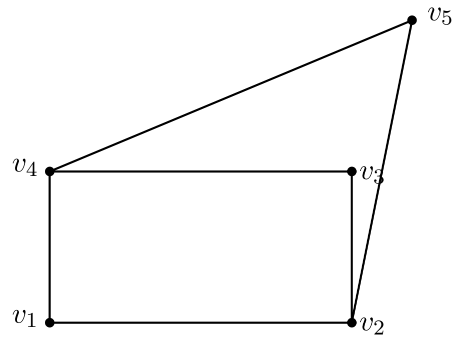

Figure 2.2. A connected graph with its median or vertex-to-edge median disconnected.

First we present some graphs G for which $M(G) = M_1(G)$.

Theorem 2.3. *If T is a tree with p vertices, then $d_1(v) = d(v) - (p-1)$ for every vertex v in T.*

Proof. We prove the result by induction on p. For $p = 1$ or 2, the result is obvious. Assume the result is true for every tree of order $p-1$, where $p \geq 3$. Let $T_1 = T - u$, where u is an end vertex of T. It follows by induction hypothesis that $D_1(v) = D(v) - (p-2)$ for every vertex v of T_1, where $D(v)$ and $D_1(v)$ denote respectively, the distance sum and the vertex-to-edge distance sum of v in T_1. Since $D(v) = d(v) - d(v,u)$ and $D_1(v) = d_1(v) - (d(v,u) - 1)$, it follows that $d_1(v) = d(v) - (p-1)$. Hence the proof is complete by induction. ∎

Corollary 2.4. *For any tree T, $M(T) = M_1(T)$.*

Corollary 2.5. *For any tree T, $M_1(T)$ consists of a single vertex or two adjacent vertices.*

Proof. This follows from Theorem 1.3 and Corollary 2.4. ∎

Proposition 2.6. *If G is the complete graph or a cycle, then $M(G) = M_1(G) = V$.*

Proof. Since both the graphs are symmetric, the result follows from the fact that the value of $d(v)$ (and $d_1(v)$) is equal for every vertex v of G. ∎

Proposition 2.7. *For the complete bipartite graph $G = K_{m,n}$, $M(G) = M_1(G)$.*

Proof. Let X and Y be the partite sets of G with $|X| = m$ and $|Y| = n$. If $m = n$, then $d(x) = d(y) = n+2(n-1) = 3n-2$; and $d_1(x) = d_1(y) = n(n-1)$ for $x \in X$ and $y \in Y$. Thus $M(G) = M_1(G) = V$. If $m < n$, then $d(x) = n+2(m-1)$; $d(y) = m + 2(n-1)$; $d_1(x) = n(m-1)$ and $d_1(y) = m(n-1)$ for $x \in X$ and $y \in Y$. It follows that $M(G) = M_1(G) = X$. If $m > n$, it follows similarly that $M(G) = M_1(G) = Y$. ∎

Remark 2.8. For a bipartite graph G, $M(G)$ need not be equal to $M_1(G)$. For the graph G in Figure 2.1, $M(G) = \{v_3, v_5\}$ and $M_1(G) = \{v_3\}$.

Theorem 2.9. *If G is a non-complete graph with $\Delta(G) = p-1$, then $M(G) = M_1(G)$.*

Proof. Let $S = \{v \in V : deg\ v = p-1\}$. Then it is clear that $d(v) = p-1$ for $v \in S$ and $d(v) \geq p$ for $v \in V-S$. Thus $M(G) = S$. Also, for $v \in S$, $deg\ v = p-1$ and there are $q-p+1$ edges not incident at v. Hence $d_1(v) = q-p+1$. If $v \in V-S$, let $deg\ v = k < p-1$. Then there are $q-k > q-p+1$ edges which are not incident at v. Since $d(v,e) \geq 1$ for each edge e not incident at v (in fact $d(v,e) = 1$ or 2 for such e), it follows that $d_1(v) > q-p+1$. Hence $M_1(G) = S$. ∎

Corollary 2.10. *Let $G = (K_{n_1} \cup K_{n_2} \cup \cdots \cup K_{n_r} \cup kK_1) + v$ be a block graph of order $p \geq 4$, $n_i \geq 2$ $(1 \leq i \leq r)$ and $n_1 + n_2 + \cdots + n_r + k = p-1$. Then $M(G) = M_1(G)$.*

Problem 2.11. Characterize graphs G for which $M(G) = M_1(G)$.

Definition. A graph G is a *self median graph* if $M(G) = V$ and a *self vertex-to-edge median graph* if $M_1(G) = V$.

Example 2.12. Complete graphs and cycles are self vertex-to-edge median graphs (Proposition 2.6). A complete bipartite graph $K_{m,n}$ is self vertex-to-edge median graph if and only if $m = n$ (Proposition 2.7). A non-trivial tree T is self vertex-to-edge median if and only if $T = K_2$ (Corollary 2.5).

We leave the following problem as an open question.

Problem 2.13. Characterize self vertex-to-edge median graphs.

3. EDGE-TO-VERTEX MEDIAN AND EDGE-TO-EDGE MEDIAN

Definition. For an edge e in a connected graph G, the *edge-to-vertex distance sum* $d_2(e)$ of e is $d_2(e) = \sum_{v \in V} d(e, v)$. An edge e for which $d_2(e)$ is minimum is called an *edge-to-vertex median* of G and the set of all edge-to-vertex median edges of G is the *edge-to-vertex median* $M_2(G)$ of G.

Definition. For an edge e in a connected graph G, the *edge-to-edge distance sum* $d_3(e)$ of e is $d_3(e) = \sum_{f \in E} d(e, f)$. An edge e for which $d_3(e)$ is minimum is called an *edge-to-edge median* of G and the set of all edge-to-edge median edges of G is the *edge-to-edge median* $M_3(G)$ of G.

Example 3.1. For the graph G given in Figure 3.1(a), $M_2(G) = \{v_2v_3, v_3v_4\}$ and $M_3(G) = \{v_2v_3, v_3v_4, v_3v_5, v_3v_6\}$ so that $M_2(G) \subsetneq M_3(G)$; and for the graph G given in Figure 3.1(b), $M_2(G) = \{v_3v_5, v_5v_6\}$ and $M_3(G) = \{v_3v_5\}$ so that $M_3(G) \subsetneq M_2(G)$. The Tables 3.1(a) and 3.1(b) show the two types of distance sums of the graphs in Figures 3.1(a) and 3.1(b) respectively.

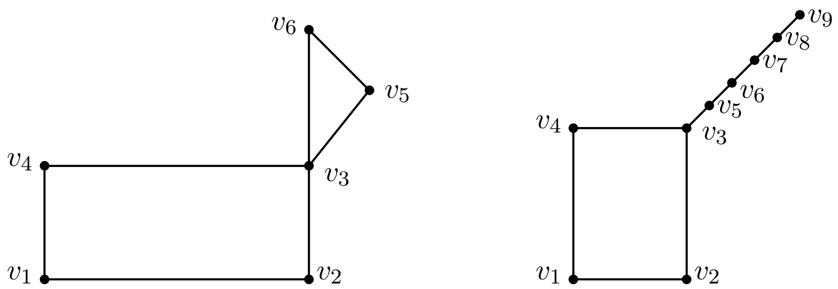

Figure 3.1(a) Figure 3.1(b)

Graphs G in Example 3.1 with $M_2(G) \subsetneq M_3(G)$ or $M_3(G) \subsetneq M_2(G)$.

e	v_1v_2	v_1v_4	v_2v_3	v_3v_4	v_3v_5	v_3v_6	v_5v_6
$d_2(e)$	6	6	4	4	5	5	8
$d_3(e)$	5	5	2	2	2	2	6

Table 3.1(a). The edge-to-vertex and the edge-to-edge distance sums of the edges of the graph in Figure 3.1(a).

e	v_1v_2	v_1v_4	v_2v_3	v_3v_4	v_3v_5	v_5v_6	v_6v_7	v_7v_8	v_8v_9
$d_2(e)$	22	22	17	17	14	14	16	20	26
$d_3(e)$	16	16	11	11	8	9	12	17	24

Table 3.1(b). The edge-to-vertex and the edge-to-edge distance sums of the edges of the graph in Figure 3.1(b).

Theorem 3.2. *If T is a nontrivial tree with p vertices, then $d_3(e) = d_2(e) - (p-2)$ for every edge e in T.*

Proof. We prove the theorem by induction on p. Let $p = 2$. Then $T = K_2$ and so $d_2(e) = d_3(e) = 0$ for every edge e in T. Thus $d_3(e) = d_2(e) - (p - 2)$. Let $p \geq 3$ and assume that the theorem is true for all trees with $p - 1$ vertices. Let $T_1 = T - u$, where u is an end vertex of T. By induction hypothesis, we have $D_3(e) = D_2(e) - (p - 3)$ for every edge e in T_1, where $D_2(e)$ and $D_3(e)$ denote respectively, the edge-to-vertex distance sum and the edge-to-edge distance sum of an edge e in T_1. Now, for any edge e in T_1, $D_2(e) = d_2(e) - d(e, u)$ and $D_3(e) = d_3(e) - d(e, f) = d_3(e) - (d(e, u) - 1)$, where f is the unique edge of T incident at u. Hence it follows that $d_3(e) = d_2(e) - (p - 2)$ and the proof is complete by induction. ■

Corollary 3.3. *For any nontrivial tree T, $M_2(T) = M_3(T)$.*

Proposition 3.4. *If G is the complete graph of order at least 2 or a cycle, then $M_2(G) = M_3(G) = E$.*

Proof. Let $G = K_p$. Then for any edge e of G, there are $p - 2$ vertices that are adjacent to the ends of e and so $d_2(e) = p - 2$. Hence $M_2(G) = E$. Also, the number of edges incident with the ends of e is $(p - 1) + (p - 1) - 1 = 2p - 3$ and so the number of edges which are not incident with any of the ends of e is $\frac{p(p-1)}{2} - (2p - 3) = \frac{p^2-5p+6}{2}$. Hence it follows that $d_3(e) = \frac{p^2-5p+6}{2}$ for every edge e of G and so $M_3(G) = E$.

It is easy to see that for an even cycle C_{2n}, $d_2(e) = n(n - 1)$ and $d_3(e) = (n - 1)^2$; and for an odd cycle $C_{2n+1} (n \geq 2)$, $d_2(e) = n^2$ and $d_3(e) = n(n - 1)$ for any edge e in G. Thus $M_2(G) = M_3(G) = E$ for a cycle G. ■

Proposition 3.5. *For the complete bipartite graph $G = K_{m,n}$,*

$$M_2(G) = M_3(G) = E.$$

Proof. It is easy to see that $d_2(e) = m + n - 2$ and $d_3(e) = (m-1)(n-1)$ for every edge e of G and the result follows. ■

Remark 3.6. For a bipartite graph G, it is not true that $M_2(G) = M_3(G)$. For the graph G given in Figure 3.1(b), $M_2(G) \neq M_3(G)$.

Problem 3.7. Characterize graphs G for which $M_2(G) = M_3(G)$.

Definition. A graph G is a *self edge-to-vertex median graph* if $M_2(G) = E$ and a *self edge-to-edge median* graph if $M_3(G) = E$.

Example 3.8. Complete graphs, cycles and complete bipartite graphs are both self edge-to-vertex and self edge-to-edge median graphs (Propositions 3.4 and 3.5).

Theorem 3.9. *Let G be any connected graph and L its line graph. Let d_L denote the distance metric on L. Then $d_3(e) = d_L(e) - q + 1$ for every edge e of G and $M_3(G) = M(L)$.*

Proof. Let $e = xy$ and $f = zw$ be two distinct edges of G such that $d(e,f) = n$. Let $P : x = u_0, u_1, \ldots, u_n = z$ be a shortest $e - f$ path in G. Then y and w do not lie on P. Let $e_i = u_{i-1}u_i$ $(1 \leq i \leq n)$. Since P is a shortest path in G, it follows that the edges $e, e_1, e_2, \ldots, e_n, f$ all are distinct and $Q : e, e_1, e_2, \ldots, e_n, f$ is a $e - f$ shortest path in L. Hence $d_L(e,f) = n+1 = d(e,f)+1$ and so $d_3(e) = \sum_{f\in E} d(e,f) = \sum_{f\in L, f\neq e}(d_L(e,f) - 1) = \sum_{f\in L, f\neq e} d_L(e,f) - \sum_{f\in L, f\neq e} 1 = d_L(e) - (q-1) = d_L(e) - q + 1$. It follows that $M_3(G) = M(L)$. ■

Corollary 3.10. *A graph G is self edge-to-edge median if and only if its line graph is self-median.*

The next theorem shows that the center and the median; the vertex-to-edge center and the vertex-to-edge median; the edge-to-vertex center and the edge-to-vertex median; and the edge-to-edge center and the edge-to-edge median of a graph are not only different but can be arbitrarily far apart.

Theorem 3.11. *For any positive integer k, there is a connected graph G such that*

(i) $d(C(G), M(G)) = k$,

(ii) $d(C_1(G), M_1(G)) = k$,

(iii) $d(C_2(G), M_2(G)) = k$,

(iv) $d(C_3(G), M_3(G)) = k$.

Proof. (i) Let G be the tree of order $4k+3$ given in Figure 3.2. Then $e(x) = k+1$ and $e(z) > k+1$ for all $x \neq z$ so that $C(G) = \{x\}$. Also, $d(y) = (1+2+3+\cdots+k+(k+1)+\cdots+(2k+1)) + (2k+1) = 2k^2+5k+2$ and $d(z) > 2k^2+5k+2$ for all $z \neq y$ so that $M(G) = \{y\}$. Hence $d(C(G), M(G)) = k$.

(ii) First we prove that $C_1(G) = C(G)$ for any tree G. Let v be a vertex of G. Then it is clear that the eccentricity $e_1(v)$ is attained at a pendant edge $e = xy$ of G with y the end vertex of G. Hence $e_1(v) = d(v,x)$ and $e(v) = d(v,y) = d(v,x)+1 = e_1(v)+1$. Thus $e_1(v) = e(v)-1$ for every vertex v of G and so $C_1(G) = C(G)$. Now, the result follows from Corollary 2.4 and Theorem 3.11(i) for the tree G given in Figure 3.2.

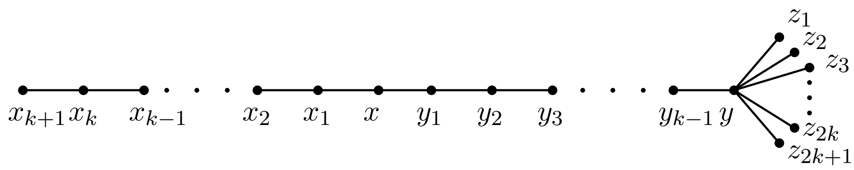

Figure 3.2. The graph G in the proof of Theorem 3.11 (i) and (ii).

(iii) Let G be the tree of order $4k+8$ given in Figure 3.3. It is clear that $e_2(e) = k+2$ and $e_2(f) > k+2$ for the edge $e = xy_1$ and for any $f \neq e$. Hence $C_2(G) = \{e\}$. Also, it is clear that $d_2(g) = (1+2+3+\cdots+(2k+3)) + (2k+3) = 2k^2+9k+9$ for $g = y_{k+1}y$ and $d_2(h) > 2k^2+9k+9$ for any edge $h \neq g$ so that $M_2(G) = \{g\}$. Hence $d(C_2(G), M_2(G)) = k$.

(iv) First we prove that $C_2(G) = C_3(G)$. Let f be an edge of G. Then it is clear that the eccentricity $e_3(f)$ is attained at a pendant edge $h = ab$ of G with b the end vertex of h. Hence $e_3(f) = d(f,h) = d(f,a)$ and $e_2(f) = d(f,b) = d(f,a)+1 = e_3(f)+1$. Thus $e_3(f) = e_2(f)-1$ for any edge f of G and so $C_3(G) = C_2(G)$. Now the result follows from Corollary 3.3 and Theorem 3.11(iii) for the tree G given in Figure 3.3. ■

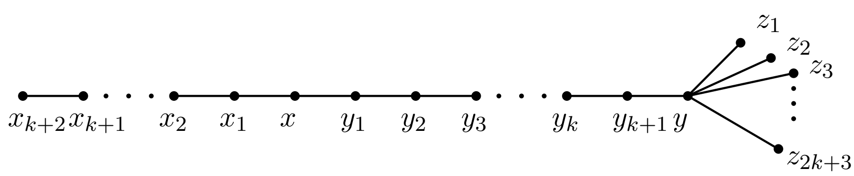

Figure 3.3. The graph G in the proof of Theorem 3.11 (iii) and (iv).

Aknowledgment

The author is thankful to the referee for his useful suggestions.

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Received 4 June 2010
Revised 25 December 2010
Accepted 27 December 2010