

ON DISTANCE EDGE COLOURINGS OF A CYCLIC MULTIGRAPH

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We shall use the distance chromatic index defined by the present author in early nineties, cf. [5] or [4] of 1993. The *edge distance* of two edges in a multigraph M is defined to be their distance in the line graph $L(M)$ of M . Given a positive integer d , define the d^+ -chromatic index of the multigraph M , denoted by $q^{(d)}(M)$, to be equal to the chromatic number χ of the d th power of the line graph $L(M)$,

$$q^{(d)}(M) = \chi(L(M)^d).$$

Then the colour classes are matchings in M with edges at edge distance larger than d apart.

Call C to be a *cyclic multigraph* if C consists of a cycle on n vertices with possibly more than one edge between two consecutive vertices.

The following problem was presented in [6].

Problem. Given an integer $d \geq 2$ and a cyclic multigraph C , find (or estimate) $q^{(d)}(C)$, the d^+ -chromatic index of C .

In other words, generalize the following formula due to Berge [1] for the ordinary chromatic index ($q = q^1$)

$$q(C) = \begin{cases} \max \left\{ \Delta(C), \left\lceil \frac{e(C)}{\lfloor \frac{n}{2} \rfloor} \right\rceil \right\} & \text{for odd } n, \\ \Delta(C) & \text{for even } n, \end{cases}$$

where $\Delta(C)$ and $e(C)$ are the maximum degree among vertices and the size of C , respectively.

Remarks 1. 2^+ -chromatic index $q^{(2)}$ is known under the name *strong chromatic index*, estimations of $q^{(2)}(C)$ being studied in [2, 3].

2. In [5] it is proved that

$$q^{(d)}({}^pC_n) = \begin{cases} pn & \text{if } n \leq 2d + 1, \\ \left\lceil \frac{pn}{\lfloor \frac{n}{d+1} \rfloor} \right\rceil & \text{if } n \geq d + 1 \end{cases}$$

where pC_n is the cyclic multigraph C with all edge multiplicities equal to p .

3. Let M be a loopless multigraph whose underlying graph is a forest. Then $q^{(d)}(M)$, the d^+ -chromatic index of M , can be seen to be equal to the diameter- d cluster (or diameter- d edge-clique) number of M (i.e., the density of the d th power, $L(M)^d$, of the line graph of M). This extends the known corresponding results on a tree [5] and on $q^{(2)}(M)$ in [2].

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