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ROTATION AND JUMP DISTANCES BETWEEN GRAPHS

Gary Chartrand[1]

Department of Mathematics and Statistics
Western Michigan University, Kalamazoo, MI 49008, USA

Heather Gavlas

Smiths Industries, Defense Systems North America
Grand Rapids, MI 49518-3469, USA

Héctor Hevia

Escuela de Ingenieria Comercial, Universidad Adolfo Ibanez
Balmaceda 1625, Vina del Mar, CHILE

and

Mark A. Johnson

Pharmacia & Upjohn, 7247-267-133
301 Henrietta Street, Kalamazoo, MI 49007, USA

Abstract

A graph H is obtained from a graph G by an edge rotation if G contains three distinct vertices u, v, and w such that $uv \in E(G)$, $uw \notin E(G)$, and $H = G - uv + uw$. A graph H is obtained from a graph G by an edge jump if G contains four distinct vertices u, v, w, and x such that $uv \in E(G)$, $wx \notin E(G)$, and $H = G - uv + wx$. If a graph H is obtained from a graph G by a sequence of edge jumps, then G is said to be j-transformed into H. It is shown that for every two graphs G and H of the same order (at least 5) and same size, G can be j-transformed into H. For every two graphs G and H of the same order and same size, the jump distance $d_j(G, H)$ between G and H is defined as the minimum number of edge jumps required to j-transform G into H. The rotation distance $d_r(G, H)$ between two graphs G and H of the same order and same size is the minimum number of edge rotations needed to transform G into H. The jump

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and rotation distances of two graphs of the same order and same size are compared. For a set $\mathcal{S}$ of graphs of a fixed order at least 5 and fixed size, the jump distance graph $D_j(\mathcal{S})$ of $\mathcal{S}$ has $\mathcal{S}$ as its vertex set and where G_1 and G_2 in $\mathcal{S}$ are adjacent if and only if $d_j(G_1, G_2) = 1$. A graph G is a jump distance graph if there exists a set $\mathcal{S}$ of graphs of the same order and same size with $D_j(\mathcal{S}) = G$. Several graphs are shown to be jump distance graphs, including all complete graphs, trees, cycles, and cartesian products of jump distance graphs.

Keywords: edge rotation, rotation distance, edge jump, jump distance, jump distance graph.

1991 Mathematics Subject Classification: Primary: 05C12, Secondary: 05C75.

1. Introduction

Several distances between graphs involve the idea of transformations. In this paper, we consider two elementary transformations previously introduced, namely, edge moves and edge rotations, and a new transformation referred to as an edge jump. We indicate that two graphs G and H are isomorphic by writing $G = H$.

Let G and H be two graphs having the same order and the same size. In [2] H is said to be obtained from G by an edge move if G contains (not necessarily distinct) vertices u, v, w, and x such that $uv \in E(G)$, $wx \notin E(G)$, and $H = G - uv + wx$. A graph G is *m-transformed* into a graph H if H is (isomorphic to the graph) obtained from G by a sequence of edge moves, i.e., if there is a sequence $G = G_0, G_1, ..., G_n = H$ $(n \geq 0)$ of graphs such that G_{i+1} is obtained from G_i by an edge move for $i = 0, 1, ..., n-1$. In [4] H is said to be obtained from G by an edge rotation if G contains distinct vertices u, v, and w such that $uv \in E(G)$, $uw \notin E(G)$, and $H = G - uv + uw$. More generally, G is *r-transformed* into H if H is (isomorphic to the graph) obtained from G by a sequence of edge rotations.

We now say that a graph H is obtained from a graph G by an *edge jump* if G contains four distinct vertices u, v, w, and x such that $uv \in E(G)$, $wx \notin E(G)$, and $H = G - uv + wx$. If H is (isomorphic to the graph) obtained from G by a sequence of edge jumps, we say that G is *j-transformed* into H.

While an edge move is an unrestricted transfer of an edge uv of a graph G to an edge wx, where, previously, $wx \notin E(G)$, in both an edge rotation and an edge jump, there is a restricted edge transfer. In an edge move, the vertices u, v, w, and x may or may not be distinct. With an edge rotation, the vertices u, v, w, and x are not distinct; while in an edge jump, these vertices must be distinct. That is, each edge move is either an edge rotation or an edge jump, but not both.

For example, the graph H of Figure 1 is obtained from the graph G by an edge rotation, but H is not obtained from G by an edge jump. On the other hand, the graph H' is obtained from G by an edge jump (as well as by an r-transformation that is a sequence of two edge rotations). For all three graphs G, H, and H', each is obtained from either of the others by a single edge move.

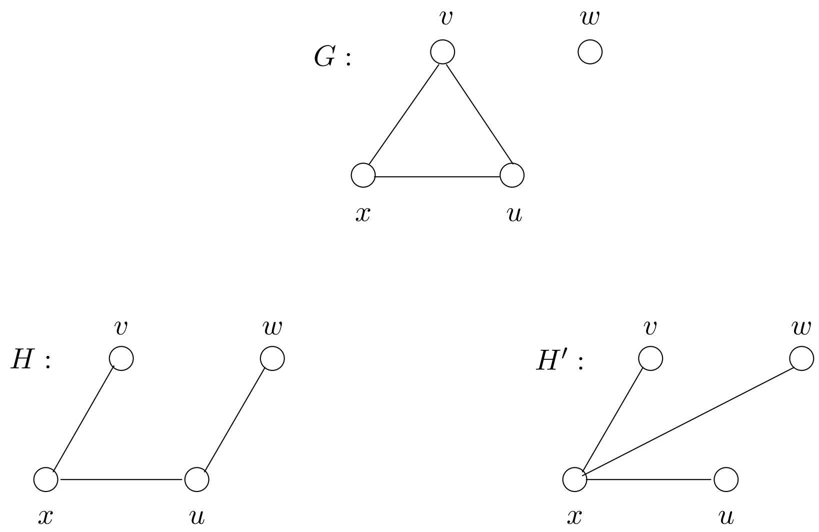


Figure 1

Our first result will have some interesting consequences.

Theorem 1. *Let G and H be graphs having the same order* (*at least* 5) *and the same size. If H is obtained from G by an edge move, then H is obtained from G by an edge jump or a sequence of two edge jumps.*

Proof. Since H is obtained from G by an edge move, G contains (not necessarily distinct) vertices u, v, w, and x such that $uv \in E(G)$, $wx \notin E(G)$, and $H = G - uv + wx$. If the vertices u, v, w, and x are distinct, then H is obtained from G by an edge jump. Suppose, then, that the vertices u, v, w, and x are not distinct. Without loss of generality, we may assume that $v = x$. Hence $uv \in E(G)$ and $vw \notin E(G)$. Since the order of G is at least 5, G contains two vertices y and z distinct from u, v, and w. Suppose first that $yz \notin E(G)$. In this case, define $F = G - uv + yz$ and $H = F - yz + vw$. Then

H is obtained from F by an edge jump and F is obtained from G by an edge jump. Next suppose that $yz \in E(G)$. Here, we define $F = G - yz + vw$ and $H = F - uv + yz$. Once again, H is obtained from F by an edge jump and F is obtained from G by an edge jump. Hence, in either case, H is obtained from G by a j-transformation that is a sequence of two edge jumps. ■

In [2] it was shown that for every two graphs G and H of the same order and same size, G can be m-transformed into H. This gives us an immediate corollary of Theorem 1.

Corollary 2. *If G and H are two graphs of the same order* (*at least* 5) *and the same size, then G can be j-transformed into H.*

In [2] the *move distance* $d_m(G, H)$ between two graphs G and H of the same order and same size is defined as the minimum number of edge moves required to m-transform G into H. The *rotation distance* $d_r(G, H)$ is defined analogously. Both distances are metrics on the space of all graphs of a fixed order and fixed size.

It is now natural to define the *jump distance* $d_j(G, H)$ between two graphs G and H of the same order (at least 5) and same size as the minimum number of edge jumps required to j-transform G into H. By Corollary 2, this distance is well-defined. It is also straightforward to see that this distance too is a metric on the space of all graphs of fixed order at least 5 and fixed size. For the graphs F, G, and H of Figure 2, $d_j(F, G) = d_j(F, H) = 1$ and $d_j(G, H) = 2$.

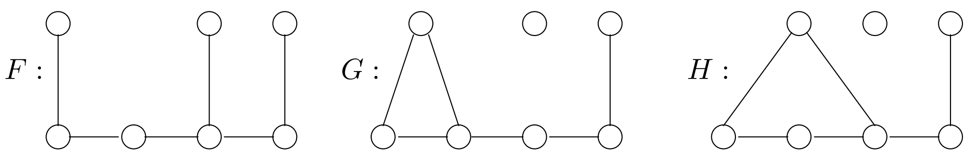


Figure 2

The Dugundji-Ugi principle of minimum chemical distance provides an interesting illustration of a possible application of the distinction between rotation and jump distances. In this formalism, the reactants and products of a chemical reaction are represented as graphs with the possible inclusion of loops and multiple edges [7, 9]. Two distances are involved. The first, the "experimental distance", is the sum over the individual steps of the reaction of the number of valence electrons that participate in each step. The second, called the chemical distance, can be shown to correspond to the move distance between the graph of the reactants and the graph of the products. The principle asserts the equality of the experimental and chemical distances. Counterexamples are known to exist, however. Such discrepancies are often attributed to the formation of a bond in a reaction intermediate that is subsequently cleaved in the course of the reaction [7].

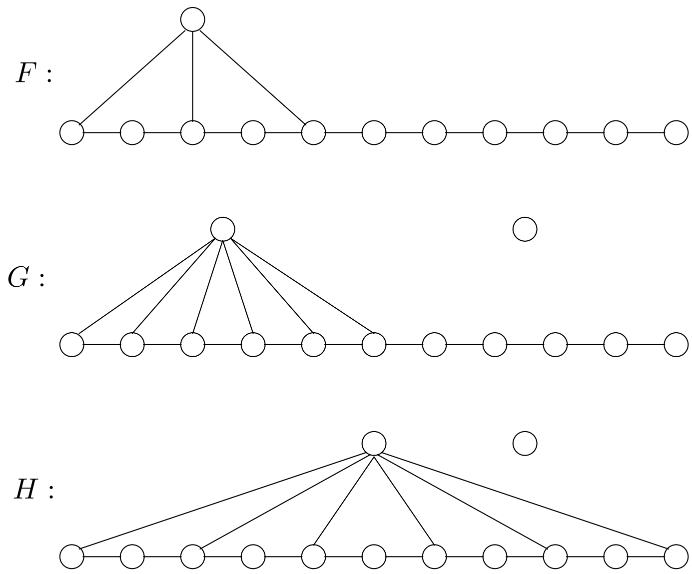
Another way for generating such a discrepancy emerges if it can be assumed that the basic mechanisms of chemical reactions are appropriately represented by the arrows used to describe electron flow in chemical reaction diagrams. Each of these arrows typically represents the equivalent of an edge rotation. It follows that whenever the rotation distance between the reactants and products exceeds the move distance, then the experimental distance must exceed the chemical distance. It may be that some of the counterexamples to the principle of minimum chemical distance would cease to exist if the principle of minimum chemical distance were reformulated in terms of rotation distance. One such possibility is explored by Kvasnička and Pospíchal [2]. An extension of the move distance to all graphs is given in Johnson [8] along with some applications of the metric to medicinal chemistry.

A greatest common subgraph of two graphs G and H is a graph of maximum size without isolated vertices that is a common subgraph of G and H. For graphs G and H of the same order and same size q having a greatest common subgraph of size s, it was shown in [2] that $d_m(G,H) = q - s$. The following is now a corollary of Theorem 1.

Corollary 3. *Let G and H be two graphs of order $p \geq 5$ and size q having a greatest common subgraph of size s. Then $d_j(G,H) \leq 2(q-s)$.*

The result presented in Corollary 3 is sharp in the sense that for every two positive integers q and s with $s \leq q$, there exist graphs G and H of size q containing a greatest common subgraph of size s such that $d_j(G,H) = 2(q-s)$. For a positive integer n, let P denote the path $v_1, v_2, ..., v_{4n-1}$. The graph G is constructed by adding a vertex v_0 to P together with the edges $v_0v_i (1 \leq i \leq 2n)$ and then adding an isolated vertex v_{4n}. The graph H is constructed by adding a vertex v_0 to P together with the edges $v_0v_{2i-1} (1 \leq i \leq 2n)$ and then adding an isolated vertex v_{4n}. Each of the graphs G and H has size $q = 6n - 2$. Note that a triangle-free subgraph of G has size at most $5n - 2$. Therefore, the greatest common subgraph of G and H is the graph F composed of P, the vertex v_0, and the edges $v_0v_{2i-1} (1 \leq i \leq n)$.

Thus F has size $s = 5n - 2$; so $q - s = n$. This construction is illustrated in Figure 3 for the case $n = 3$.



$d_j(G, H) = 6$

Figure 3

2. Comparisons of the Rotation and Jump Distances

In this section we study the two restricted edge transfer metrics defined on the space of graphs of a fixed order and fixed size. If two graphs G and G' have the same order and same size but have distinct degree sequences, then certainly $d_r(G, G') > 0$ and $d_j(G, G') > 0$. We next describe a lower bound for these two distances. For the purpose of doing this, it is useful to introduce a parameter defined on every two such graphs.

Let G and G' be graphs of the same order p and the same size with degree sequences $d_1, d_2, ..., d_p$ and $d'_1, d'_2, ..., d'_p$, respectively, where we assume that $d_1 \geq d_2 \geq ... \geq d_p$ and $d'_1 \geq d'_2 \geq ... \geq d'_p$. Thus $\Sigma d_i = \Sigma d'_i$. We define

$$\delta(G, G') = \min \sum_{i=1}^{p} |d_i - d'_{f(i)}|$$

over all permutations f on $\{1, 2, ..., p\}$. Indeed, it is straightforward to show that this minimum is obtained when f is the identity permutation, i.e.,

$$\delta(G, G') = \sum_{i=1}^{p} |d_i - d'_i|.$$

Although δ is not a metric since many nonisomorphic graphs with the same degree sequence exist, it is a pseudometric, that is, δ is symmetric and satisfies the triangle inequality. There is one other basic characteristic of δ that is worthy of mention.

Lemma 4. *For every two graphs G and G' (of the same order and same size), $\delta(G, G')$ is even.*

Proof. Let G and G' have degree sequences $s : d_1 \geq d_2 \geq ... \geq d_p$ and $s' : d'_1 \geq d'_2 \geq ... \geq d'_p$. Since there is an even number of odd terms in each sequence, it follows that in the sum $\sum_{i=1}^{p} |d_i - d'_i|$, the number of odd terms in s that are paired with even terms of s' and the number of odd terms of s' that are paired with even terms in s are of the same parity. Thus $\delta(G, G')$ is even. ∎

We are now prepared to present a lower bound for both $d_r(G, G')$ and $d_j(G, G')$ in terms of $\delta(G, G')$.

Theorem 5. *For every two graphs G and G' of the same order (at least 5) and same size,*

$$d_r(G, G') \geq \delta(G, G')/2 \quad \textit{and} \quad d_j(G, G') \geq \delta(G, G')/4.$$

Proof. Suppose that $\delta(G, G') = 2n$. If H is any graph obtained from G by an edge rotation, then the degree of some vertex of H is increased by 1 while the degree of another vertex of H is decreased by 1, so $\delta(G, H) = 2$ or $\delta(G, H) = 0$. In any case, at least n edge rotations are required to r-transform G into a graph with the same degree sequence as G'. Hence

$$d_r(G, G') \geq n = \delta(G, G')/2.$$

In the case of an edge jump, the degrees of four vertices are changed. Thus at least $n/2$ edge jumps are required to j-transform G into a graph with the same degree sequence as G'. So

$$d_j(G, G') \geq n/2 = \delta(G, G')/4.$$

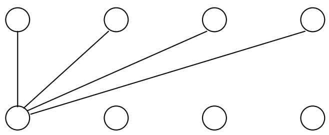
∎

The next result shows that the bounds presented in Theorem 5 are sharp.

Theorem 6. *For every positive integer n, there exist*
(a) *graphs F and F' such that $d(F, F') = 2n$ and $d_r(F, F') = n$,*
(b) *graphs G and G' such that $d(G, G') = 4n$ and $d_j(G, G') = n$, and*
(c) *graphs H and H' such that $d(H, H') = 4n + 2$ and $d_j(H, H') = n + 1$.*

Proof. Define $F = K_{1,n+1} \cup nK_1$ and $F' = (n + 1)K_2$. (Figure 4 shows F and F' when $n = 3$.) Here $\delta(F, F') = 2n$. Thus $d_r(F, F') \geq n$ by Theorem 5. Certainly F can be r-transformed into F' by a sequence of n edge rotations. Therefore, $d_r(F, F') \leq n$; so $d_r(F, F') = n$ and (a) is proved.

F :



F' :

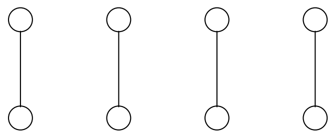
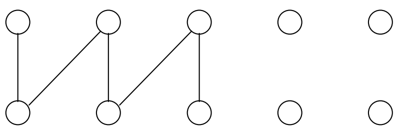


Figure 4

Next, define $G = P_{2n+2} \cup 2nK_1$ and $G' = (2n + 1)K_2$. (Figure 5 shows G and G' when $n = 2$.) Now $\delta(G, G') = 4n$. Therefore, $d_j(G, G') \geq n$ by Theorem 5. Since G can be j-transformed into G' by a sequence of n edge jumps, $d_j(G, G') \leq n$. Consequently, $d_j(G, G') = n$ and (b) is verified.

G :



G' :

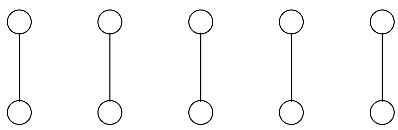


Figure 5

Finally, define $H = P_{2n+3} \cup (2n + 1)K_1$ and $H' = (2n + 2)K_2$. In this case, $\delta(H, H') = 4n+2$; so $d_j(H, H') \geq n+1$ by Theorem 5. The graph H can be j-transformed into H' by a sequence of $n+1$ edge jumps (although some care must be taken in the order of edge jumps chosen). Thus $d_j(H, H') \leq n + 1$; so $d_j(H, H') = n + 1$ and (c) is proved. ■

We now compare the rotation distance and jump distance and show that for two graphs on which both metrics are defined, each distance is at most twice the other.

Theorem 7. *For any two graphs G and H having the same order at least 5 and the same size,*

$$d_r(G,H) \le 2d_j(G,H) \quad \textit{and} \quad d_j(G,H) \le 2d_r(G,H).$$

Proof. According to Theorem 1, if a graph H is obtained from a graph G by a sequence of $d_r(G,H)$ edge rotations, then H is obtained from G by a sequence of at most $2d_r(G,H)$ edge jumps. Thus $d_j(G,H) \le 2d_r(G,H)$ and the second inequality is established.

We now verify the first inequality. Suppose that a graph G_2 is obtained from a graph G_1 by an edge jump. Then there exist distinct vertices u, v, w, and x such that $uv \in E(G_1)$, $wx \notin E(G_1)$, and $G_2 = G_1 - uv + wx$. We show that G_2 can be obtained from G_1 by two edge rotations.

Suppose first that $ux \notin E(G_2)$. Let $G_3 = G_1 - uv + ux$. Thus G_3 is obtained from G_1 by an edge rotation. Then $G_2 = G_3 - ux + wx$ is obtained from G_3 by an edge rotation. Hence G_2 is obtained from G_1 by an r-transformation that is a sequence of two edge rotations.

Suppose next that $ux \in E(G_1)$. Let $G_0 = G_1 - ux + wx$. Then G_2 is obtained from G_0 by an edge rotation, and G_0 is obtained from G_1 by an edge rotaion. Thus G_2 is obtained from G_1 by an r-transformation that is a sequence of two edge rotaions.

Therefore, if H is a graph that is obtained from a graph G by a sequence of $d_j(G,H)$ edge jumps, then H is also obtained form G by a sequence of at most $2d_j(G,H)$ edge rotations. Hence $d_r(G,H) \le 2d_j(G,H)$. ■

The inequalities in Theorem 7 can be described below.

Corollary 8. *For any two graphs G and H having the same order at least 5 and same size,*

$$\frac{1}{2}d_j(G,H) \le d_r(G,H) \le 2d_j(G,H),$$

or, equivalently,

$$\frac{1}{2}d_r(G,H) \le d_j(G,H) \le 2d_r(G,H).$$

The bounds provided for each metric in terms of the other are the only restrictions, as the next result shows.

Theorem 9. *For every two positive integers a and b with $a/2 \le b \le 2a$, there exist graphs G and H of the same order and same size such that $d_j(G,H) = a$ and $d_r(G,H) = b$.*

Proof. Assume first that $a \leq b \leq 2a$. Represent the double star with two vertices of degree 3 by $S_{3,3}$. Define $G = aK_2 \cup bP_3$ and $H = bK_1 \cup (2a-b)K_{1,3} \cup (b-a)S_{3,3}$. (See Figure 6 for G and H when $a = 2$ and $b = 3$.) Observe that G and H both have order $2a+3b$ and size $a+2b$. It is straightforward to see $d_j(G,H) = a$ and $d_r(G,H) = b$.

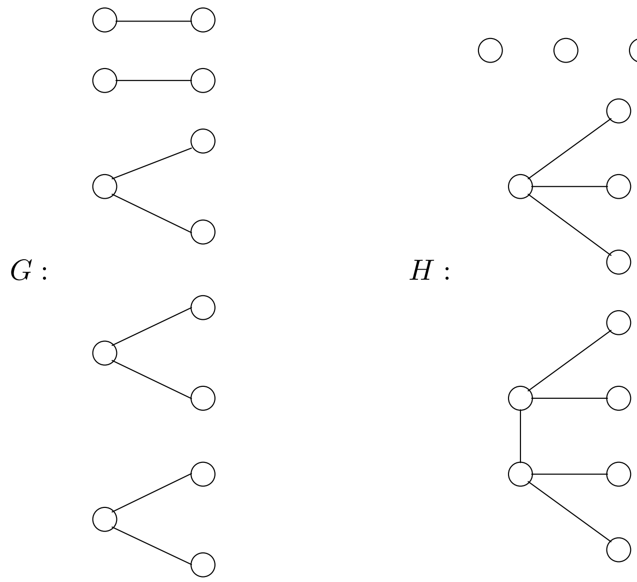


Figure 6

Next assume that $a/2 \leq b < a$. We now construct graphs G and H of order $2a+3$ and size $\binom{a+2}{2}$. The graph $G = K_{a+1} \cup K_{1,a+1}$. The graph H consists of a graph K_{a+1} and vertices $v, v_1, v_2, ..., v_{a+1}$ where v is joined to b vertices of K_{a+1} as well as to the vertices $v_1, v_2, ..., v_{a+1-b}$. (See Figure 7 for G and H when $a = 4$ and $b = 3$.)

It is clear that $d_r(G,H) = b$. To j-transform G into H, a total of $2b$ edge jumps are required for v to be the vertex of H having degree $a+1$ and not belonging to the given K_{a+1}. If $a = 2b$, then $d_j(G,H) = a$. Suppose then that $a < 2b$. Then we may j-transform G into (a graph isomorphic to) H using a total of a edge jumps by giving v_1, say, degree $a+1$. This cannot be accomplished with fewer than a edge jumps since the degree of v_1 must then be increased by a. In this case, we then have $d_j(G,H) = a$. ■

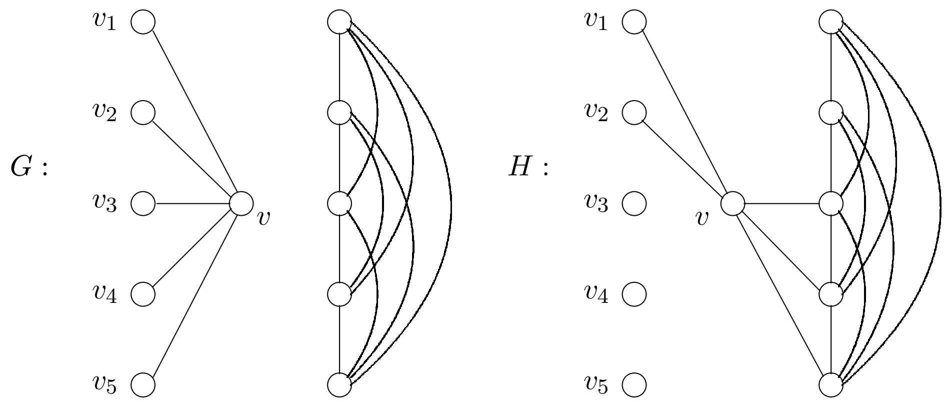


Figure 7

3. JUMP DISTANCE GRAPHS

Let $\mathcal{S}$ denote a set of graphs of the same order and same size. In [3] the *rotation distance graph* $D_r(\mathcal{S})$ of $\mathcal{S}$ is defined as that graph with vertex set $\mathcal{S}$ such that G_1 and G_2 are adjacent in $D_r(\mathcal{S})$ if and only if $d_r(G_1, G_2) = 1$. A graph G is a *rotation distance graph* if there exists a set $\mathcal{S}$ of graphs of the same order and same size such that $G = D_r(\mathcal{S})$. Many classes of graphs have been shown to be rotation distance graphs in [3], [5], and [6], including complete graphs, trees, cycles, complete bipartite graphs and all line graphs, but no graph has been found that is not a rotation distance graph. Indeed, it is conjectured in [3] that every graph is a rotation distance graph. We now turn our attention to the analogous concept for jump distance.

Let $\mathcal{S}$ denote a set of graphs of the same order at least 5 and same size. *The jump distance graph* $D_j(\mathcal{S})$ of $\mathcal{S}$ is that graph whose vertices are the graphs of $\mathcal{S}$ and where G_1 and G_2 in $\mathcal{S}$ are adjacent if and only if $d_j(G_1, G_2) = 1$. A graph G is a *jump distance graph* if there exists a set $\mathcal{S}$ of graphs of the same order at least 5 and same size such that $D_j(\mathcal{S}) = G$. We now show that several classes of graphs are jump distance graphs.

Theorem 10. *Every complete graph is a jump distance graph.*

Proof. Let H be a graph isomorphic to $P_{2p} \cup K_1$ such that $V(H) = \{v_0, v_1, ..., v_{2p}\}$ and $E(H) = \{v_i v_{i+1} | 1 \leq i \leq 2p-1\}$. Let $e = v_{2p-1}v_{2p}$ and $H_i = H - e + v_0 v_i$, $1 \leq i \leq p$. (See Figure 8 for the case $p = 4$.) Then, $d_j(H_i, H_k) = 1$, for each $i \neq k$, $1 \leq i, k \leq p$, and hence

$$D_j(\{H_1, H_2, ..., H_p\}) = K_p. \qquad \blacksquare$$

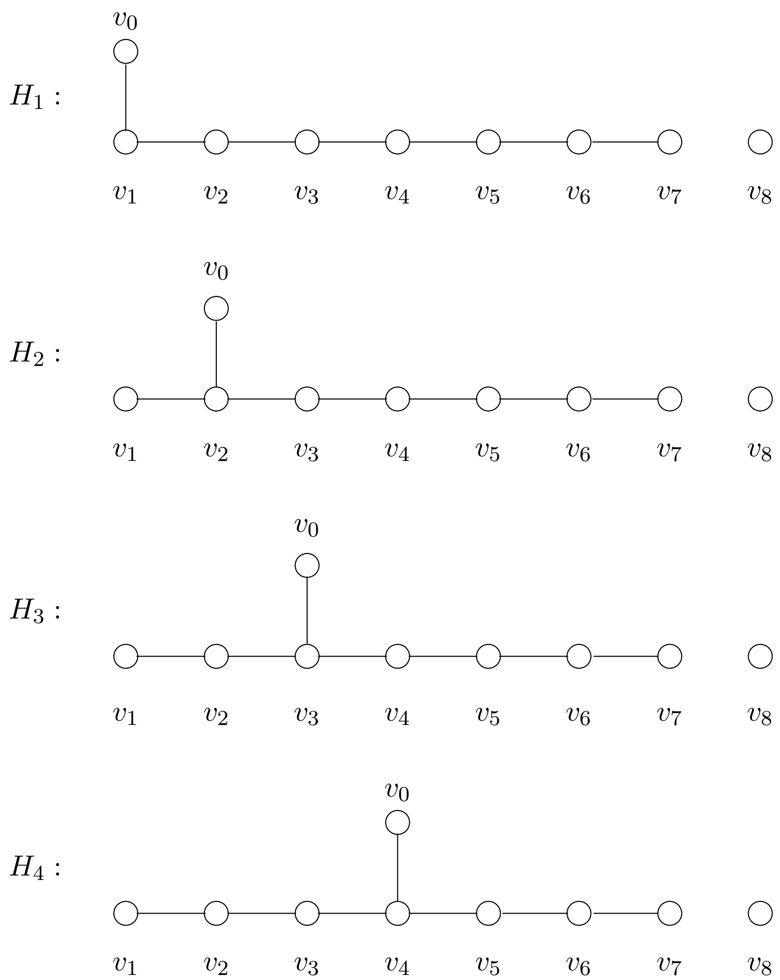


Figure 8

If G and H are graphs of the same order and same size such that $d_j(G, H) = 1$, then certainly $d_j(G + K_1, H + K_1) = 1$. This observation will prove to be useful to us.

We now recall a few definitions from graph theory. A graph G is *n-connected* if the removal of fewer than n vertices from G results in neither a disconnected graph nor a trivial graph. Similarly, a graph G is *n-edge-connected* if the removal of fewer than n edges from G results in neither a disconnected graph nor a trivial graph. For example, the complete graph K_p $(p \geq 2)$ is both k-connected and k-edge-connected for $1 \leq k \leq p - 1$.

Lemma 11. *Let G be a jump distance graph and n a positive integer. Then there is a set $\mathcal{S}$ of n-connected graphs such that $G = D_j(\mathcal{S})$.*

Proof. Let $\mathcal{T}$ be a set of graphs with $G = D_j(\mathcal{T})$. By our earlier observation, it follows that if $\mathcal{S} = \{H + K_n | H \in \mathcal{T}\}$, then $G = D_j(\mathcal{S})$. ■

Clearly, if a graph G is n-connected, $n \geq 1$, then G is n-edge-connected. This observation will be useful in the following theorem.

Theorem 12. *If G and H are jump distance graphs, then the cartesian product $G \times H$ is a jump distance graph.*

Proof. By Lemma 11, there exist disjoint sets $\mathcal{S}$ and $\mathcal{T}$ of 2-connected graphs such that $D_j(\mathcal{S}) = G$ and $D_j(\mathcal{T}) = H$. Let $\mathcal{S} = \{G_u | u \in V(G)\}$ with $d_j(G_u, G_w) = 1$ if and only if $uw \in E(G)$. Similarly, $\mathcal{T} = \{H_v | v \in V(H)\}$. We show that $G \times H = D_j(\{G_u \cup H_v | u \in V(G),\ v \in V(H)\})$ by showing that $d_j(G_u \cup H_v, G_w \cup H_x) = 1$ if and only if (1) $G_u = G_w$ and $d_j(H_v, H_x) = 1$ or (2) $d_j(G_u, G_w) = 1$ and $H_v = H_x$. Clearly if (1) $G_u = G_w$ and $d_j(H_v, H_x) = 1$ or (2) $d_j(G_u, G_w) = 1$ and $H_v = H_x$, then $d_j(G_u \cup H_v, G_w \cup H_x) = 1$.

We now consider the reverse implication. Suppose that $d_j(G_u \cup H_v, G_w \cup H_x) = 1$. Then $G_w \cup H_x = (G_u \cup H_v) - e + f$ for edges e and f, where $e \in E(G_u \cup H_v)$ and $f \in E(G_w \cup H_x)$. Suppose first that $e \in E(G_u)$. Since G_u and H_v are 2-edge-connected for $u \in V(G)$ and $v \in V(H)$, it follows that $f \in E(G_w)$ or $f \in E(H_x)$. If $f \in E(G_w)$, then $d_j(G_u, G_w) = 1$ and $H_v = H_x$. On the other hand, if $f \in E(H_x)$, then $d_j(G_u, H_x) = 1$ and $G_w = H_v$, which is impossible since $\mathcal{S}$ and $\mathcal{T}$ are disjoint. Similarly, if $e \in E(H_v)$, then $G_u = G_w$ and $d_j(H_v, H_x) = 1$. Thus $d_j(G_u \cup H_v,\ G_w \cup H_x) = 1$ if and only if (1) $u = w$ and $vx \in E(H)$ or (2) $v = x$ and $uw \in E(G)$. Therefore $G \times H = D_j(\{G_u \cup H_v | u \in V(G),\ v \in V(H)\})$. ■

Next, since the graph obtained by identifying a vertex u of a graph G with a vertex v of a graph H is an induced subgraph of $G \times H$, we have the following.

Corollary 13. *Let G and H be jump distance graphs. Then the graph obtained by identifying a vertex u of G with a vertex v of H is a jump distance graph.*

Finally, if the blocks of a connected graph G are jump distance graphs, then, from repeated applications of Corollary 13, G is also a jump distance graph. Consequently, we have the following corollary.

Corollary 14. *Every tree is a jump distance graph.*

Using a construction similar to the one given in [3] which shows that every cycle is a rotation distance graph, we now show that every cycle is a jump distance graph.

Theorem 15. *Every cycle is a jump distance graph.*

Proof. Let $n \geq 3$ be a positive integer and let $C: v_1, v_2, ..., v_{2n+4}, v_1$ be a $(2n+4)$-cycle. For $i = 1, 2, ..., n$, let $F_i = C + v_i v_{2i+1}$. Since F_i $(1 \leq i \leq n)$ contains a cycle of length $i + 2$, it follows that the graphs $F_1, F_2, ..., F_n$ are pairwise nonisomorphic. For $n = 4$, the graphs F_1, F_2, F_3, F_4 are shown in Figure 9.

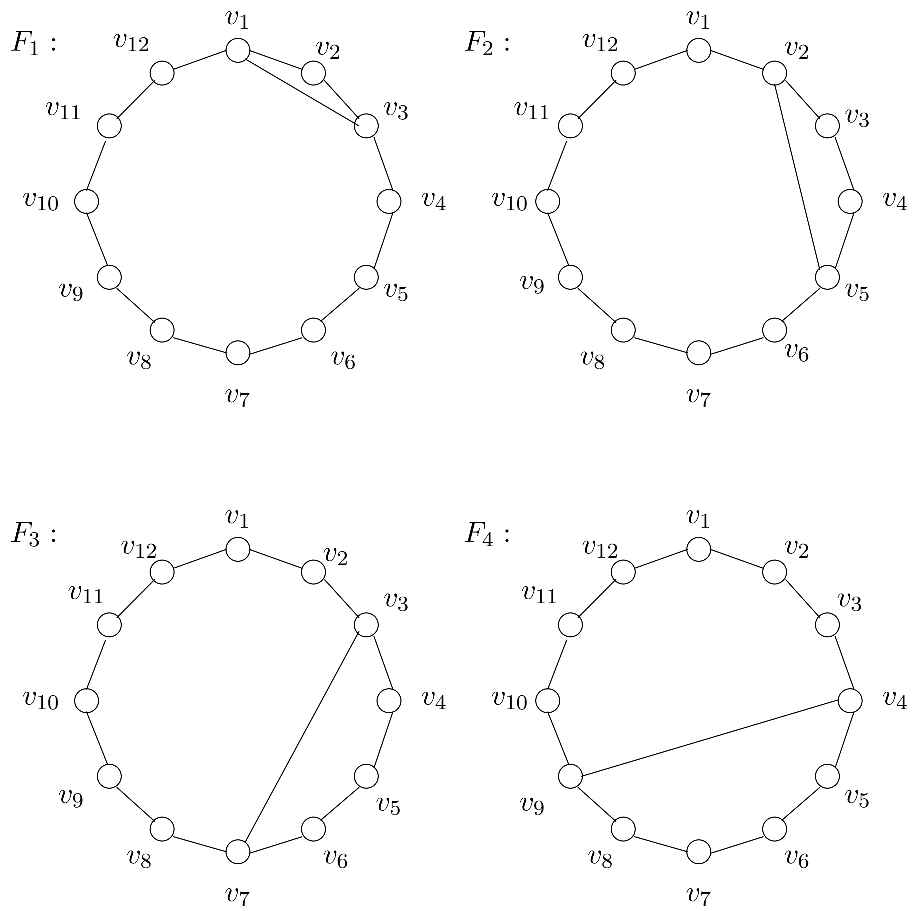


Figure 9

Next, for $i = 1, 2, ..., n-1$, let $G_i = F_i \cup F_{i+1}$, and let $G_n = F_n \cup F_1$. Clearly, the graphs G_i and $G_k (1 \le i \le k \le n)$ differ in exactly one edge when $k = i+1$ or when $i = 1$ and $k = n$, and differ in two edges otherwise. Thus since $d_j(G_i, G_{i+1}) = 1$ $(1 \le i \le n-1)$ and $d_j(G_n, G_1) = 1$, it follows that $D_j(\{G_1, G_2, ..., G_n\}) = C_n$. ■

Recall that the line graph $L(G)$ of a graph G is that graph whose vertices are the edges of G and two vertices e and f of $L(G)$ are adjacent if and only if the edges e and f are adjacent in G. Thus the graph $\overline{L(G)}$ can be described as that graph whose vertices are the edges of G and two vertices e and f of $\overline{L(G)}$ are adjacent if and only if the edges e and f are not adjacent in G. We now show that the complement of every line graph is a jump distance graph.

Theorem 16. *For every graph G, the graph $\overline{L(G)}$ is a jump distance graph.*

Proof. Let G be a graph of size q where $e_1, e_2, ..., e_q$ denote the edges of G. Next for $i = 1, 2, ..., q$, let $G_i = G - e_i$. Let i and k be positive integers such that $i \ne k$ and $1 \le i,\ k \le q$. Now $E(G_i) \cap E(G_k) = E(G) - e_i - e_k$ and thus $d_j(G_i, G_k) = 1$ if and only if the edges e_i and e_k are not adjacent. Thus $D_j(\{G_1, G_2, ..., G_q\}) = \overline{L(G)}$. ■

We have seen many graphs that are jump distance graphs, such as complete graphs, trees, cycles, cartesian products of jump distance graphs, and complements of line graphs. In fact, we know of no graph that is not a jump distance graph and thus we conclude with the following.

Conjecture. Every graph is a jump distance graph.

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