CORRIGENDUM TO: BOUNDS ON THE NUMBER OF EDGES OF EDGE-MINIMAL, EDGE-MAXIMAL AND $l$-HYPERTREES [DISCUSSIONES MATHEMATICAE GRAPH THEORY 36 (2016) 259–278]

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Abstract

In this corrigendum, we correct the proof of Theorem 10 from our paper titled „Bounds on the number of edges of edge-minimal, edge-maximal and $l$-hypertrees”.

Keywords: hypertree, chain in hypergraph, edge-minimal hypertree, edge-maximal hypertree, 2-hypertree, Steiner system.

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1. The Corrected Proof of Theorem 10

In the original proof of Theorem 10, we stated that

\[
\frac{1}{k-1} \binom{n}{k-1} - \frac{1}{(k-1)(n-k+1)} |E| \\
\leq \frac{1}{k-1} \binom{n}{k-1} - \frac{1}{(k-1)^2(n-k+1)} \binom{n}{k-1}.
\]

This can only be true if $|E| \geq \frac{1}{k-1} \binom{n}{k-1}$, but in reality, the exact opposite is true, i.e., $|E| \leq \frac{1}{k-1} \binom{n}{k-1}$ (see Theorem 9).

Below, we present the corrected proof of Theorem 10.

Theorem 10. If $\mathcal{H} = (V, E)$ is a $k$-uniform 2-hypertree, then $|E| \leq \frac{1}{k-1} \binom{n}{k-1} - \frac{1}{(k-1)^2} \binom{n}{k-2}$. 

Proof. We use the simple fact that $\sum_{i=1}^{n-k+1} C_i \geq \frac{1}{n-k+1} |E|$, which follows from $|E| = \sum_{i=1}^{n-k+1} iC_i \leq (n-k+1) \sum_{i=1}^{n-k+1} C_i$.

Comparing it to the Star-equation (Theorem 9), we get

$$|E| \leq \frac{1}{k-1} \left( \binom{n}{k-1} - \frac{1}{(k-1)(n-k+1)} |E| - \frac{1}{k-1} \right),$$

which implies that

$$|E| \leq \left( (k-1) + \frac{1}{(n-k+1)} \right)^{-1} \left( \binom{n}{k-1} \right).$$

$$= \left( \frac{1}{k-1} - \frac{1}{(k-1)^2(n-k+1) + (k-1)} \right) \left( \binom{n}{k-1} \right),$$

$$\leq \left( \frac{1}{k-1} - \frac{1}{(k-1)^2(n-k+2)} \right) \left( \binom{n}{k-1} \right),$$

$$= \frac{1}{k-1} \left( \binom{n}{k-1} - \frac{1}{(k-1)^3} \binom{n}{k-2} \right).$$

\[\square\]