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PARTITIONS OF A GRAPH INTO CYCLES CONTAINING A SPECIFIED LINEAR FOREST

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Abstract

In this note, we consider the partition of a graph into cycles containing a specified linear forest. Minimum degree and degree sum conditions are given, which are best possible.

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1. Introduction

In this paper, we consider only finite undirected graphs without loops or multiple edges. We will generally follow notation and terminology of [2]. For a vertex x of a graph G, the neighborhood of x is denoted by $N_G(x)$ and $d_G(x) = |N_G(x)|$ is the degree of x in G. For a subgraph H of G and

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a vertex $x \in V(G) - V(H)$, we also denote $N_H(x) = N_G(x) \cap V(H)$ and $d_H(x) = |N_H(x)|$. For a subset S of $V(G)$, we write $\langle S \rangle$ for the subgraph induced by S. For a subgraph H of G and a subset S of $V(G)$, $d_H(S) = \sum_{x \in S} d_H(x)$, $N_H(S) = \bigcup_{x \in S} N_H(x)$ and define $G - H = \langle V(G) - V(H) \rangle$ and $G - S = \langle V(G) - S \rangle$. For a graph G, $|G| = |V(G)|$ is the order of G, $\delta(G)$ is the minimum degree of G, and

$$\sigma_2(G) = \min\{d_G(x) + d_G(y) \mid xy \notin E(G), x, y \in V(G), x \neq y\}$$

is the minimum degree sum of nonadjacent vertices. (When G is complete, we define $\sigma_2(G) = \infty$.)

A forest is a graph each of whose components is a tree and a linear forest is a forest consisting of paths. We regard a single vertex as a path of order 1. For a path $P = v_1 v_2 \cdots v_p$, we call v_i an internal vertex for $2 \leq i \leq p-1$. If P is contained in a cycle C as a subgraph, we denote it by $P \subset C$.

For graphs G and H, $G \cup H$ is the union of G and H, and $G + H$ is the join of G and H. K_n is a complete graph of order n.

Suppose that $H_1, \ldots, H_k$ are vertex-disjoint subgraphs such that $V(G) = \bigcup_{i=1}^{k} V(H_i)$. Then we say G can be partitioned into $H_1, \ldots, H_k$ and $\{H_1, \ldots, H_k\}$ is a partition of G.

Research on partitions of a graph into cycles with a specified number of components was started by Brandt *et al.*

Theorem 1 (Brandt *et al.* [1]). *Suppose that $|G| \geq 4k$ and $\sigma_2(G) \geq |G|$. Then G can be partitioned into k cycles.*

In this paper, we consider partitions into cycles each of which contains exactly one component of a specified linear forest as a subgraph. In the following, n always denotes the order of a graph G, and 'disjoint' means 'vertex-disjoint' because we only deal with partitions of the vertex set.

The special cases where each component of a specified linear forest is a vertex or an edge were considered in several papers [3–11]. In particular, the following theorem was obtained in [7].

Theorem 2 (Enomoto and Matsumura [7]). *Suppose that $n \geq 10p + 10q$, $p + q \geq 1$ and either*

$$\delta(G) \geq \max\left\{\frac{n+q}{2}, \frac{n+p+2q-3}{2}\right\},$$

or

$$\sigma_2(G) \geq \max\{n+q, n+2p+2q-2\}.$$

Then for any linear forest with components $P_1, \ldots, P_{p+q}$ such that $|P_i| = 1$ for $1 \leq i \leq p$ and $|P_i| = 2$ for $p+1 \leq i \leq p+q$, G can be partitioned into cycles $H_1, \ldots, H_{p+q}$ such that $P_i \subset H_i$.

In this paper, we consider a more general case, that is, we specify not only vertices and edges but also paths of order at least 3. The main result of this paper is the following.

Theorem 3. *Suppose that $n \geq 10p+10q'$, $p+q \geq 1$, $p \geq 0$, $q' \geq q \geq 0$, and either*

$$\delta(G) \geq \max\left\{\frac{n+q'}{2}, \frac{n+p+q+q'-3}{2}\right\},$$

or

$$\sigma_2(G) \geq \max\{n+q', n+2p+q+q'-2\}.$$

Then for any linear forest with components $P_1, \ldots, P_{p+q}$ such that $|P_i| = 1$ for $1 \leq i \leq p$, $|P_i| \geq 2$ for $p+1 \leq i \leq p+q$ and $\sum_{i=p+1}^{p+q} |E(P_i)| = q'$, G can be partitioned into cycles $H_1, \ldots, H_{p+q}$ such that $P_i \subset H_i$.

The minimum degree condition in Theorem 3 is sharp in the following sense. (In the following five examples, we let m be a sufficiently large integer.)

Example 1. Suppose that $q' \geq q \geq 1$ and $p+q \geq 2$. Let $G_1 = (K_m^1 \cup K_m^2) + K_{p+q+q'-2}$, where K_m^i is a complete graph of order m for $i = 1, 2$. Take p distinct vertices $P_1, \ldots, P_p$ and $q-1$ disjoint paths $P_{p+1}, \ldots, P_{p+q-1}$ in $K_{p+q+q'-2}$ such that $|E(P_i)| \geq 1$ and $\sum_{i=p+1}^{p+q-1} |E(P_i)| = q_0 < q'$. Moreover, we take a path P_{p+q} which connects K_m^1 and K_m^2, $|E(P_{p+q})| = q' - q_0$ and all internal vertices are contained in $K_{p+q+q'-2}$. (If $q' - q_0 = 1$, we add an edge e which connects K_m^1 and K_m^2 directly and let $P_{p+q} = e$.) Then we cannot take a cycle passing through P_{p+q} without using vertices in $\bigcup_{i=1}^{p+q-1} V(P_i)$. Hence G_1 cannot have the desired partition, while $\delta(G_1) = (|G_1| + p + q + q' - 4)/2$.

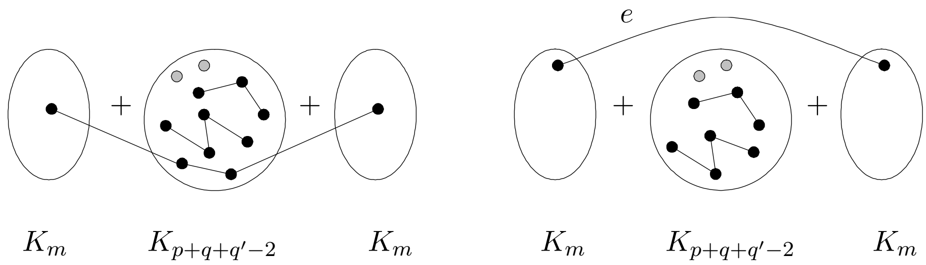

Figure 1. The graph G_1.

Example 2. Suppose that $q = 0$ and let $G_2 = K_{m,m+1}$, a complete bipartite graph with partite sets of order m and $m+1$. Clearly, G_2 cannot have the desired partition, while $\delta(G) = (|G_2| - 1)/2$.

Example 3. Suppose that $p = 0$ and $q' \geq q \geq 1$ and let $G_3 = K_{m+q'} + (m+1)K_1$. Take q disjoint paths $P_1, \ldots, P_q$ in $K_{m+q'}$ so that $|E(P_i)| \geq 1$ and $\sum_{i=1}^{q} |E(P_i)| = q'$. Then G_3 does not have the desired partition, while $\delta(G_3) = (|G_3| + q' - 1)/2$.

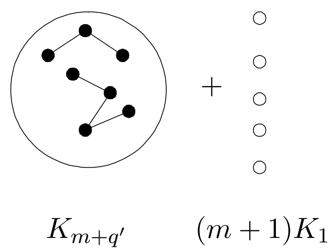

Figure 2. The graph G_3.

The degree sum condition in Theorem 3 is also sharp when there exists some component P_i such that $|P_i| \leq 2$.

Example 4. Suppose that $p \geq 1$. Let $G_4 = (K_p \cup K_m) + K_{2p+q+q'-1}$. Take p distinct vertices $P_1, \ldots, P_p$ in K_p and q disjoint paths $P_{p+1}, \ldots, P_{p+q}$ in $K_{2p+q+q'-1}$ so that $\sum_{i=p+1}^{p+q} |E(P_i)| = q'$. To make a cycle through P_i for

$1 \leq i \leq p$, we have to use at least 2 vertices in $K_{2p+q+q'-1}$ but only $2p-1$ vertices are available. Then G_4 cannot have the desired partition, while $\sigma_2(G_4) = |G_4| + 2p + q + q' - 3$.

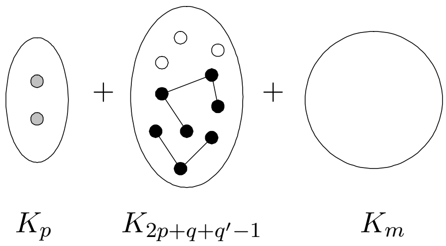

Figure 3. The graph G_4.

Example 5. Suppose that $p = 0$ and let $G_5 = (K_1 \cup K_m) + K_{q+q'-1}$. Take $q-1$ disjoint paths $P_1, \ldots, P_{q-1}$ in $K_{q+q'-1}$ so that $\sum_{i=1}^{q-1} |E(P_i)| = q' - 1$ and an edge P_q connecting K_1 and $K_{q+q'-1}$. Then we cannot take a cycle through P_q without using the vertices of other specified paths. Hence G_5 cannot be partitioned into cycles $H_1, \ldots, H_{p+q}$ such that $P_i \subset H_i$, while $\sigma_2(G_5) = |G_5| + q + q' - 3$.

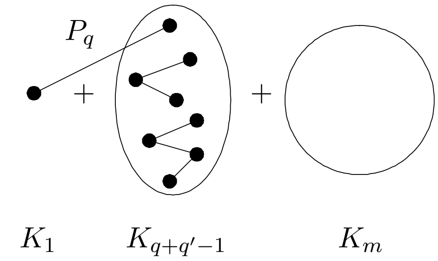

Figure 4. The graph G_5.

The graphs G_2 and G_3 show that the condition '$\sigma_2(G) \geq n + q'$' cannot be dropped because $\sigma_2(G_2) = |G_2| - 1$ and $\sigma_2(G_3) = |G_3| + q' - 1$.

For the case where each component of a specified linear forest is a path of order at least 3, the degree sum condition of Theorem 3 is not sharp and we prove the following.

Theorem 4. *Suppose that $n \geq 3q + q'$, $q \geq 1$, $q' \geq 2q$ and*

$$\sigma_2(G) \geq \max\{n + q', n + q + q' - 3\}.$$

Then for any disjoint paths of order at least 3 $P_1, \ldots, P_q$ such that $\sum_{i=1}^{q} |E(P_i)| = q'$, G can be partitioned into cycles $H_1, \ldots, H_q$ such that $P_i \subset H_i$.

The graph G_1 shows the sharpness of the degree sum condition in Theorem 4, because $\sigma_2(G_1) = |G_1| + p + q + q' - 4$.

To prove Theorem 4, we prove the following theorem, which deals with the case where all paths are of order 3.

Theorem 5. *Suppose that $n \geq 5q$, $q \geq 1$ and*

$$\sigma_2(G) \geq \max\{n + 2q, n + 3q - 3\}.$$

Then for any disjoint paths of order 3 $P_1, \ldots, P_q$, G can be partitioned into cycles $H_1, \ldots, H_q$ such that $P_i \subset H_i$.

We can prove Theorems 3 and 4 similarly. The proof of Theorem 3 is given in the next section. Before proving Theorem 4, we will give a proof of Theorem 5 in Section 3. We will prove Theorem 4 in Section 4.

2. Proof of Theorem 3

Let $\{p_i\} = V(P_i)$ for $1 \leq i \leq p$ and x_i and y_i be endvertices of P_i for $p + 1 \leq i \leq p + q$.

We generate a new graph G' from G by deleting all internal vertices of P_i and adding the edge $x_i y_i$ if $x_i y_i \notin E(G)$ for $p + 1 \leq i \leq p + q$. Then

$$\begin{aligned}
\delta(G') &\geq \max\left\{\frac{n + q'}{2}, \frac{n + p + q + q' - 3}{2}\right\} - (q' - q) \\
&= \max\left\{\frac{(n - q' + q) + q}{2}, \frac{(n - q' + q) + p + 2q - 3}{2}\right\} \\
&= \max\left\{\frac{|G'| + q}{2}, \frac{|G'| + p + 2q - 3}{2}\right\},
\end{aligned}$$

and

$$\begin{aligned}\sigma_2(G') &\geq \max\{n+q', n+2p+q+q'-2\} - 2(q'-q)\\ &= \max\{(n-q'+q)+q, (n-q'+q)+2p+2q-2\}\\ &= \max\{|G'|+q, |G'|+2p+2q-2\}.\end{aligned}$$

Moreover, $|G'| \geq 10p + 10q' - (q'-q) = 10p + 9q' + q \geq 10p + 10q$. Hence by Theorem 2, G' can be partitioned into cycles $H'_1, \ldots, H'_{p+q}$ such that $p_i \in V(H'_i)$ for $1 \leq i \leq p$ and $x_i y_i \in E(H'_i)$ for $p+1 \leq i \leq p+q$.

If we replace $x_i y_i$ by P_i, then we get a cycle H_i from H'_i for $p+1 \leq i \leq p+q$ and $\{H_1, \ldots, H_{p+q}\}$ is the desired partition of G.

3. Proof of Theorem 5

To prove Theorem 5, we first prove the following theorem.

Theorem 6. *Suppose that $n \geq 5q$, $q \geq 1$ and $\sigma_2(G) \geq n + 3q - 3$. Then for any disjoint paths of order 3 $P_1, \ldots, P_q$, G contains q disjoint cycles $C_1, \ldots, C_q$ such that $P_i \subset C_i$ and $|C_i| \leq 5$.*

To complete the proof of Theorem 5, we use the following theorem.

Theorem 7 (Egawa *et al.* [4])**.** *Suppose that $q \geq 1$, $\sigma_2(G) \geq n + q$ and $C_1, \ldots, C_q$ are disjoint subgraphs such that C_i is a cycle or K_2 and $e_i \in E(C_i)$ for $1 \leq i \leq q$. Then there exist disjoint subgraphs $H_1, \ldots, H_q$ such that $V(G) = \bigcup_{i=1}^{q} V(H_i)$, $e_i \in E(H_i)$ and H_i is a cycle if C_i is a cycle and H_i is a cycle or K_2 if C_i is K_2 for $1 \leq i \leq q$.*

3.1. Proof of Theorem 6

A cycle C is called *admissible* if $P_i \subset C$ for some i, $1 \leq i \leq q$, $|V(C) \cap \bigcup_{i=1}^{q} V(P_i)| = 3$ and $|C| \leq 5$. For $1 \leq r \leq q$, a set of cycles $\{C_1, \ldots, C_r\}$ is *admissible* if each C_i is admissible, and C_i and C_j are disjoint if $i \neq j$. If we say 'r admissible cycles', then it means that the set of these r cycles is admissible. A set of admissible cycles $\{C_1, \ldots, C_r\}$ is *minimal* if there exist no r admissible cycles $D_1, \ldots, D_r$ such that $|\bigcup_{i=1}^{r} V(D_i)| < |\bigcup_{i=1}^{r} V(C_i)|$.

Let G be an edge-maximal counterexample and $P_i = x_i y_i z_i$ for $1 \leq i \leq q$. Clearly, G is not complete. Let x and y be nonadjacent vertices of G and define $G' = G + xy$, the graph obtained from G by adding the edge xy. Then G' is no longer a counterexample and G' has q admissible cycles. Since G is a counterexample, the edge xy is contained in some admissible cycle. This implies that G contains $q-1$ admissible cycles and we take minimal admissible cycles $C_1, \ldots, C_{q-1}$. Without loss of generality, we may assume that $P_i \subset C_i$ for $1 \leq i \leq q-1$. Let $L = \langle \bigcup_{i=1}^{q-1} V(C_i) \rangle$, $M = G - L$ and $D = M - V(P_q)$. Note that $x_q z_q \notin E(G)$ and $N_D(x_q) \cap N_D(z_q) = \emptyset$. If possible, we take $C_1, \ldots, C_{q-1}$ so that $d_D(x_q) > 0$ and $d_D(z_q) > 0$.

Claim 1. We have $d_D(x_q) > 0$ and $d_D(z_q) > 0$.

Proof. We first remark that we can take $C_1, \ldots, C_{q-1}$ so that $d_D(x_q) > 0$. To see this, suppose that $d_D(x_q) = 0$ and take any $y \in V(D)$. Since

$$d_M(x_q) + d_M(y) \leq 1 + |M| - 2 = |M| - 1,$$

we have

$$\begin{aligned} d_L(x_q) + d_L(y) &\geq n + 3q - 3 - (|M| - 1) = |L| + 3q - 2 \\ &= \sum_{i=1}^{q-1} |C_i| + 3q - 2 > \sum_{i=1}^{q-1} (|C_i| + 3). \end{aligned}$$

Hence

$$d_{C_i}(x_q) + d_{C_i}(y) \geq |C_i| + 4$$

holds for some i, $1 \leq i \leq q-1$.

If $|C_i| = 3$, then this inequality cannot hold. Hence $|C_i| \geq 4$. Without loss of generality, we may assume that $i = 1$.

Suppose that $|C_1| = 4$ and let $C_1 = x_1 y_1 z_1 v x_1$. Note that $N_{C_1}(x_q) = N_{C_1}(y) = V(C_1)$. If we take $D_1 = x_1 y_1 z_1 y x_1$ and let $D_i = C_i$ for $2 \leq i \leq q-1$, then $\{D_1, \ldots, D_{q-1}\}$ is also minimal admissible and x_q can have a neighbor in $G - \bigcup_{i=1}^{q-1} V(D_i)$ because $x_q v \in E(G)$.

Next suppose that $|C_1| = 5$ and let $C_1 = x_1 y_1 z_1 v u x_1$. If $\{x_1, z_1\} \subset N_{C_1}(y)$, then we can find a shorter admissible cycle passing through P_1. Hence we have $d_{C_1}(y) = 4$. By symmetry, we may assume that $N_{C_1}(y) = \{y_1, z_1, v, u\}$. Then $N_{C_1}(x_q) = V(C_1)$. If we take $D_1 = z_1 y_1 x_1 u y z_1$ and let $D_i = C_i$ for $2 \leq i \leq q-1$, then $\{D_1, \ldots, D_{q-1}\}$ is minimal admissible and

x_q can have a neighbor in $G - \bigcup_{i=1}^{q-1} V(D_i)$ because $x_q v \in E(G)$. Hence we may assume that $d_D(x_q) > 0$.

Now suppose that the claim is false. In view of the remark made at the beginning of the proof, we may assume that $d_D(x_q) > 0$ and $d_D(z_q) = 0$. Take $z \in N_D(x_q)$ and $y \in V(D) - \{z\}$. Arguing as above, we see that there exists j such that $d_{C_j}(z_q) + d_{C_j}(y) \geq |C_j| + 4$ and we can take admissible cycles $D_1, \ldots, D_{q-1}$ so that $\{D_1, \ldots, D_{q-1}\}$ is minimal admissible and z_q can have a neighbor in $G - \bigcup_{i=1}^{q-1} V(D_i)$. But this contradicts the choice of $C_1, \ldots, C_{q-1}$ mentioned immediately before the statement of Claim 1.

Take any $z \in N_D(x_q)$ and $w \in N_D(z_q)$. Note that $\{zw, x_q w, z_q z\} \cap E(G) = \emptyset$, $N_D(x_q) \cap N_D(w) = \emptyset$, and $N_D(z_q) \cap N_D(z) = \emptyset$. (It may occur $\{y_q z, y_q w\} \cap E(G) \neq \emptyset$.)

Let $S = \{x_q, z_q, z, w\}$. Since

$$d_M(S) \leq 8 + 2(|M| - 5) = 2|M| - 2,$$

we have

$$\begin{aligned} d_L(S) &\geq 2(n + 3q - 3) - (2|M| - 2) = 2|L| + 6q - 4 \\ &= \sum_{i=1}^{q-1} 2|C_i| + 6q - 4 > \sum_{i=1}^{q-1} (2|C_i| + 6). \end{aligned}$$

This means that

$$d_{C_i}(S) \geq 2|C_i| + 7$$

for some i, $1 \leq i \leq q$.

If $|C_i| = 3$, then this inequality cannot hold. Hence $|C_i| \geq 4$.

Suppose that $|C_i| = 4$ and let $C_i = x_i y_i z_i v x_i$. By symmetry, we may assume that $N_{C_i}(x_q) = N_{C_i}(z) = V(C_i)$. Then $v \notin N_{C_i}(z_q) \cup N_{C_i}(w)$, because otherwise we can find two admissible cycles. But this means that $d_{C_i}(S) \leq 14$, a contradiction.

Next, suppose that $|C_i| = 5$ and let $C_i = x_i y_i z_i v u x_i$. If $d_{C_i}(z) = 5$, then we can find an admissible cycle $x_i y_i z_i z x_i$, which is shorter than C_i. Hence $d_{C_i}(z) \leq 4$. Similarly, $d_{C_i}(w) \leq 4$. If $(N_{C_i}(x_q) \cap N_{C_i}(z_q)) \cap \{v, u\} \neq \emptyset$, we can also find shorter admissible cycle passing through P_q. Hence $d_{C_i}(x_q) + d_{C_i}(z_q) \leq 8$. But this implies that $d_{C_i}(S) \leq 16$, a contradiction.

This completes the proof of Theorem 6.

3.2. Proof of Theorem 5

By Theorem 6, there exist disjoint cycles $C_1, \dots, C_q$ such that $P_i \subset C_i$. Let $P_i = x_i y_i z_i$ for $1 \le i \le q$.

We make G' from G by deleting $\{y_1, \dots, y_q\}$ and adding the edge $x_i z_i$ for $1 \le i \le q$ if $x_i z_i \notin E(G)$. Then we have disjoint subgraphs $C'_1, \dots, C'_q$ of G' such that $x_i z_i \in E(C'_i)$, and C'_i is a cycle if $|C_i| \ge 4$, and C'_i is K_2 if $|C_i| = 3$. Moreover,

$$\begin{aligned}\sigma_2(G') &\ge \max\{n + 3q - 3, n + 2q\} - 2q \\ &= \max\{(n - q) + 2q - 3, (n - q) + q\} \\ &= \max\{|G'| + 2q - 3, |G'| + q\} \ge |G'| + q.\end{aligned}$$

Hence by Theorem 7, there exist disjoint subgraphs $H'_1, \dots, H'_q$ satisfying $V(G') = \bigcup_{i=1}^{q} V(H_i)$, $x_i z_i \in E(H'_i)$ for $1 \le i \le q$ and H'_i is a cycle if C'_i is a cycle and H'_i is a cycle or K_2 if C'_i is K_2.

By replacing the edge $x_i z_i$ by P_i, we make a cycle H_i from H'_i for $1 \le i \le q$. Then $\{H_1, \dots, H_k\}$ is the desired partition of G.

This completes the proof of Theorem 5.

4. Proof of Theorem 4

Let $P_i = x_i z_i \cdots y_i$ for $1 \le i \le q$. We make G' from G by deleting all internal vertices except z_i of P_i and adding the edge $z_i y_i$ if $z_i y_i \notin E(G)$ for $1 \le i \le q$. Then

$$\begin{aligned}\sigma_2(G) &\ge \max\{n + q', n + q + q' - 3\} - 2(q' - 2q) \\ &\ge \max\{(n - q' + 2q) + 2q, (n - q' + 2q) + 3q - 3\} \\ &\ge \max\{|G'| + 2q, |G'| + 3q - 3\}.\end{aligned}$$

Moreover, $|G'| \ge 3q + q' - (q' - 2q) = 5q$. Hence by Theorem 5, G' can be partitioned into cycles $H'_1, \dots, H'_q$ such that $P'_i \subset H'_i$ for $1 \le i \le q$, where $P'_i = x_i z_i y_i$.

We replace P'_i by P_i and get a cycle H_i from H'_i for $1 \le i \le q$. Then $\{H_1, \dots, H_k\}$ is the desired partition of G.

This completes the proof of Theorem 4.

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