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PROBLEM PRESENTED AT THE WORKSHOP IN KRYNICA 2004

This is a problem by Michael Kubesa, Technical University Ostrava, presented by Dalibor Froncek.

Let K_{2n} be a complete graph and T a tree, both with $2n$ vertices. A T-*factorization* of K_{2n} is a collection of edge disjoint spanning subgraphs (i.e., factors) $T_1, T_2, \ldots, T_n$ of K_{2n}, all isomorphic to T. Every edge of K_{2n} then appears in exactly one copy of T.

M. Kubesa asked the following question: Suppose that there exists a T-factorization of K_{2n}. Is it then true that the vertex set of T can be decomposed into two subsets, X and Y, such that

(1) $|X| = |Y| = n$,

(2) $\sum_{x \in X} \deg(x) = \sum_{y \in Y} \deg(y)$?

Notice that the sets X, Y in general are *not* the partite sets of the bipartition of T.