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SOME RESULTS ON SEMI-TOTAL SIGNED GRAPHS [1]

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Abstract

A *signed graph* (or *sigraph* in short) is an ordered pair $S = (S^u, \sigma)$, where S^u is a graph $G = (V, E)$, called the *underlying graph* of S and $\sigma : E \to \{+, -\}$ is a function from the edge set E of S^u into the set $\{+, -\}$, called the *signature* of S. The $\times$-*line sigraph* of S denoted by $L_\times(S)$ is a sigraph defined on the line graph $L(S^u)$ of the graph S^u by assigning to each edge ef of $L(S^u)$, the product of signs of the adjacent edges e and f in S. In this paper, first we define *semi-total line sigraph* and *semi-total point sigraph* of a given sigraph and then characterize balance and consistency of semi-total line sigraph and semi-total point sigraph.

Keywords: sigraph, semi-total line sigraph, semi-total point sigraph, balanced sigraph, consistent sigraph.

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1. Introduction

For standard terminology and notation in graph theory we refer Harary [14] and West [21] and Zaslavsky [22, 23] for sigraphs. Throughout the text, we consider finite, undirected graph with no loops or multiple edges.

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A *signed graph* (or sigraph in short; see [7, 11] is an ordered pair $S = (S^u, \sigma)$, where S^u is a graph $G = (V, E)$, called the *underlying graph* of S and $\sigma : E \rightarrow \{+, -\}$ is a function from the edge set E of S^u into the set $\{+, -\}$, called the *signature* of S. Alternatively, the sigraph can be written as $S = (V, E, \sigma)$, with V, E, σ in the above sense. Let $E^+(S) = \{e \in E(G) : \sigma(e) = +\}$ and $E^-(S) = \{e \in E(G) : \sigma(e) = -\}$. The elements of $E^+(S)$ and $E^-(S)$ are called *positive* and *negative* edges of S, respectively. A sigraph is said to be *homogeneous* if all its edges are of the same sign and *heterogeneous* otherwise.

A sigraph S is called a *regular sigraph* if the number of positive edges, $d^+(v)$ incident at a vertex v in S, is independent of the choice of v in S and the number of negative edges, $d^-(v)$ incident at a vertex v in S is also independent of the choice of v in S, i.e., S is a sigraph of order n and regular of degree $k = i + j$, where i=$d^+(v)$ is the positive degree of v in S and j=$d^-(v)$ is the negative degree of v in S.

For a sigraph S, Behzad and Chartrand [7] defined its *line sigraph* $L(S)$ as the sigraph in which the edges of S are represented as vertices, two of these vertices are defined adjacent whenever the corresponding edges in S have a vertex in common and any such edge ef is defined to be negative whenever both e and f are negative edges in S. In [12], the author introduced a variation of the above standard notion of line sigraph $L(S)$ of a given sigraph S as follows: $L_\times(S)$ is a sigraph defined on the line graph $L(S^u)$ of the graph S^u by assigning to each edge ef of $L(S^u)$, the product of signs of the adjacent edges e and f of S. $L_\times(S)$ is called the $\times$-*line sigraph* of S.

A path in a sigraph S is said to be *all-negative* if each of its edge is negative. A cycle in a sigraph S is said to be *all-positive*(*all-negative*) if each of its edge is positive (negative). A cycle in a sigraph S is said to be *positive* if it contains an even number of negative edges. A given sigraph S is said be *balanced* if every cycle in S is positive, i.e., it contains an even number of negative edges [4, 10, 13]. A spectral characterization of balanced sigraphs was given by Acharya [2]. Harary and Kabell [15, 16] developed a simple algorithm to get balanced sigraphs and also enumerated them. The following important lemma on balanced sigraph is given by Zaslavsky.

Lemma 1 [24]**.** *A signed graph in which every chordless cycle is positive, is balanced.*

A *marked signed graph* is an ordered pair $S_\mu = (S, \mu)$, where $S = (S^u, \sigma)$ is a sigraph and $\mu : V(S^u) \rightarrow \{+, -\}$ is a function from the vertex set $V(S^u)$

of S^u into the set $\{+,-\}$, called a *marking* of S. A cycle Z in S_μ is said to be *consistent* if it contains an even number of negative vertices. A given sigraph S is said be *consistent* if every cycle in it is consistent [8, 9]. To this end, we define the following *canonical marking* on S: for each vertex $v \in V(S)$,

$$\mu(v) = \prod_{e \in E_v} \sigma(e)$$

where E_v is set of edges e incident at v in S.

The *semi-total line graph* $T_1(G)$ of a graph G [19] is the graph whose vertex set is $V(G) \cup E(G)$ where $V(G)$ and $E(G)$ are vertex set and edge set of G, respectively and in $T_1(G)$ two vertices are adjacent if and only if (i) they are adjacent edges in G (ii) one is a vertex and the other is an edge in G incident to it.

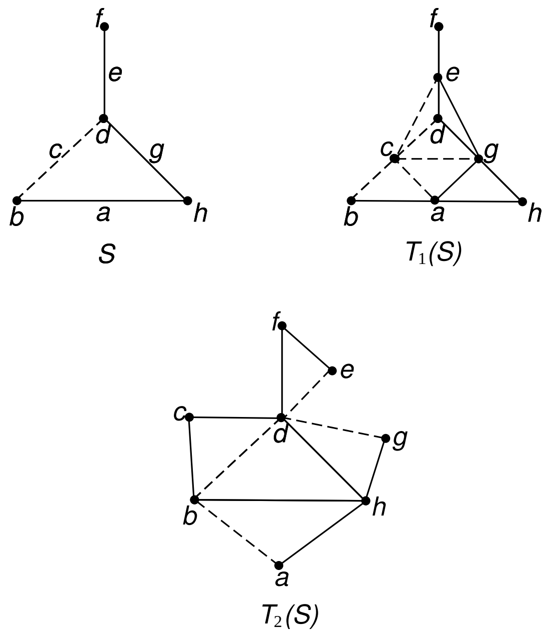

Figure 1. Showing $T_1(S)$ and $T_2(S)$ of a sigraph S.

The *semi-total point graph* $T_2(G)$ of a graph G [19] is the graph whose vertex set is $V(G) \cup E(G)$ where $V(G)$ and $E(G)$ are vertex set and edge set of G, respectively and in $T_2(G)$ two vertices are adjacent if and only if (i) they are adjacent vertices in G, (ii) one is a vertex and the other is an edge in G incident to it.

Let $S = (V, E, \sigma)$ be any sigraph. Its *semi-total line sigraph* $T_1(S)$ [as shown in Figure 1] has $T_1(S^u)$ as its underlying graph and for any edge uv of $T_1(S^u)$

$$\sigma_{T_1}(uv) = \begin{cases} \sigma(u)\sigma(v) & \text{if } u, v \in E, \\ \sigma(v) & \text{if } u \in V \text{ and } v \in E. \end{cases}$$

Let $S = (V, E, \sigma)$ be any sigraph. Its *semi-total point sigraph* $T_2(S)$ [as shown in Figure 1] has $T_2(S^u)$ as its underlying graph and for any edge uv of $T_2(S^u)$

$$\sigma_{T_2}(uv) = \begin{cases} \sigma(uv) & \text{if } u, v \in V, \\ \sigma(u)\prod_{e \in E_v} \sigma(e) & \text{if } u \in E \text{ and } v \in V. \end{cases}$$

We observe that the $\times$-line sigraph, $L_\times(S)$ is an induced subsigraph of $T_1(S)$ and S is an induced subsigraph of $T_2(S)$.

2. Balanced Semi-Total Line Sigraph

In this section, we obtain a characterization of balanced semi-total line sigraph.

Theorem 2 [6]**.** *The $\times$-line sigraph $L_\times(S)$ of a sigraph S is a balanced sigraph.*

Theorem 3 [18]**.** *A sigraph $S = (S^u, \sigma)$ is balanced if and only if there exists a marking μ of its vertices such that for each edge uv in S one has $\sigma(uv) = \mu(u)\mu(v)$.*

Theorem 4. *The semi-total line sigraph $T_1(S)$ of a sigraph S is a balanced sigraph.*

Proof. By the definition of $T_1(S)$, it contains $L_\times(S)$ as induced subsigraph, triangles formed by the adjacent edges e and f in S and the vertex v such that $e \cap f = \{v\}$ and cycles formed by the symmetric difference of these triangles and cycles in $L_\times(S)$. Since $L_\times(S)$ is a balanced sigraph due to Theorem 2, we have to only show that triangles and cycles formed as above in $T_1(S)$ are positive.

Case (i). Suppose e and f both are positive adjacent edges with v as their common vertex in S, then ef will be a positive edge in $T_1(S)$. Now by the definition of $T_1(S)$, the triangle formed by the vertices e, f and v does not contain any negative edge. Thus, such triangles are positive.

Case (ii). Suppose e and f both are negative adjacent edges with v as their common vertex in S, then ef will be a positive edge in $T_1(S)$. Now by the definition of $T_1(S)$, the triangle formed by the vertices e, f and v contain two negative edges. Thus, such triangles are positive too.

Case (iii). Suppose e and f are edges of opposite parity and they are adjacent with v as their common vertex in S, then ef will be a negative edge in $T_1(S)$. Now by the definition of $T_1(S)$, the triangle formed by the vertices e, f and v contain two negative edges. Thus, such triangles are also positive.

Now, due to Lemma 1, it follows that $T_1(S)$ is a balanced sigraph. Hence the theorem. ■

3. Consistent Semi-Total Line Sigraph

Beineke and Harary [8, 9] were the first to pose the problem of characterizing consistent marked graphs, which was subsequently settled by Acharya [1, 2] and Hoede [17]. Acharya and Sinha obtained consistency of sigraphs that satisfy certain sigraph equations in [20, 5]. In this section, first we define a μ_1-marking and then obtain a characterization of μ_1-consistent semi-total line sigraph.

For any sigraph $S = (S^u, \sigma)$, we define μ_1-*marking* in semi-total line sigraph $T_1(S)$ as $\mu_1 : V(T_1(S)) \to \{+, -\}$ such that

$$\mu_1(v_i) = \prod_{e_j \in E_{v_i}} \sigma(e_j)$$

and

$$\mu_1(e_i) = \sigma(e_i).$$

Theorem 5 [17]**.** *A marked graph G_μ is consistent if and only if for any spanning tree T of G all fundamental cycles with respect to T are consistent and all common paths of pairs of those fundamental cycles have end vertices carrying the same marks.*

Theorem 6. *The semi-total line sigraph $T_1(S)$ of a sigraph S is μ_1-consistent if and only if the following conditions hold in S:*

(i) *each cycle Z in S is homogeneous and positive,*

(ii) *if $d(v) \geq 3$, then $d^-(v) = 0$ for every vertex v in S.*

Proof. *Necessity*: Suppose $T_1(S)$ of a sigraph S is μ_1-consistent. Since $L_\times(S)$ is an induced subsigraph of $T_1(S)$ and $T_1(S)$ is μ_1-consistent, it follows that $L_\times(S)$ is μ_1-consistent. Let Z be a cycle in S and $v \in V(Z)$. Let $d(v) = 2$. If possible $d^-(v) = 1$, then let a positive edge e and a negative edge f be incident at v. Due to μ_1-marking in $T_1(S)$, there is a μ_1-consistent cycle Z_1 having one positively marked vertex e and two negatively marked vertices v and f in $T_1(S)$. Let Z_2 be a μ_1-consistent cycle in $L_\times(S)$ having the edge ef. Now, taking the symmetric difference of the edge sets of Z_1 and Z_2, we get a μ_1-inconsistent cycle in $T_1(S)$, since the end vertices of the common edge ef are oppositely marked. Thus, a contradiction to the assumption that $T_1(S)$ is μ_1-consistent. That means, each cycle Z in S is homogeneous. Again, the edges of each cycle Z in S create a cycle in $L_\times(S)$ and each cycle in $L_\times(S)$ has an even number of negatively marked vertices. So, each cycle Z in S has an even number of negative edges. That means, each cycle Z in S is positive. Thus, (i) follows.

If for a vertex v in S, $d^-(v) \geq 3$, then any of the three negative edges incident to v will form a μ_1-inconsistent triangle in $L_\times(S)$, a contradiction that $L_\times(S)$ is μ_1-consistent. So, $d^-(v) < 3$. Now, if $d(v) > 3$, then $d^-(v)$ being equal to one or two would contradict the fact that $L_\times(S)$ is μ_1-consistent. If $d(v) = 3$, then $d^-(v)$ being equal to one again contradicts the fact that $L_\times(S)$ is μ_1-consistent. If $d(v) = 3$ and $d^-(v) = 2$, then let a positive edge e and two negative edges f and g be incident on v. Now due to μ_1-marking in $T_1(S)$, there is a μ_1-inconsistent cycle Z having two positively marked vertices v and e and one negatively marked vertex f in $T_1(S)$, a contradiction to the assumption that $T_1(S)$ is μ_1-consistent. Thus, (ii) follows.

Sufficiency: Suppose both the conditions (i) and (ii) hold for a given sigraph S. We have to show that $T_1(S)$ is μ_1-consistent. By the definition of $T_1(S)$, it contains $L_\times(S)$ as an induced subsigraph, triangles due to the adjacent edges e and f in S and the vertex v such that $e \cap f = \{v\}$ and cycles formed by the symmetric difference of these triangles and cycles in $L_\times(S)$. By these conditions, S is either an all-negative cycle of even length or the sigraph containing all-positive cycles and the end vertices of induced all-negative path do not lie on any cycle.

Case (i). Suppose S is an all-negative cycle of even length. That means, $L_\times(S)$ has an even number of negatively marked vertices. That means, $L_\times(S)$ is μ_1-consistent. Now we have to see the μ_1-consistency of the triangles formed by the edges of S and $L_\times(S)$. Suppose both e and f are negative adjacent edges with v as their common vertex in S. Then due to the μ_1-marking of $T_1(S)$, $\mu_1(e) = \mu_1(f) = -$ and $\mu_1(v) = +$. Hence, the triangle formed by the vertices e, f and v in $T_1(S)$ contains two negatively marked vertices e and f. That means, such triangles are μ_1-consistent. Now, since the vertices e and f in $T_1(S)$ have the same marks, so due to Theorem 5, cycles formed by the symmetric difference of these triangles and cycles in $L_\times(S)$ will be μ_1-consistent. Hence $T_1(S)$ is μ_1-consistent.

Case (ii). Suppose S is the graph containing all-positive cycles and by the condition (ii), such cycles will be adjacent with positive edges only. That means the end vertices of induced all-negative path do not lie on any cycle and due to condition (ii), the end vertices of these induced all-negative path are of degree two. Let e and f be positive and negative adjacent edges, respectively with v as their common vertex in S, then by the μ_1-marking of $T_1(S)$, there is a μ_1-consistent cycle Z having one positively marked vertex e and two negatively marked vertices v and f in $T_1(S)$. Again, let both e and f be negative adjacent edges with v as their common vertex in S. Then, by the μ_1-marking of $T_1(S)$, there is a consistent cycle Z having one positively marked vertex v and two negatively marked vertices e and f in $T_1(S)$. Hence $T_1(S)$ is μ_1-consistent. ∎

4. Balanced Semi-Total Point Sigraph

In this section, we define a μ_1-marking in semi-total point sigraph and obtain a characterization of balanced semi-total point sigraph.

For any sigraph $S = (S^u, \sigma)$, we define μ_1-*marking* in semi-total point sigraph $T_2(S)$ as $\mu_1 : V(T_2(S)) \to \{+,-\}$ such that $\mu_1(v_i) = \prod_{e_j \in E_{v_i}} \sigma(e_j)$ and $\mu_1(e_i) = \sigma(e_i)$.

Theorem 7. *The semi-total point sigraph $T_2(S)$ of a sigraph S is balanced if and only if the following conditions hold in S:*

(i) *if e is a positive edge in S and u, v are the end vertices of e, then the number of negative edges incident at u and v are of the same parity,*

(ii) *if e is a negative edge in S and u, v are the end vertices of e, then the number of negative edges incident at u and v are of the opposite parity.*

Proof. *Necessity*: Suppose $T_2(S)$ is a balanced sigraph, then every cycle in $T_2(S)$ must have an even number of negative edges. The vertex e being adjacent to the vertices u and v in $T_2(S)$, where uv is an edge e in S, we get a triangle Z in $T_2(S)$ due to the vertices u, v and e which is balanced due to hypothesis. Now,

Case (i). If e is a positive edge in Z, then the edges ue and ve must be of the same parity. That means,

$$\prod_{e_i \in E_u} \sigma(e_i) = \prod_{e_j \in E_v} \sigma(e_j),$$

whence, the number of negative edges incident at u and v are of the same parity. Thus, (i) follows.

Case (ii). If e is a negative edge in Z, then the edges ue and ve must be of the opposite parity. That means,

$$\prod_{e_i \in E_u} \sigma(e_i) \neq \prod_{e_j \in E_v} \sigma(e_j),$$

whence, the number of negative edges incident at u and v are of the opposite parity. Thus, (ii) follows.

Sufficiency: Suppose conditions (i) and (ii) hold for a given sigraph S. We have to show that $T_2(S)$ is a balanced sigraph. Let e be an edge in S whose end vertices are u and v. By the definition of $T_2(S)$, $T_2(S)$ contains S as an induced subsigraph, triangles due to the vertices u, v and e and the cycles due to the symmetric difference of these triangles and cycles in S.

By condition (i) and (ii), the sign of each edge in S is the product of μ_1-marking of corresponding end vertices in S. So, using Theorem 3, S is balanced. Now, we have to only show that the triangles and cycles formed as above in $T_2(S)$ are also positive.

By condition (i), e is a positive edge in S whose end vertices are u and v and the number of negative edges incident at u and v are of the same parity. That means,

$$\prod_{e_i \in E_u} \sigma(e_i) = \prod_{e_j \in E_v} \sigma(e_j)$$

and

$$\sigma(e) = +.$$

Hence, by the definition of $T_2(S)$, the triangle due to the vertices u, v and e, contains either no negative edge or two negative edges. Thus, such triangles are positive.

By condition (ii), e is a negative edge in S, whose end vertices are u and v and the number of negative edges incident at u and v are of the opposite parity. That means,

$$\prod_{e_i \in E_u} \sigma(e_i) \neq \prod_{e_j \in E_v} \sigma(e_j)$$

and

$$\sigma(e) = -.$$

Hence, by the definition of $T_2(S)$, the triangle due to the vertices u, v and e, contains two negative edges. Thus, such triangles are positive.

Thus, due to Lemma 1, it follows that $T_1(S)$ is a balanced sigraph. Hence the theorem. ■

Corollary 8. *The semi-total point sigraph $T_2(S)$ of a regular heterogeneous sigraph S is not balanced.*

5. Consistent Semi-Total Point Sigraph

In this section, we obtain a characterization of μ_1-consistent semi-total point sigraph.

Theorem 9. *The semi-total point sigraph $T_2(S)$ of a sigraph $S = (S^u, \sigma)$ is μ_1-consistent if and only if the following conditions hold in S:*

(i) *if e is a positive edge in S and u, v are the end vertices of e, then the vertices u and v are of the same parity,*

(ii) *if e is a negative edge in S and u, v are the end vertices of e, then the vertices u and v are of the opposite parity,*

(iii) *each cycle Z in S is all-positive and if for any $v \in V(Z)$*

$$\prod_{e \in E_v} \sigma(e) = -,$$

then Z is of even length.

Proof. *Necessity*: Suppose $T_2(S)$ is μ_1-consistent, then every cycle in $T_2(S)$ must have an even number of negative vertices. Since $T_2(S)$ has S as an induced subgraph and $T_2(S)$ is μ_1-consistent, it follows that the induced S in $T_2(S)$ is μ_1-consistent. Now, the vertex e being adjacent to the vertices u and v in $T_2(S)$, where uv is an edge e in S, we get a triangle Z in $T_2(S)$ due to the vertices u, v and e which is μ_1-consistent by hypothesis.

Case (i). Let e be a positive edge in Z and $\mu_1(e) = \sigma(e)$, then the number of negative edges incident at u and v are of the same parity. That means,

$$\prod_{e_i \in E_u} \sigma(e_i) = \prod_{e_j \in E_v} \sigma(e_j).$$

This implies, the vertices u and v are of the same parity. Thus, (i) follows.

Case (ii). Let e be a negative edge in Z and $\mu_1(e) = \sigma(e)$, then the number of negative edges incident at u and v are of the opposite parity. That means,

$$\prod_{e_i \in E_u} \sigma(e_i) \neq \prod_{e_j \in E_v} \sigma(e_j).$$

This implies, the vertices u and v are of the opposite parity. Thus, (ii) follows.

Now, let e be a negative edge contained in a cycle Z in S and u, v are the end vertices of e, then due to condition (ii), u and v are of the opposite parity. That means,

$$\mu_1(u) \neq \mu_1(v)$$

and

$$\mu_1(e) = \sigma(e)$$

whence, we get a triangle Z_1 in $T_2(S)$ due to the vertices u, v and e which is μ_1-consistent by hypothesis. Let Z_2 be the consistent cycle in $T_2(S)$ containing the edge $e = uv$. Now, if we take the symmetric difference of Z_1 and Z_2, then by Theorem 5, $T_2(S)$ will not be μ_1-consistent, a contradiction of our hypothesis. Thus, e can not be contained in any cycle in S. This implies, each cycle Z in S is all-positive. Now, let for any $v \in V(Z)$

$$\prod_{e \in E_v} \sigma(e) = -.$$

Since Z is all-positive, then by condition (i), for each $v \in V(Z)$

$$\prod_{e \in E_v} \sigma(e) = -,$$

whence, for each $v \in V(Z)$

$$\mu_1(v) = -.$$

Since S is an induced subsigraph of $T_2(S)$, the cycle Z will be the cycle in $T_2(S)$ and Z will be μ_1-consistent due to hypothesis. It follows that, Z is of even length. Thus (iii) follows.

Sufficiency: Suppose conditions (i), (ii) and (iii) hold for a given sigraph S. We have to show that $T_2(S)$ is μ_1-consistent. Let e be an edge in S whose end vertices are u and v. By the definition of $T_2(S)$, $T_2(S)$ contains S as an induced subsigraph, triangles due to the vertices u, v and e and cycles due to the symmetric difference of these triangles and cycles in S.

By the condition (i), e is a positive edge in S whose end vertices are u and v and the vertices u and v are of the same parity. That means,

$$\mu_1(u) = \mu_1(v)$$

and

$$\mu_1(e) = \sigma(e) = +.$$

By the definition of $T_2(S)$, the triangle due to the vertices u, v and e, contains either no negatively marked vertex or two negatively marked vertices. Therefore, such triangles are μ_1-consistent.

By the condition (ii), e is a negative edge in S whose end vertices are u and v and the vertices u and v are of the opposite parity. That means,

$$\mu_1(u) \neq \mu_1(v)$$

and

$$\mu_1(e) = \sigma(e) = -.$$

By the definition of $T_2(S)$, the triangle due to the vertices u, v and e, contains two negatively marked vertices. Therefore, such triangles are μ_1-consistent.

By the condition (iii), each cycle Z in S is all-positive. Let e be a positive edge of such a cycle and u, v be the end vertices of e. Then, by condition (i) and the definition of $T_2(S)$, we get a μ_1-consistent triangle Z_1

due to the vertices u, v and e, containing the edge e. Let Z_2 be the μ_1-consistent cycle in S containing the edge e. Now, we take the symmetric difference of Z_1 and Z_2. Then, by Theorem 5, we get a μ_1-consistent cycle. Since cycles in S are cycles in $T_2(S)$, Z will be the cycle in $T_2(S)$ and if for any $v \in V(Z)$

$$\prod_{e \in E_v} \sigma(e) = -$$

then Z is of even length. Thus, Z is a μ_1-consistent cycle. Hence the theorem.

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Corollary 10. *The semi-total point sigraph $T_2(S)$ of a regular heterogeneous sigraph S is not μ_1-consistent.*

Corollary 11. *The semi-total point sigraph $T_2(S)$ of a heterogeneous cycle S is not μ_1-consistent.*

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