# ON TRACEABILITY AND 2-FACTORS <br> IN CLAW-FREE GRAPHS 

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#### Abstract

If $G$ is a claw-free graph of sufficiently large order $n$, satisfying a degree condition $\sigma_{k}>n+k^{2}-4 k+7$ (where $k$ is an arbitrary constant), then $G$ has a 2 -factor with at most $k-1$ components. As a second main result, we present classes of graphs $\mathcal{C}_{1}, \ldots, \mathcal{C}_{8}$ such that every sufficiently large connected claw-free graph satisfying degree condition $\sigma_{6}(k)>n+19$ (or, as a corollary, $\delta(G)>\frac{n+19}{6}$ ) either belongs to $\cup_{i=1}^{8} \mathcal{C}_{i}$ or is traceable.


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## 1. Introduction

We consider finite undirected graphs $G=(V(G), E(G))$ without loops and multiple edges. We follow the most common terminology and notation and for concepts not defined here we refer e.g. to [1]. For any set $A \subset V(G)$ we denote by $\langle A\rangle_{G}$ the subgraph of $G$ induced on $A$ and $G-A$ stands for $\langle V(G) \backslash A\rangle$. A graph $G$ is $H$-free (where $H$ is a graph), if $G$ does not contain an induced subgraph isomorphic to $H$. In the special case $H=$ $K_{1,3}$ we say that $G$ is claw-free. The independence number of $G$ is denoted by $\alpha(G)$ and the clique covering number of $G$ (i.e., the minimum number of cliques necessary for covering $V(G))$ by $\theta(G)$. We denote by $\delta(G)$ the minimum degree of $G$ and by $\sigma_{k}(G)(k \geq 1)$ the minimum degree sum over all independent sets of $k$ vertices in $G$ (for $k>\alpha(G)$ we set $\sigma_{k}(G)=\infty$ ). The circumference of $G$, i.e., the length of a longest cycle in $G$, is denoted by $c(G)$, and the length of a longest path in $G$ is denoted by $p(G)$. A graph $G$ of order $n$ is hamiltonian or traceable if $c(G)=n$ or $p(G)=n$, respectively.

The line graph of a graph $H$ is denoted by $L(H)$. If $G=L(H)$, then we also denote $H=L^{-1}(G)$ and say that $H$ is the line graph preimage of $G$ (recall that for any line graph $G$ nonisomorphic to $K_{3}$, its line graph preimage is uniquely determined).

A vertex $x \in V(G)$ is said to be locally connected if its neighborhood $N(x)$ induces a connected graph. The closure of a claw-free graph $G$ (introduced in [12] by the first author) is defined as follows: the closure $\operatorname{cl}(G)$ of $G$ is the (unique) graph obtained by recursively completing the neighborhood of any locally connected vertex of $G$, as long as this is possible. The closure $\mathrm{cl}(G)$ remains a claw-free graph and its connectivity is at least equal to the
connectivity of $G$. The following basic properties of the closure $\operatorname{cl}(G)$ were proved in [12], [3] and [13].

Theorem A. Let $G$ be a claw-free graph and $\operatorname{cl}(G)$ its closure. Then
(i) [12] there is a triangle-free graph $H_{G}$ such that $\operatorname{cl}(G)=L\left(H_{G}\right)$,
(ii) [12] $c(G)=c(\mathrm{cl}(G))$,
(iii) $[3] p(G)=p(\mathrm{cl}(G))$,
(iv) [13] $G$ has a 2 -factor with at most $k$ components if and only if $\mathrm{cl}(G)$ has a 2 -factor with at most $k$ components.

Consequently, $G$ is hamiltonian (traceable) if and only if $\mathrm{cl}(G)$ is hamiltonian (traceable). If $G$ is a claw-free graph such that $G=\operatorname{cl}(G)$, then we say that $G$ is closed. It is apparent that a claw-free graph $G$ is closed if and only if every vertex $x \in V(G)$ is either simplicial (i.e., $\langle N(x)\rangle_{G}$ is a clique), or is locally disconnected (i.e., $\langle N(x)\rangle_{G}$ consists of two vertex disjoint cliques).

In [12], the closure concept was used to answer an old question by showing that every 7 -connected claw-free graph is hamiltonian. H. Li [10] extended this result as follows.

Theorem B [10]. Every 6 -connected claw-free graph with at most 34 vertices of degree 6 is hamiltonian.

In [5], the following result on 2-factors with limited number of components was proved.

Theorem C [5]. If $G$ is a claw-free graph of order $n$ and minimum degree $\delta \geq 4$, then $G$ contains a 2 -factor with at most $\frac{6 n}{\delta+2}-1$ components.

This result was improved by Gould and Jacobson [8].
Theorem D [8]. Let $k \geq 2$ be an integer and let $G$ be a claw-free graph of order $n \geq 16 k^{3}$ and minimum degree $\delta \geq \frac{n}{k}$. Then $G$ contains a 2 -factor with at most $k$ components.

In the first main result of this paper, Theorem 3, we give a strengthening of this result.

A trail $T$ (closed or not) in a graph $H$ is said to be dominating if every edge of $H$ has at least one vertex on $T$. Harary and Nash-Williams [11]
proved the following result, showing that hamiltonicity of a line graph is equivalent to the existence of a dominating closed trail in its preimage.

Theorem E [11]. Let $H$ be a graph without isolated vertices. Then $L(H)$ is hamiltonian if and only if either $H$ is isomorphic to $K_{1, r}$ (for some $r \geq 3$ ) or $H$ contains a dominating closed trail.

It is straightforward to verify the following analogue of Theorem E for traceability.

Theorem F. Let $H$ be a graph without isolated vertices. Then $L(H)$ is traceable if and only if either $H$ is isomorphic to $K_{1, r}$ (for some $r \geq 3$ ) or $H$ contains a dominating trail.

Using the closure concept in claw-free graphs [12], Favaron, Flandrin, Li and Ryjáček [6] observed that there is a close relation between the minimum degree sum $\sigma_{k}(G)$ (or the minimum degree $\delta(G)$, respectively) of a closed claw-free graph $G$ and its clique covering number. These connections are established in the following results [6].

Theorem G [6]. Let $k \geq 2$ be an integer and let $G$ be a claw-free graph of order $n$ such that $\delta(G)>3 k-5$ and $\sigma_{k}(G)>n+k^{2}-2 k$. Then $\theta(\operatorname{cl}(G)) \leq$ $k-1$.

Corollary H [6]. Let $k \geq 2$ be an integer and let $G$ be a claw-free graph of order $n \geq 2 k^{2}-3 k$ and minimum degree $\delta(G)>\frac{n}{k}+k-2$. Then $\theta(\operatorname{cl}(G)) \leq$ $k-1$.
The bounds on $\sigma_{k}(G)(\delta(G))$ in the previous results are sharp (this can be easily seen considering the cartesian product of cliques).

It was shown in [6] and [9] that these results can be slightly strengthened under an additional assumption that $G$ is not hamiltonian, and this result was used to obtain degree conditions for hamiltonicity (by characterizing the classes of all 2-connected nonhamiltonian closed claw-free graphs with small clique covering number). In the second main result of this paper, Theorem 6, we follow up with this study by considering analogous questions for traceability.

## 2. Main Results

We begin with a structural result that can be considered, in a sense, as a strengthening of Theorem G.

Theorem 1. Let $k \geq 2$ be an integer, let $G$ be a claw-free graph of order $n$ and let $\kappa=\kappa(\mathrm{cl}(G))$. Suppose that $G$ is such that $n \geq 3 k^{2}+k-(k+1) \kappa-2$, $\delta(G) \geq 3 k-4$ and

$$
\sigma_{k}(G)>n+k^{2}-4 k+2+\kappa
$$

Then either $\theta(\operatorname{cl}(G)) \leq k-1$, or $\alpha(\operatorname{cl}(G)) \leq \kappa$.

Before proving Theorem 1, we first recall the following auxiliary results that were proved in [6].

Lemma I [6]. Let $G$ be a closed claw-free graph of order $n$ and $\left\{a_{1}, a_{2}, \ldots\right.$, $\left.a_{t}\right\} \subset V(G)$ an independent set. Then
(i) $\left|N\left(a_{i}\right) \cap N\left(a_{j}\right)\right| \leq 2, \quad 1 \leq i<j \leq t$,
(ii) $\sum_{i=1}^{t} d\left(a_{i}\right) \leq n+t^{2}-2 t$.

## Lemma J [6].

(i) Any triangle-free graph $H$ whose matching number $\nu(H)$ and vertex covering number $\tau(H)$ satisfy $\nu(H)<\tau(H)$, contains an edge xy such that $d(x)+d(y) \leq \nu(H)+\tau(H)$.
(ii) Let $G$ be a closed claw-free graph. If $\alpha(G)<\theta(G)$, then $\delta(G) \leq \alpha(G)+$ $\theta(G)-2$.

Lemma K [6]. Let $G$ be a closed claw-free graph. Then $\theta(G) \leq 2 \alpha(G)$.

Lemma L [6]. Let $G$ be a closed claw-free graph of order $n$ and connectivity $\kappa(G)$ such that $1 \leq \kappa(G)<\alpha(G)$ and let $A=\left\{a_{1}, \ldots, a_{\alpha}\right\}$ be a maximum independent set in $G$. Then

$$
\sum_{i=1}^{\alpha} d\left(a_{i}\right) \leq n+\alpha^{2}-4 \alpha+2+\kappa(G)
$$

Proof of Theorem 1. If $G$ is a counterexample to Theorem 1 such that $G$ satisfies the assumptions but $\kappa<\alpha(\operatorname{cl}(G))$ and $\theta(\operatorname{cl}(G)) \geq k$, then so is the closure $\operatorname{cl}(G)$. Hence we can suppose that $G$ is closed.

If $\alpha(G) \geq k+1$, then by Lemma I we have $\sigma_{k+1}(G) \leq n+(k+1)^{2}-$ $2(k+1)=n+k^{2}-1$, implying $\sigma_{k}(G) \leq \frac{k}{k+1}\left(n+k^{2}-1\right) \leq n+k^{2}-4 k+2+\kappa$ for $n \geq 3 k^{2}+k-(k+1) \kappa-2$, a contradiction. Hence $\alpha(G) \leq k$.

If $\alpha(G) \leq k-1$, then $\alpha(G)<\theta(G)$ and, by Lemma J and Lemma K, $\delta(G) \leq \alpha(G)+\theta(G)-2 \leq(k-1)+2(k-1)-2=3 k-5$, a contradiction.

Hence we have $\alpha(G)=k$. Since $\kappa(G)<\alpha(G)$, Lemma L gives $\sigma_{k}(G) \leq$ $n+k^{2}-4 k+\kappa+2$, a contradiction.
From Theorem 1 we obtain the following minimum degree result.
Corollary 2. Let $k \geq 2$ be an integer, let $G$ be a claw-free graph of order $n$ and let $\kappa=\kappa(\mathrm{cl}(G))$. Suppose that $G$ is such that $n \geq 3 k^{2}-k-\kappa-2$ and

$$
\delta(G)>\frac{n+k^{2}-4 k+2+\kappa}{k} .
$$

Then either $\theta(\operatorname{cl}(G)) \leq k-1$, or $\alpha(\operatorname{cl}(G)) \leq \kappa$.
Proof. We can again suppose that $G$ is closed. If $n \geq 3 k^{2}-k-\kappa-2$, then obviously $\delta(G)>\frac{n+k^{2}-4 k+2+\kappa}{k} \geq 3 k-5$ and hence $\delta(G) \geq 3 k-4$. The rest of the proof follows immediately from Theorem 1.
Now we can prove our first main result that gives a degree condition for the existence of a 2 -factor with limited number of components.

Theorem 3. Let $k \geq 2$ be an integer, let $G$ be a claw-free graph of order $n$ and let $\kappa=\kappa(\mathrm{cl}(G))$. Suppose that $G$ is such that $n \geq 3 k^{2}+k-(k+1) \kappa-2$, $\delta(G) \geq 3 k-4$ and

$$
\sigma_{k}(G)>n+k^{2}-4 k+2+\kappa .
$$

Then $G$ has a 2 -factor with at most $k-\kappa$ components.
Proof. If $G$ satisfies the assumptions of the theorem but has no 2-factor with at most $k-\kappa$ components, then, since $\delta(G) \leq \delta(\mathrm{cl}(G))$ and by Theorem A(iv), so does its closure $\operatorname{cl}(G)$. Since $\operatorname{cl}(G)$ is nonhamiltonian, by the well-known theorem of Chvátal and Erdős (see [2]), $\alpha(\operatorname{cl}(G))>\kappa(\operatorname{cl}(G))$. By Theorem 1, we have $\theta(\operatorname{cl}(G)) \leq k-1$.

Let $\mathcal{P}=\left\{K_{1}, \ldots, K_{\theta}\right\}$ be a minimum clique covering of $\operatorname{cl}(G)$ such that each of the cliques $K_{1}, \ldots, K_{\theta}$ is maximal. We show that each clique of $\mathcal{P}$ has at least $2 k-1$ vertices. This follows immediately from $\delta(\operatorname{cl}(G)) \geq 3 k-4$ for those $K_{i}$ 's that contain at least one simplicial vertex. Thus, suppose that (say) $K_{1}$ contains no simplicial vertex. By the minimality of $\mathcal{P}$, there is a clique $K^{\prime}$ with $K_{1} \cap K^{\prime} \neq \emptyset$ and $K^{\prime} \notin \mathcal{P}$ (otherwise $\mathcal{P} \backslash\left\{K_{1}\right\}$ is also a clique covering of $\operatorname{cl}(G))$. Since clearly every clique in $\operatorname{cl}(G)$ that contains a simplicial vertex must be in $\mathcal{P}, K^{\prime}$ has no simplicial vertex. By the structure of the closure, $\left|K_{1} \cap K^{\prime}\right|=1$.

Denote $K_{1} \cap K^{\prime}=\{x\},\left|K_{1}\right|=t$ and $\left|K^{\prime}\right|=r$. Then we have $d(x)=$ $t-1+r-1 \geq \delta(\operatorname{cl}(G)) \geq 3 k-4$, implying $t+r \geq 3 k-2$. Since $K^{\prime} \notin \mathcal{P}$, there are $r-1$ further cliques $K_{i_{1}}, \ldots, K_{i_{r-1}} \in \mathcal{P}$ having a common vertex with $K^{\prime}$. By the structure of the closure, $K_{i_{j}} \neq K_{i \ell}, j \neq \ell, j, \ell=1, \ldots, r-1$, implying $\theta \geq r$. Since $\theta \leq k-1$, we have $3 k-2 \leq t+r \leq t+k-1$, from which $t \geq 2 k-1$.

Now, since $\theta \leq k-1$, each clique of $\mathcal{P}$ contains at least $2 k-1-(k-2)=$ $k+1$ vertices that are in no other clique of $\mathcal{P}$. Since $k \geq 2$, every $K_{i} \in \mathcal{P}$ contains a cycle $C_{i}$ that is vertex-disjoint from all other cliques of $\mathcal{P}$. Let $x_{i} \in K_{i}, i=1, \ldots, \theta$ be such that each $x_{i}$ is in no other clique of $\mathcal{P}$. Since $\kappa=\kappa(\operatorname{cl}(G))$, by a well-known theorem by Dirac [4], there is a cycle $C$ in $\operatorname{cl}(G)$ containing all the vertices $x_{1}, \ldots, x_{\kappa}$. Let $\mathcal{C}$ be the collection of those of the cycles $C_{\kappa+1}, \ldots, C_{\theta}$, which are vertex-disjoint with $C$. Then the collection of cycles $\{C\} \cup \mathcal{C}$ can be easily extended to a 2 -factor of $\operatorname{cl}(G)$ with at most $k-\kappa$ components. The result then follows by Theorem A(iv).

Corollary 4. Let $k \geq 4$ be an integer and $G$ be a connected claw-free graph of order $n \geq 3 k^{2}-3, \delta(G) \geq 3 k-4$ and

$$
\sigma_{k}(G)>n+k^{2}-4 k+7 .
$$

Then $G$ has a 2-factor with at most $k-1$ components.

Proof. We can again suppose that $G$ is closed. If $\kappa(\operatorname{cl}(G)) \geq 6$, then $G$ has a required 2 -factor since $G$ is hamiltonian by Theorem B (note that $\delta(G) \geq 3 k-4 \geq 8)$. Hence suppose $1 \leq \kappa(\operatorname{cl}(G)) \leq 5$. Then we have $n \geq 3 k^{2}-3 \geq 3 k^{2}-(k+1) \kappa+2$ since $\kappa \geq 1$ and $\sigma_{k}(G)>n+k^{2}-4 k+7 \geq$ $n+k^{2}-4 k+\kappa+2$ since $\kappa \leq 5$. Then $G$ has a 2 -factor with at most $k-1$ components by Theorem 3 .

Remark. It would be possible to formulate minimum degree results corresponding to Theorem 3 and Corollary 4. Details are left to the reader.

Next we turn our attention to traceability. Let $\mathcal{C}_{i}, i=1, \ldots, 8$, be the class of all spanning subgraphs of the graphs $G_{i}, i=1, \ldots, 8$, shown in Figure 1 (where the circular and elliptical parts represent cliques of arbitrary order). Using a technique similar to that of [6], we can prove the following result.

Theorem 5. Let $G$ be a connected closed claw-free graph with clique covering number $\theta \leq 5$. Then either $G \in \cup_{i=1}^{8} \mathcal{C}_{i}$, or $G$ is traceable.

Proof of Theorem 5 is lengthy and it is therefore postponed to Section 3.

$\mathcal{C}_{i}$ is the class of all spanning subgraphs of $G_{i}, i=1, \ldots, 8$.

Figure 1
Combining Theorem 5 and Theorem 1, we can now obtain the following theorem, which is the second main result of this paper.

Theorem 6. Let $G$ be a connected claw-free graph of order $n \geq 112$ $7 \kappa(\operatorname{cl}(G))$ such that $\delta(G) \geq 14$ and

$$
\sigma_{6}(G)>n+14+\kappa(\operatorname{cl}(G)) .
$$

Then either $\operatorname{cl}(G) \in \cup_{i=1}^{8} \mathcal{C}_{i}$, or $G$ is traceable.

Proof. If $G$ is a nontraceable graph satisfying the asumptions of the theorem, then clearly so is $\operatorname{cl}(G)$. Thus, suppose that $G$ is closed. By a wellknown consequence of a theorem by Chvátal and Erdős [2] (see e.g. [7], Part I, Corollary 4.17), nontraceability of $G$ implies $\alpha(G)>\kappa+1$. From Theorem 1 (for $k=6$ ) we then obtain $\theta(G) \leq 5$. The rest of the proof follows from Theorem 5 .

Corollary 7. Let $G$ be a connected claw-free graph of order $n \geq 105$ such that $\delta(G) \geq 14$ and

$$
\sigma_{6}(G)>n+19
$$

Then either $\operatorname{cl}(G) \in \cup_{i=1}^{8} \mathcal{C}_{i}$, or $G$ is traceable.
Proof. We can again suppose that $G$ is closed. If $G$ is nontraceable, then Theorem B implies $1 \leq \kappa(G) \leq 5$. Rest of the proof follows immediately from Theorem 6.

Corollary 8. Let $G$ be a connected claw-free graph of order $n \geq 105$ with minimum degree

$$
\delta(G)>\frac{n+19}{6} .
$$

Then either $\operatorname{cl}(G) \in \cup_{i=1}^{8} \mathcal{C}_{i}$, or $G$ is traceable.

## 3. Proof of Theorem 5

We basically follow the terminology and notation introduced in [6] and [9]. Let $\mathcal{G}_{\theta}$ be the class of all connected nontraceable closed claw-free graphs with clique covering number $\theta$. By Theorem A, every $G \in \mathcal{G}_{\theta}$ is the line graph of some (unique) triangle-free graph $H$. Let $D_{1}(H)$ be the set of all degree 1 vertices of $H$ and put $H^{\prime}=H-D_{1}(H)$. Set $\mathcal{H}_{\theta}=\left\{L^{-1}(G) \mid G \in \mathcal{G}_{\theta}\right\}$ and $\mathcal{H}_{\theta}^{\prime}=\left\{H-D_{1}(H) \mid H \in \mathcal{H}_{\theta}\right\}$.

In every $G \in \mathcal{G}_{\theta}$ choose a fixed minimum clique covering $\mathcal{P}_{G}=\left\{B_{1}, \ldots\right.$, $\left.B_{\theta}\right\}$ of $G$ such that each clique $B_{i}$ is maximal. Since $\mathcal{P}_{G}$ is minimum, every $B_{i}$ contains at least one proper vertex, i.e., a vertex belonging to no other clique of $\mathcal{P}_{G}$. The centers $B_{1}, \ldots, B_{\theta}$ of the stars of $H=L^{-1}(G)$ that correspond to the cliques of $G$ will be called the black vertices of $H$. The other vertices of $H$ are called white. The set of black (white) vertices of $H$ is denoted by $B(H)(W(H))$, respectively. Since $B(H)$ is a vertex covering
of $H$ (i.e., every edge of $H$ has at least one vertex in $B(H)$ ), the set $W(H)$ is independent.

It is easy to see that for any $G \in \mathcal{G}_{\theta}$, any graph obtained from $G$ by adding/removing simplicial vertices to/from cliques of $\mathcal{P}_{G}$ also belongs to $\mathcal{G}_{\theta}$ as long as (in the case of removal) at least one simplicial vertex in the clique remains (while the removal of the last simplicial vertex of a clique can turn $G$ into a traceable graph). Hence we can without loss of generality denote for any $H^{\prime} \in \mathcal{H}_{\theta}^{\prime}$ by $L(H)$ the graph obtained from the line graph of $H^{\prime}$ by adding one simplicial vertex to every clique corresponding to a black vertex of $H^{\prime}$.

Let $G_{1}, G_{2} \in \mathcal{G}_{\theta}$. We say that $G_{1}$ is an ss-subgraph of $G_{2}$, if $G_{1}$ is isomorphic to a spanning subgraph of a graph, which is obtained from $G_{2}$ by adding an appropriate number of simplicial vertices to some cliques of $\mathcal{P}_{G_{2}}$, and that $G_{1}$ is a proper ss-subgraph of $G_{2}$ if $G_{1}$ is an ss-subgraph of $G_{2}$ and $G_{1}, G_{2}$ are nonisomorphic. In the following we present a method for finding a subset $\mathcal{F}_{\theta} \subset \mathcal{H}_{\theta}^{\prime}$ such that
(i) every $G \in \mathcal{G}_{\theta}$ is an $s s$-subgraph of $L(F)$ for some $F \in \mathcal{F}_{\theta}$,
(ii) for any $F_{1}, F_{2} \in \mathcal{F}_{\theta}, L\left(F_{1}\right)$ is not an $s s$-subgraph of $L\left(F_{2}\right)$.

By the previous observations, the class $\mathcal{G}_{\theta}$ is fully characterized by $\mathcal{F}_{\theta}$.
If, for some $H \in \mathcal{H}_{\theta}$, the corresponding $H^{\prime} \in \mathcal{H}_{\theta}^{\prime}$ has a black trail (abbreviated BT), i.e., a trail containing all black vertices of $H^{\prime}$, then clearly $H$ has a dominating trail. Since, by Theorem F , no $H \in \mathcal{H}_{\theta}$ has a dominating trail, no $H^{\prime} \in \mathcal{H}_{\theta}^{\prime}$ has a BT.

For a trail $T$ in $H^{\prime} \in \mathcal{H}_{\theta}^{\prime}$ we denote by bla $(T)$ the black length of $T$, i.e., the number of black vertices of $H^{\prime}$ that are on $T$, and by $\operatorname{cro}(T)$ the number of "crossings" of $T$, i.e., the number of vertices of $H^{\prime}$ that are visited by $T$ at least twice.

Two vertices of $H^{\prime}$ are said to be related if they are adjacent or if they are both black and have a common white neighbor. If $T$ is a (fixed) trail in $H^{\prime}$ and $x, y$ are vertices of $H^{\prime}$, then we say that $x, y$ are $\bar{T}$-related (denoted $x \sim y)$ if $x y \in E\left(H^{\prime}\right) \backslash E(T)$ or $x$ and $y$ have a white common neighbor outside $T$.

Let now $H^{\prime} \in \mathcal{H}_{\theta}^{\prime}$, and let $T$ be a trail in $H^{\prime}$ such that
(i) $\mathrm{bla}(T)$ is maximum,
(ii) subject to $(i), \operatorname{cro}(T)$ is minimum,
(iii) subject to $(i)$ and $(i i)$, the length of $T$ is minimum.

Then $T$ has two black vertices of degree 1 . We will always denote by $b_{1}, \ldots, b_{k}$ the black vertices of $T$ labelled along $T$, and by $w_{i}$ the white successor of $b_{i}$ on $T$, if it exists. Note that, since $T$ is a trail, possibly $b_{i}=b_{j}$ or $w_{i}=w_{j}$ for some $i \neq j$. If $b_{i} \sim b_{j}$, then the (possible) white common neighbor of $b_{i}, b_{j}$ outside $T$ will be denoted by $w_{i j}$.

Case $\theta=3$.
Let $B=\left\{b_{1}, b_{2}, b_{3}\right\}$. Then, clearly, $\operatorname{bla}(T)=2, \operatorname{cro}(T)=0$ and $T=$ $b_{1}\left(w_{1}\right) b_{2}$. Since $H^{\prime}$ is connected and the set $\left\{b_{1}, b_{2}, b_{3}\right\}$ is dominating, $b_{3}$ is $\bar{T}$-related to some vertex of $T$. Clearly both $b_{3} \sim b_{1}$ and $b_{3} \sim b_{2}$ imply traceability of $L\left(H^{\prime}\right)$, hence $b_{3} \sim w_{1}$, implying $b_{3} w_{1} \in E\left(H^{\prime}\right)$. The existence of any further relation implies traceability of $L\left(H^{\prime}\right)$, hence $V\left(H^{\prime}\right)=$ $\left\{b_{1}, b_{2}, b_{3}, w_{1}\right\}$ and $E\left(H^{\prime}\right)=\left\{w_{1} b_{1}, w_{1} b_{2}, w_{1} b_{3}\right\}$, implying $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$.

Case $\theta=4$.
Let $B=\left\{b_{1}, b_{2}, b_{3}, b_{4}\right\}$. Then obviously $2 \leq \operatorname{bla}(T) \leq 3$ and $\operatorname{cro}(T)=0$, i.e., $T$ is a path. We have two subcases.

Subcase $\operatorname{bla}(T)=3$.
Then $T=b_{1}\left(w_{1}\right) b_{2}\left(w_{2}\right) b_{3}$. Suppose first that $b_{4}$ is $\bar{T}$-related to some black vertex. Then necessarily $b_{4} \sim b_{2}$. If $b_{4} \sim x$ for some $x \in\left\{b_{1},\left(w_{1}\right), b_{3},\left(w_{2}\right)\right\}$, then we immediately have a trail $T^{\prime}$ with $\operatorname{bla}\left(T^{\prime}\right)=4$. Similarly, $b_{1} \sim b_{2}$ yields $T^{\prime}=b_{3}\left(w_{2}\right) b_{2}\left(w_{1}\right) b_{1}\left(w_{12}\right) b_{2}\left(w_{24}\right) b_{4}, b_{1} \sim w_{2}$ gives $T^{\prime}=b_{3} w_{2} b_{1}\left(w_{1}\right)$ $b_{2}\left(w_{24}\right) b_{4}$, and for $b_{1} \sim b_{3}$ we have $T^{\prime}=b_{3}\left(w_{13}\right) b_{1}\left(w_{1}\right) b_{2}\left(w_{24}\right) b_{4}$. By symmetry and since $H^{\prime}$ is triangle-free, these are all possibilities. Hence there are no further $\bar{T}$-relations, implying $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$.

Hence $b_{4}$ is related to white vertices only. If both $b_{4} \sim w_{1}$ and $b_{4} \sim w_{2}$, then $H^{\prime}$ contains no more relations, implying $L\left(H^{\prime}\right) \in \mathcal{C}_{2}$. If (by symmetry) $b_{4} \sim w_{1}$ and $b_{4} \nsim w_{2}$, then the only possible additional relation that does not create a trail $T^{\prime}$ with $\operatorname{bla}(T)=4$ is $b_{1} \sim w_{2}$. Then for $b_{1} \nsim w_{2}$ we have $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$ and for $b_{1} \sim w_{2}$ we have $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$.

Subcase bla $(T)=2$.
Then $T=b_{1}\left(w_{1}\right) b_{2}$. Then immediately $b_{3} \sim w_{1}$ and $b_{4} \sim w_{1}$. If $b_{3} \sim b_{4}$, then $L\left(H^{\prime}\right)$ is traceable; hence $b_{3} \nsim b_{4}$, implying $L\left(H^{\prime}\right) \in \mathcal{C}_{2}$.

Case $\theta=5$.
Let $B=\left\{b_{1}, b_{2}, b_{3}, b_{4}, b_{5}\right\}$. We have obviously $2 \leq \operatorname{bla}(T) \leq 4$. If $2 \leq$ $\operatorname{bla}(T) \leq 3$, then $\operatorname{cro}(T)=0$ (since $H^{\prime}$ is triangle-free), for $\operatorname{bla}(T)=4$
we have $0 \leq \operatorname{cro}(T) \leq 1$. We will denote these subcases by $k / \ell$, where $k=\operatorname{bla}(T)$ and $\ell=\operatorname{cro}(T)$. Thus, we have subcases $4 / 0,4 / 1,3 / 0$ and $2 / 0$. The subcase $4 / 1$ splits into two subcases $4 / 1 \mathrm{w}$ and $4 / 1 \mathrm{~b}$ according to whether the vertex visited twice by $T$ is white or black, respectively. We consider these subcases separately.

Subcase 4/0.
Then $T=b_{1}\left(w_{1}\right) b_{2}\left(w_{2}\right) b_{3}\left(w_{3}\right) b_{4}$ is a path. It is straightforward to check that $b_{5}$ can be $\bar{T}$-related to at most one black vetex of $T$ (for otherwise $L\left(H^{\prime}\right)$ is traceable). Thus, we have two possibilities.

Subcase 4/0-1. $b_{5}$ is $\bar{T}$-related to exactly one black vertex of $T$. By symmetry, we can suppose $b_{5} \sim b_{2}$. We distinguish two subcases.

Subcase 4/0-1-1. $b_{5}$ is $\bar{T}$-related to some white vertex on $T$.
Then the only possibility that does not imply $L\left(H^{\prime}\right)$ is traceable is $b_{5} \sim w_{3}$. Then it is straightforward to check that any further $\bar{T}$-relation between vertices of $T$ implies $L\left(H^{\prime}\right)$ is traceable, but then $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$.

Subcase 4/0-1-2. $b_{2}$ is the only $\bar{T}$-relation of $b_{5}$ on $T$. We consider possible $\bar{T}$-relations between vertices of $T$.

If $b_{1} \sim w_{3}$, then we are in a situation symmetric to the subcase 4/0-1-1 and hence $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$. All the other relations of $b_{1}$ on $T$ imply $L\left(H^{\prime}\right)$ is traceable. Hence we can assume $b_{1}$ has no $\bar{T}$-relation on $T$. Now, if also $b_{4}$ has no $\bar{T}$-relation on $T$, then we have $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$. Hence we can suppose $b_{4} \sim x$ for some $x \in V(T)$. If $x \in\left\{b_{1}, w_{1}, b_{2}\right\}$, then $L\left(H^{\prime}\right)$ is traceable. Hence $x \in\left\{w_{2}, b_{3}\right\}$. Now, if there is no $\bar{T}$-relation $y \sim z$ for any $y \in\left\{w_{1}, b_{2}\right\}$, $z \in\left\{w_{2}, b_{3}, w_{3}, b_{4}\right\}$, then we have $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$. It is straightforward to check that all such relations $y \sim z$ imply $L\left(H^{\prime}\right)$ is traceable.

Subcase 4/0-2. $b_{5}$ is $\bar{T}$-related only to white vertices on $T$. In this subcase, we have three further possibilities.

Subcase 4/0-2-1. $b_{5}$ is $\bar{T}$-related to $w_{1}, w_{2}$ and $w_{3}$. Then there is no further $\bar{T}$-relation on $T$ and $L\left(H^{\prime}\right) \in \mathcal{C}_{6}$.

Subcase 4/0-2-2. $b_{5}$ is $\bar{T}$-related to two white vertices on $T$.
By symmetry, we can suppose that $b_{5} \sim w_{1}$ and either $b_{5} \sim w_{2}$ or $b_{5} \sim w_{3}$.
Let first $b_{5} \sim w_{2}$. If no vertices on $T$ are $\bar{T}$-related, then $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$ (with $b_{3},\left(w_{3}\right), b_{4}$ in one clique of $L\left(H^{\prime}\right)$ ). Hence suppose there is a $\bar{T}$-relation between some vertices of $T$. Clearly $b_{1} \nsim b_{2}, b_{1} \nsim w_{2}, b_{1} \nsim b_{3}, b_{1} \nsim b_{4}, w_{1} \nsim b_{3}$, $w_{1} \nsim b_{4}, b_{2} \nsim b_{3}, b_{2} \nsim b_{4}$ and $w_{2} \nsim b_{4}$, since any of these relations implies $L\left(H^{\prime}\right)$ is traceable. It remains to consider the possibilities $b_{1} \sim w_{3}, b_{2} \sim w_{3}$ and $b_{3} \sim b_{4}$.

If $b_{1} \sim w_{3}$, then both $b_{2} \nsim w_{3}$ and $b_{3} \nsim b_{4}$ (otherwise $L\left(H^{\prime}\right)$ is traceable), and then $L\left(H^{\prime}\right) \in \mathcal{C}_{5}$; if $b_{2} \sim w_{3}$, then $b_{1} \nsim w_{3}$ and $b_{3} \nsim b_{4}$, implying $L\left(H^{\prime}\right) \in$ $\mathcal{C}_{6}$; and if $b_{3} \sim b_{4}$, then similarly $b_{1} \nsim w_{3}, b_{2} \nsim w_{3}$ and $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$ (in which $b_{3}, b_{4}$ and their common neighbors are in one clique).

Hence suppose $b_{5} \sim w_{3}$. Similarly as before, no $\bar{T}$-relation between vertices of $T$ implies $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$ with $b_{2},\left(w_{2}\right), b_{3}$ in one clique. Thus, suppose some vertices of $T$ are $\bar{T}$-related. Immediately $b_{1} \nsim b_{2}, b_{1} \nsim b_{3}, b_{1} \nsim w_{3}, b_{1} \nsim b_{4}$ and $w_{1} \nsucc b_{3}$, since any of these relations implies $L\left(H^{\prime}\right)$ is traceable. By symmetry, it remains to consider the possibilities $b_{1} \sim w_{2}$ and $b_{2} \sim b_{3}$. If $b_{1} \sim w_{2}$, then $b_{2} \nsim b_{3}$ (otherwise $L\left(H^{\prime}\right)$ is traceable), implying $L\left(H^{\prime}\right) \in \mathcal{C}_{5}$; if $b_{2} \sim b_{3}$, then similarly $b_{1} \nsim w_{2}$ and $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$ (with $b_{2}, b_{3}$ and their common neighbors in one clique).

Subcase 4/0-2-3. $b_{5}$ is $\bar{T}$-related to exactly one white vertex on $T$.
By symmetry, either $b_{5} \sim w_{1}$ or $b_{5} \sim w_{2}$.
Let first $b_{5} \sim w_{1}$. If $b_{1}$ is not $\bar{T}$-related to any of $b_{2}, w_{2}, b_{3}, w_{3}, b_{4}$, then $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$ (with $b_{2}, b_{3}$ and $b_{4}$ in one clique). The relations $b_{1} \sim b_{2}$ and $b_{1} \sim b_{4}$ immediately imply traceability. Hence $b_{1}$ is $\bar{T}$-related to $w_{2}, b_{3}$ or $w_{3}$.

If $b_{1} \sim b_{3}$ and, at the same time, $b_{1} \sim w_{2}$ or $b_{1} \sim w_{3}$, then $L\left(H^{\prime}\right)$ is traceable, and if $b_{1} \sim w_{2}$ and $b_{1} \sim w_{3}$, then we are in Subcase 4/0-2-1 (where $b_{1}$ plays the role of $b_{5}$ ). Hence $b_{1}$ is $\bar{T}$-related to exactly one of $b_{3}$, $w_{2}, w_{3}$.

If $b_{1} \sim b_{3}$, then any additional relation implies $L\left(H^{\prime}\right)$ is traceable, and hence we have $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$.

If $b_{1} \sim w_{2}$, then for $b_{2} \sim w_{3}$ we are in Subcase 4/0-2-1 (where $b_{2}$ plays the role of $b_{5}$ ) and $L\left(H^{\prime}\right) \in \mathcal{C}_{6}$. Any other additional relation except $b_{3} \sim b_{4}$
implies $L\left(H^{\prime}\right)$ is traceable. If $b_{3} \sim b_{4}$, or if there is no additional relation, we have $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$ (with $b_{3}, b_{4}$ in one clique).

If $b_{1} \sim w_{3}$, then any additional relation except for $b_{2} \sim b_{3}$ or $w_{2} \sim b_{4}$ implies $L\left(H^{\prime}\right)$ is traceable. For $w_{2} \sim b_{4}$ we have $L\left(H^{\prime}\right) \in \mathcal{C}_{5}$, and if $b_{2} \sim b_{3}$ or if there is no additional relation, then $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$ (with $b_{2}$ and $b_{3}$ in one clique).

Let now $b_{5} \sim w_{2}$. First observe that there is no $\bar{T}$-relation containing $w_{2}$ since $H^{\prime}$ is triangle-free and both $w_{2} \sim b_{1}$ and $w_{2} \sim b_{4}$ imply traceability. Secondly, if there is no $\bar{T}$-relation $x \sim y$ with $x \in\left\{b_{1}, w_{1}, b_{2}\right\}$ and $y \in$ $\left\{b_{3}, w_{3}, b_{4}\right\}$, then $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$. Since $b_{1} \sim b_{3}$ and $b_{1} \sim b_{4}$ imply traceability, by symmetry, we have $b_{2} \sim b_{3}, w_{1} \sim b_{3}$ or $b_{1} \sim w_{3}$. We consider these possibilities separately.

If $b_{2} \sim b_{3}$, then there is no additional $\bar{T}$-relation containing $b_{1}$ (or symmetrically $b_{4}$ ), for otherwise $L\left(H^{\prime}\right)$ is traceable. This implies $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$.

If $w_{1} \sim b_{3}$, then similarly $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$, unless there is an additional $\bar{T}$-relation containing $b_{1}$ or $b_{4}$. The only such relations that do not imply traceability are $b_{1} \sim w_{3}$ or $b_{3} \sim b_{4}$, but then $L\left(H^{\prime}\right) \in \mathcal{C}_{6}$ or $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$, respectively.

Finally, if $b_{1} \sim w_{3}$, then we already know there is no further relation, and we have $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$.

Subcase 4/1w.
Recall that in this subcase $T$ visits twice one white vertex. Choose the notation such that $T=b_{1} w_{1} b_{2} w_{2} b_{3} w_{1} b_{4}$. Clearly, $b_{5} \nsim b_{1}, b_{5} \nsim b_{4}$ and $b_{5}$ cannot be $\bar{T}$-related to both $b_{2}, b_{3}$ (since in each of these cases $L\left(H^{\prime}\right)$ is traceable). Thus, $b_{5}$ is $\bar{T}$-related to at most one black vertex on $T$. There are two subcases.

Subcase 4/1w-1. $b_{5}$ is $\bar{T}$-related to one black vertex on $T$.
By symmetry, let $b_{5} \sim b_{2}$. Then $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$ (with $w_{1}, b_{2}, w_{2}, b_{3}$ in one clique), since any additional $\bar{T}$-relation involving any of $b_{1}, b_{4}, b_{5}$ implies $L\left(H^{\prime}\right)$ is traceable.

Subcase 4/1w-2. $b_{5}$ is $\bar{T}$-related only to white vertices.
In this subcase, we distinguish three further possibilities.

Subcase 4/1w-2-1. $b_{5} \sim w_{1}, b_{5} \sim w_{2}$.
Then there is no other relation and $L\left(H^{\prime}\right) \in \mathcal{C}_{7}$.

Subcase $4 / 1 \mathrm{w}-2-2 . b_{5} \sim w_{1}, b_{5} \nsim w_{2}$.
If there is no other relation involving any of $b_{1}, b_{2}$, then we have $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$ (with $b_{2}, b_{3}$ and their common neighbors in one clique). It is straightforward to check that any further $\bar{T}$-relation involving $b_{1}$ or $b_{4}$ gives $L\left(H^{\prime}\right) \in \mathcal{C}_{8}$ (if some of $b_{1}, b_{2}$ is $\bar{T}$-related to $w_{2}$ ), or traceability of $L\left(H^{\prime}\right)$.

Subcase 4/1w-2-3. $b_{5} \sim w_{2}, b_{5} \nsim w_{1}$.
Then $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$ (with $w_{1}, b_{2}, w_{2}, b_{3}$ in one clique) and any $\bar{T}$-relation between any of $b_{2}, w_{2}, b_{3}$ and the rest implies traceability of $L\left(H^{\prime}\right)$.

Subcase 4/1b.
Choose the notation such that the vertex $b_{2}$ is visited twice by $T$, i.e., $T=$ $b_{1}\left(w_{1}\right) b_{2} w_{2} b_{3} w_{3} b_{2}\left(w_{4}\right) b_{4}$. Similarly as before, $b_{5}$ is $\bar{T}$-related to at most one black vertex on $T$, and neither to $b_{1}$ nor to $b_{4}$. We distinguish three subcases.

Subcase 4/1b-1. $b_{5} \sim b_{2}, b_{5} \nsucc b_{3}$.
In this case $b_{2}$ is the only $\bar{T}$-relation of $b_{5}$ on $T$ (since any other relation implies traceability). Now $L\left(H^{\prime}\right) \in \mathcal{C}_{2}$ and any other $\bar{T}$-relation between vertices of $T$ gives $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$ or traceability. We distinguish three possibilities.

Subcase 4/1b-2. $b_{5} \sim b_{3}, b_{5} \nsim b_{2}$.
In this subcase immediately $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$ with $\left\{b_{2}, w_{2}, b_{3}, w_{3}\right\}$ in one clique and any relation joining a vertex from this set to the rest gives traceability.

Subcase 4/1b-3. $b_{5}$ is $\bar{T}$-related only to white vertices.
Then $b_{5}$ can have $\bar{T}$-relations in at most one of the sets $\left\{w_{1}, w_{4}\right\},\left\{w_{2}, w_{3}\right\}$ (otherwise $L\left(H^{\prime}\right)$ is traceable). We have two further possibilities.

Subcase 4/1b-3-1. $b_{5} \sim w_{1}$.
For $b_{5} \nsim w_{4}$ we have $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$, and for $b_{5} \sim w_{4}$ we have $L\left(H^{\prime}\right) \in \mathcal{C}_{4}$ (with $b_{2}, w_{2}, b_{3}, w_{3}$ in one clique).

Subcase 4/1b-3-2. $b_{5} \sim w_{2}$.
If $b_{5} \nsim w_{3}$, then $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$ (with $b_{2}, w_{2}, b_{3}, w_{3}$ in one clique), and if $b_{5} \sim w_{3}$, then $L\left(H^{\prime}\right) \in \mathcal{C}_{8}$.

Subcase 3/0.
Let $T=b_{1}\left(w_{1}\right) b_{2}\left(w_{2}\right) b_{3}$.

Subcase 3/0-1. $b_{4} \sim b_{5}$.
If $b_{4}$ or $b_{5}$ is $\bar{T}$-related to any vertex on $T$, then we have a path $T^{\prime}$ with $\operatorname{bla}\left(T^{\prime}\right) \geq 4$, except for the case if $w_{45}=w_{24}=w_{25}$. In this case, $L\left(H^{\prime}\right) \in \mathcal{C}_{2}$.

Subcase 3/0-2. $b_{4} \nsim b_{5}$.
If $b_{4}$ has two relations on $T$, then the only possibility that does not create a trail $T^{\prime}$ with bla $\left(T^{\prime}\right) \geq 4$ is $b_{4} \sim w_{1}, b_{4} \sim w_{2}$, but then, for any $\bar{T}$-relation of $b_{5}$ on $T$ we again have a trail $T^{\prime}$ with $\operatorname{bla}\left(T^{\prime}\right) \geq 4$. Hence both $b_{4}$ and $b_{5}$ have one $\bar{T}$-relation on $T$. Then it is straightforward to check that in all nontraceable cases we have $L\left(H^{\prime}\right) \in \mathcal{C}_{2}$ or $L\left(H^{\prime}\right) \in \mathcal{C}_{1}$.

Subcase 2/0.
Let $T=b_{1}\left(w_{1}\right) b_{2}$. If any two of $b_{3}, b_{4}, b_{5}$ are $\bar{T}$-related, we have a trail $T^{\prime}$ with $\operatorname{bla}\left(T^{\prime}\right) \geq 3$. Hence $b_{3} \sim w_{1}, b_{4} \sim w_{1}, b_{5} \sim w_{1}$, implying $L\left(H^{\prime}\right) \in \mathcal{C}_{3}$.

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