

EXTENDING THE MAX ALGORITHM FOR MAXIMUM INDEPENDENT SET

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Abstract

The maximum independent set problem is an NP-hard problem. In this paper, we consider Algorithm MAX, which is a polynomial time algorithm for finding a maximal independent set in a graph G . We present a set of forbidden induced subgraphs such that Algorithm MAX always results in finding a maximum independent set of G . We also describe two modifications of Algorithm MAX and sets of forbidden induced subgraphs for the new algorithms.

Keywords: maximum independent set, stable set, stability number, independence number, reduction, graph transformation, MAX Algorithm, MIN Algorithm, Vertex Order Algorithm.

2010 Mathematics Subject Classification: 05C69, 05C85.

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Received 31 January 2014

Revised 7 January 2015

Accepted 7 January 2015