# THE USE OF EULER'S FORMULA IN (3,1)*-LIST COLORING 

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#### Abstract

A graph $G$ is called $(k, d)^{*}$-choosable if, for every list assignment $L$ satisfying $|L(v)|=k$ for all $v \in V(G)$, there is an $L$-coloring of $G$ such that each vertex of $G$ has at most $d$ neighbors colored with the same color as itself. Ko-Wei Lih et al. used the way of discharging to prove that every planar graph without 4 -cycles and $i$-cycles for some $i \in\{5,6,7\}$ is $(3,1)^{*}$-choosable. In this paper, we show that if $G$ is 2 connected, we may just use Euler's formula and the graph's structural properties to prove these results. Furthermore, for 2-connected planar graph $G$, we use the same way to prove that, if $G$ has no 4 -cycles, and the number of 5 -cycles contained in $G$ is at most $11+\left\lfloor\sum_{i \geq 5} \frac{5 i-24}{4}\left|V_{i}\right|\right\rfloor$, then $G$ is $(3,1)^{*}$-choosable; if $G$ has no 5 -cycles, and any planar embedding of $G$ does not contain any adjacent 3 -faces and adjacent 4 -faces, then $G$ is $(3,1)^{*}$-choosable.


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## 1. Introduction

All graphs considered in this paper are finite simple graphs. For a plane graph $G$, we denote its vertex set, edge set, face set, and minimum degree by $V(G), E(G), F(G)$ and $\delta(G)$, respectively. For $x \in V(G) \cup F(G)$, let $d(x)$ denote the degree of $x$ in $G$. A vertex (or face) of degree $k$ is called a $k$-vertex (or $k$-face). Let $N(u)$ denote the set of neighbors of a vertex $u$ in $G$. Two faces of a plane graph are said to be adjacent if they have at least one common boundary edge. A vertex $v$ and a face $f$ are said to be incident if $v$ lies on the boundary of $f$. For $x \in V(G) \cup F(G)$, we use $F_{k}(x)$ to denote the set of all $k$-faces that are incident or adjacent to $x$, and $V_{k}(x)$ to denote the set of all $k$-vertices that are incident or adjacent to $x$. For $f \in F(G)$, we write $f=\left[u_{1} u_{2} \cdots u_{n}\right]$ if $u_{1}, u_{2}, \ldots, u_{n}$ are the boundary vertices of $f$ in the clockwise order. A 3-face $\left[u_{1} u_{2} u_{3}\right]$ is called an $\left(m_{1}, m_{2}, m_{3}\right)$-face if $d\left(u_{i}\right)=m_{i}$ for $i=1,2,3$.

Let $m>1$ be an integer. A graph $G$ is $(m, d)^{*}$-colorable, if the vertices of $G$ can be colored with $m$ colors so that each vertex has at most $d$ neighbors colored with the same color as itself. An $(m, 0)^{*}$-coloring is an ordinary proper $m$-coloring. A list assignment of $G$ is a function $L$ that assigns a list $L(v)$ of colors to each vertex $v \in V(G)$. An $(L, d)^{*}$-coloring is a mapping $\phi$ that assigns a color $\phi(v) \in L(v)$ to each vertex $v \in V(G)$ such that $v$ has at most $d$ neighbors colored with $\phi(v)$. A graph is $(m, d)^{*}$-choosable, if there exist an $(L, d)^{*}$-coloring for every list assignment $L$ with $|L(v)|=m$ for all $v \in V(G)$. Obviously, $(m, 0)^{*}$-choosability is the ordinary $m$-choosability introduced by Erdös, Rubin and Taylor [2], and independently by Vizing [8].

The notion of list improper coloring was introduced independently by Škrekovski [5] and Eaton and Hull [1]. This class of problems has been studied widely $[1,3,4,5,6,7,9]$ since its introduction. Ko-Wei Lih et al. [3] used the way of discharging to prove that every planar graph without 4-cycles and $i$-cycles for some $i \in\{5,6,7\}$ is $(3,1)^{*}$-choosable. In this paper, we show that if $G$ is 2 -connected, we may just use Euler's formula and the graph's structural properties to prove these results. Furthermore, for 2-connected planar graph $G$, we use the same way to prove that, if $G$ has no 4 -cycles, and the number of 5 -cycles contained in $G$ is at most $11+\left\lfloor\sum_{i \geq 5} \frac{5 i-24}{4}\left|V_{i}\right|\right\rfloor$, then $G$ is $(3,1)^{*}$-choosable; if $G$ has no 5 -cycles, and any planar embedding of $G$ does not contain any adjacent 3 -faces and adjacent 4 -faces, then $G$ is $(3,1)^{*}$-choosable. In Section 2 we provide some lemmata, and in Section 3 we prove our main results.

## 2. Lemmata

We first cite a useful lemma from [3].
Lemma 1 (Lih et al. [3]). Let $G$ be a graph and $d \geq 1$ an integer. If $G$ is not $(k, d)^{*}$-choosable but every subgraph of $G$ with fewer vertices is, then the following facts hold:

1. $\delta(G) \geq k$;
2. If $u \in V(G)$ is a $k$-vertex and $v$ is a neighbor of $u$, then $d(v) \geq k+d$.

The following corollary is obvious.
Corollary 2 (Lih et al. [3]). If $G$ is a plane graph and is not $(3,1)^{*}$ choosable with the fewest vertices, then the following facts hold:

1. $\delta(G) \geq 3$;
2. $G$ does not contain two adjacent 3 -vertices;
3. $G$ does not contain a $(3,4,4)$-face.

Corollary 3. Let $G$ be a 2 -connected plane graph without adjacent 3 -faces. If $G$ is not $(3,1)^{*}$-choosable with the fewest vertices, then

$$
\begin{equation*}
\left|V_{3}(f)\right|+\left|F_{3}(f)\right| \leq d(f) \tag{1}
\end{equation*}
$$

for any $f \in F(G)$.
Proof. Suppose that graph $G$ is not $(3,1)^{*}$-choosable with the fewest vertices. Note that if $G$ is 2-connected, then the boundary of every face of $G$ forms a cycle, and every vertex $v \in V(G)$ is incident to exactly $d(v)$ distinct faces. Let $f$ be a face of $G$. If $d(f)=3$, then the result is obvious by 2 of Corollary 2 and the assumption. So we suppose $d(f) \geq 4$. If all the faces adjacent to $f$ are 3-faces, then $\left|F_{3}(f)\right|=d(f)$, and $\left|V_{3}(f)\right|=0$. Otherwise, there will exist two adjacent 3 -faces, which contradicts the assumption. By 2 of Corollary 2, it is easy to see that whenever $\left|F_{3}(f)\right|$ lessens $1,\left|V_{3}(f)\right|$ increases by at most 1 . So (1) holds for any $f \in F(G)$.

Given a plane graph $G$, let $V_{i}$ ( $F_{i}$, respectively) be the set of all $i$-vertices ( $i$-faces, respectively) of $G, V_{3}^{1}$ the set of all 3 -vertices of $G$ that are not incident to any 3-face, and $V_{3}^{2}=V_{3} \backslash V_{3}^{1}$.

Lemma 4. Let $G$ be a 2-connected plane graph that is not $(3,1)^{*}$-choosable with the fewest vertices.

1. If $G$ does not contain 4-cycles, then

$$
\begin{equation*}
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right| \leq 2|E(G)| \tag{2}
\end{equation*}
$$

2. If $G$ contains neither 4-cycles nor 6-cycles, then

$$
\begin{equation*}
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right|+3\left|F_{5}\right| \leq 2|E(G)| \tag{3}
\end{equation*}
$$

3. If $G$ contains neither 4 -cycles nor 7 -cycles, then

$$
\begin{equation*}
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right|+2\left|F_{5}\right|+3\left|F_{6}\right| \leq 2|E(G)| \tag{4}
\end{equation*}
$$

Proof. Case 1. Suppose that $G$ does not contain 4-cycles, then $G$ contains neither 4 -faces nor adjacent 3 -faces. So by (1),

$$
\sum_{d(f) \geq 5}\left|V_{3}(f)\right|+\sum_{d(f) \geq 5}\left|F_{3}(f)\right| \leq \sum_{d(f) \geq 5} d(f)
$$

Since $\sum_{d(f) \geq 5}\left|V_{3}(f)\right|=3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|$ and $\sum_{d(f) \geq 5}\left|F_{3}(f)\right|=3\left|F_{3}\right|$, then

$$
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+3\left|F_{3}\right| \leq \sum_{d(f) \geq 5} d(f)
$$

or

$$
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+3\left|F_{3}\right|+\sum_{d(f)=3} d(f) \leq \sum_{d(f) \geq 5} d(f)+\sum_{d(f)=3} d(f)
$$

Since $\sum_{d(f)=3} d(f)=3\left|F_{3}\right|$ and that $G$ does not contain any 4-face, then

$$
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right| \leq 2 E(G)
$$

Case 2. Suppose that $G$ is a plane graph without 4 -cycles and 6 -cycles, then any 3 -face is not adjacent to a 5 -face in $G$. So

$$
\sum_{d(f)=5}\left|V_{3}(f)\right|+\sum_{d(f)=5}\left|F_{3}(f)\right|=\sum_{d(f)=5}\left|V_{3}(f)\right| \leq 2\left|F_{5}\right|
$$

By (1),

$$
\sum_{d(f) \geq 7}\left|V_{3}(f)\right|+\sum_{d(f) \geq 7}\left|F_{3}(f)\right| \leq \sum_{d(f) \geq 7} d(f) .
$$

Combining the two equalities above,

$$
\sum_{d(f) \geq 7}\left|V_{3}(f)\right|+\sum_{d(f)=5}\left|V_{3}(f)\right|+\sum_{d(f) \geq 7}\left|F_{3}(f)\right|+\sum_{d(f)=5}\left|F_{3}(f)\right| \leq \sum_{d(f) \geq 7} d(f)+2\left|F_{5}\right| .
$$

By the same cases used in the proof of Case 1, we have

$$
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+3\left|F_{3}\right| \leq \sum_{d(f) \geq 7} d(f)+2\left|F_{5}\right|,
$$

and

$$
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right|+3\left|F_{5}\right| \leq \sum_{d(f) \geq 7} d(f)+5\left|F_{5}\right|+3\left|F_{3}\right|=2|E(G)| .
$$

Case 3. Suppose that $G$ contains neither 4 -cycles nor 7 -cycles, then any 5 -face is adjacent to at most one 3 -face in $G$. So

$$
\sum_{d(f)=5}\left|V_{3}(f)\right|+\sum_{d(f)=5}\left|F_{3}(f)\right| \leq 3\left|F_{5}\right|,
$$

and

$$
\sum_{d(f)=6}\left|V_{3}(f)\right|+\sum_{d(f)=6}\left|F_{3}(f)\right| \leq 3\left|F_{6}\right| .
$$

By (1),

$$
\sum_{d(f) \geq 8}\left|V_{3}(f)\right|+\sum_{d(f) \geq 8}\left|F_{3}(f)\right| \leq \sum_{d(f) \geq 8} d(f) .
$$

Combining these three equalities above, we have

$$
\sum_{d(f) \geq 5}\left|V_{3}(f)\right|+\sum_{d(f) \geq 5}\left|F_{3}(f)\right| \leq \sum_{d(f) \geq 8} d(f)+3\left|F_{5}\right|+3\left|F_{6}\right| .
$$

Furthermore,

$$
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+3\left|F_{3}\right| \leq \sum_{d(f) \geq 8} d(f)+3\left|F_{5}\right|+3\left|F_{6}\right| .
$$

So

$$
\begin{aligned}
& 3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right|+2\left|F_{5}\right|+3\left|F_{6}\right| \\
& \leq \sum_{d(f) \geq 8} d(f)+3\left|F_{3}\right|+5\left|F_{5}\right|+6\left|F_{6}\right|=2|E(G)| .
\end{aligned}
$$

The proof is complete.
Lemma 5. If $G$ is a plane graph without adjacent 3 -faces and is not $(3,1)^{*}$ choosable with the fewest vertices, then

$$
\begin{equation*}
\left|V_{3}^{2}\right| \leq \frac{1}{2} \sum_{i \geq 5} i\left|V_{i}\right| . \tag{5}
\end{equation*}
$$

Proof. By 2 and 3 of Corollary 2, if $v$ is a 3 -vertex of $G$ incident to a 3 -face, then $v$ must be adjacent to a vertex whose degree is at least 5 . So for a vertex $v \in V(G), d(v) \geq 5$, let

$$
V_{3}^{*}(v)=\left\{u \mid u \in N(v) \cap V_{3}^{2}, \text { and } u v \text { is a triangle's edge }\right\},
$$

then $V_{3}^{2}=\bigcup_{d(v) \geq 5} V_{3}^{*}(v)$. Since $G$ does not contain adjacent 3-faces and adjacent 3 -vertices, then $\left|V_{3}^{*}(v)\right| \leq \frac{1}{2} d(v)$. Therefore

$$
\left|V_{3}^{2}\right| \leq \sum_{d(v) \geq 5}\left|V_{3}^{*}(v)\right| \leq \frac{1}{2} \sum_{d(v) \geq 5} d(v)=\frac{1}{2} \sum_{i \geq 5} i\left|V_{i}\right| .
$$

## 3. Main Results

In this section we just use Euler's formula and the lemmata provided in the previous section to prove the theorems.

Theorem 6 (Lih et al. [3]). If $G$ is a 2-connected planar graph without 4 -cycles and $i$-cycles for some $i \in\{5,6,7\}$, then $G$ is $(3,1)^{*}$-choosable.

Proof. Suppose that $G$ is a counterexample with the fewest vertices, and we consider the planar embeddings of $G$. By Euler's formula

$$
|V(G)|+|F(G)|=|E(G)|+2
$$

or

$$
\sum_{i \geq 3}\left|V_{i}\right|+\sum_{i \geq 3}\left|F_{i}\right|=|E(G)|+2,
$$

we have

$$
\begin{aligned}
& \frac{1}{4}\left|V_{3}\right|+\frac{1}{4} \sum_{i \geq 3} i\left|V_{i}\right|-\frac{1}{4} \sum_{i \geq 5}(i-4)\left|V_{i}\right|+\frac{3}{6}\left|F_{3}\right| \\
& +\frac{2}{6}\left|F_{4}\right|+\frac{1}{6}\left|F_{5}\right|+\frac{1}{6} \sum_{i \geq 3} i\left|F_{i}\right| \geq|E(G)|+2
\end{aligned}
$$

i.e.,

$$
\begin{aligned}
& \frac{1}{4}\left|V_{3}\right|+\frac{2|E(G)|}{4}-\frac{1}{4} \sum_{i \geq 5}(i-4)\left|V_{i}\right|+\frac{1}{2}\left|F_{3}\right| \\
& +\frac{1}{3}\left|F_{4}\right|+\frac{1}{6}\left|F_{5}\right|+\frac{2|E(G)|}{6} \geq|E(G)|+2
\end{aligned}
$$

or
(6) $\quad 3\left|V_{3}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right|+6\left|F_{3}\right|+4\left|F_{4}\right|+2\left|F_{5}\right| \geq 2|E(G)|+24$.

Case 1. $G$ has no 4 -cycles and 5 -cycles. By (2) and (6),

$$
3\left|V_{3}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right|+6\left|F_{3}\right| \geq 3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right|+24,
$$

i.e.,

$$
\left|V_{3}^{2}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right| \geq 24 .
$$

By (5),

$$
\frac{1}{2} \sum_{i \geq 5} i\left|V_{i}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right| \geq 24,
$$

i.e.,

$$
\sum_{i \geq 5}\left(12-\frac{5}{2} i\right)\left|V_{i}\right| \geq 24
$$

which is impossible, since $12-\frac{5}{2} i<0$ when $i \geq 5$.
Case 2. $G$ has no 4 -cycles and 6 -cycles. By (3) and (6),
$3\left|V_{3}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right|+6\left|F_{3}\right|+2\left|F_{5}\right| \geq 3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right|+3\left|F_{5}\right|+24$, i.e.,

$$
\left|V_{3}^{2}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right| \geq\left|F_{5}\right|+24
$$

By (5),

$$
\frac{1}{2} \sum_{i \geq 5} i\left|V_{i}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right| \geq\left|F_{5}\right|+24,
$$

i.e.,

$$
\sum_{i \geq 5}\left(12-\frac{5}{2} i\right)\left|V_{i}\right| \geq\left|F_{5}\right|+24
$$

which is impossible.
Case 3. $G$ has no 4 -cycles and 7 -cycles. By (4) and (6),

$$
\begin{aligned}
& 3\left|V_{3}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right|+6\left|F_{3}\right|+2\left|F_{5}\right| \\
& \geq 3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right|+2\left|F_{5}\right|+3\left|F_{6}\right|+24,
\end{aligned}
$$

i.e.,

$$
\left|V_{3}^{2}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right| \geq 3\left|F_{6}\right|+24 .
$$

By (5),

$$
\frac{1}{2} \sum_{i \geq 5} i\left|V_{i}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right| \geq 3\left|F_{6}\right|+24
$$

i.e.,

$$
\sum_{i \geq 5}\left(12-\frac{5}{2} i\right)\left|V_{i}\right| \geq 3\left|F_{6}\right|+24
$$

which is impossible. The proof is complete.

## Theorem 7. Let $G$ be a 2-connected planar graph.

1. If $G$ has no 4-cycles, and the number of 5 -cycles contained in $G$ is at most $11+\left\lfloor\sum_{i \geq 5} \frac{5 i-24}{4}\left|V_{i}\right|\right\rfloor$, then $G$ is $(3,1)^{*}$-choosable.
2. If $G$ has no 5 -cycles, and any planar embedding of $G$ does not contain any adjacent 3 -faces and adjacent 4 -faces, then $G$ is (3,1)*-choosable.

Proof. 1. Suppose that $G$ is a 2 -connected planar graph without 4-cycles and is not $(3,1)^{*}$-choosable with the fewest vertices. We consider the planar embeddings of $G$. By (2) and (6),

$$
3\left|V_{3}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right|+6\left|F_{3}\right|+2\left|F_{5}\right| \geq 3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right|+24,
$$

i.e.,

$$
\left|V_{3}^{2}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right|+2\left|F_{5}\right| \geq 24
$$

By (5),

$$
\frac{1}{2} \sum_{i \geq 5} i\left|V_{i}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right|+2\left|F_{5}\right| \geq 24
$$

or

$$
\left|F_{5}\right| \geq 12+\sum_{i \geq 5} \frac{5 i-24}{4}\left|V_{i}\right|,
$$

a contradiction.
2. Suppose that $G$ is a counterexample with the fewest vertices, and we consider the planar embeddings of $G$. Since $G$ has no 5 -cycles, then there is no 3 -face adjacent to a 4 -face in any planar embedding of $G$. By 2 of Corollary 2, $G$ does not contain adjacent 3 -vertices. So for any $f \in F(G)$, we have

$$
\begin{equation*}
\left|V_{3}(f)\right|+\left|F_{3}(f)\right|+\left|F_{4}(f)\right| \leq d(f) \tag{7}
\end{equation*}
$$

When $d(f)=3$,4, (7) trivially holds. So we suppose $d(f) \geq 6$. If $F_{i}(f)=\phi$ for $i \geq 5$, then $\left|F_{3}(f)\right|+\left|F_{4}(f)\right|=d(f)$ and $\left|V_{3}(f)\right|=0$ by the conditions of the theorem. It is easy to see that whenever $\left|F_{3}(f)\right|+\left|F_{4}(f)\right|$ lessens 1, $\left|V_{3}(f)\right|$ increases by at most 1 . So (7) holds for any $f \in F(G)$.

By (7),

$$
\sum_{d(f) \geq 4}\left|V_{3}(f)\right|+\sum_{d(f) \geq 4}\left|F_{3}(f)\right|+\sum_{d(f) \geq 4}\left|F_{4}(f)\right| \leq \sum_{d(f) \geq 4} d(f)
$$

or

$$
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+3\left|F_{3}\right|+4\left|F_{4}\right| \leq \sum_{i \geq 4} i\left|F_{i}\right| .
$$

Therefore

$$
\begin{equation*}
3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right|+4\left|F_{4}\right| \leq 2|E(G)| . \tag{8}
\end{equation*}
$$

By (6) and (8),
$3\left|V_{3}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right|+6\left|F_{3}\right|+4\left|F_{4}\right| \geq 3\left|V_{3}^{1}\right|+2\left|V_{3}^{2}\right|+6\left|F_{3}\right|+4\left|F_{4}\right|+24$,
i.e.,

$$
\left|V_{3}^{2}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right| \geq 24 .
$$

By (5),

$$
\frac{1}{2} \sum_{i \geq 5} i\left|V_{i}\right|-3 \sum_{i \geq 5}(i-4)\left|V_{i}\right| \geq 24 .
$$

i.e.,

$$
\sum_{i \geq 5}\left(12-\frac{5}{2} i\right)\left|V_{i}\right| \geq 24
$$

which is impossible, since $12-\frac{5}{2} i<0$ when $i \geq 5$.

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